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DELHI**



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In each chapter, for better comprehension, questions have been categorized according to the typology issued by CBSE as follows :

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 U - Understanding,
 A - Analysing,
 Ap - Applying
 C - Creating
 E - Evaluating

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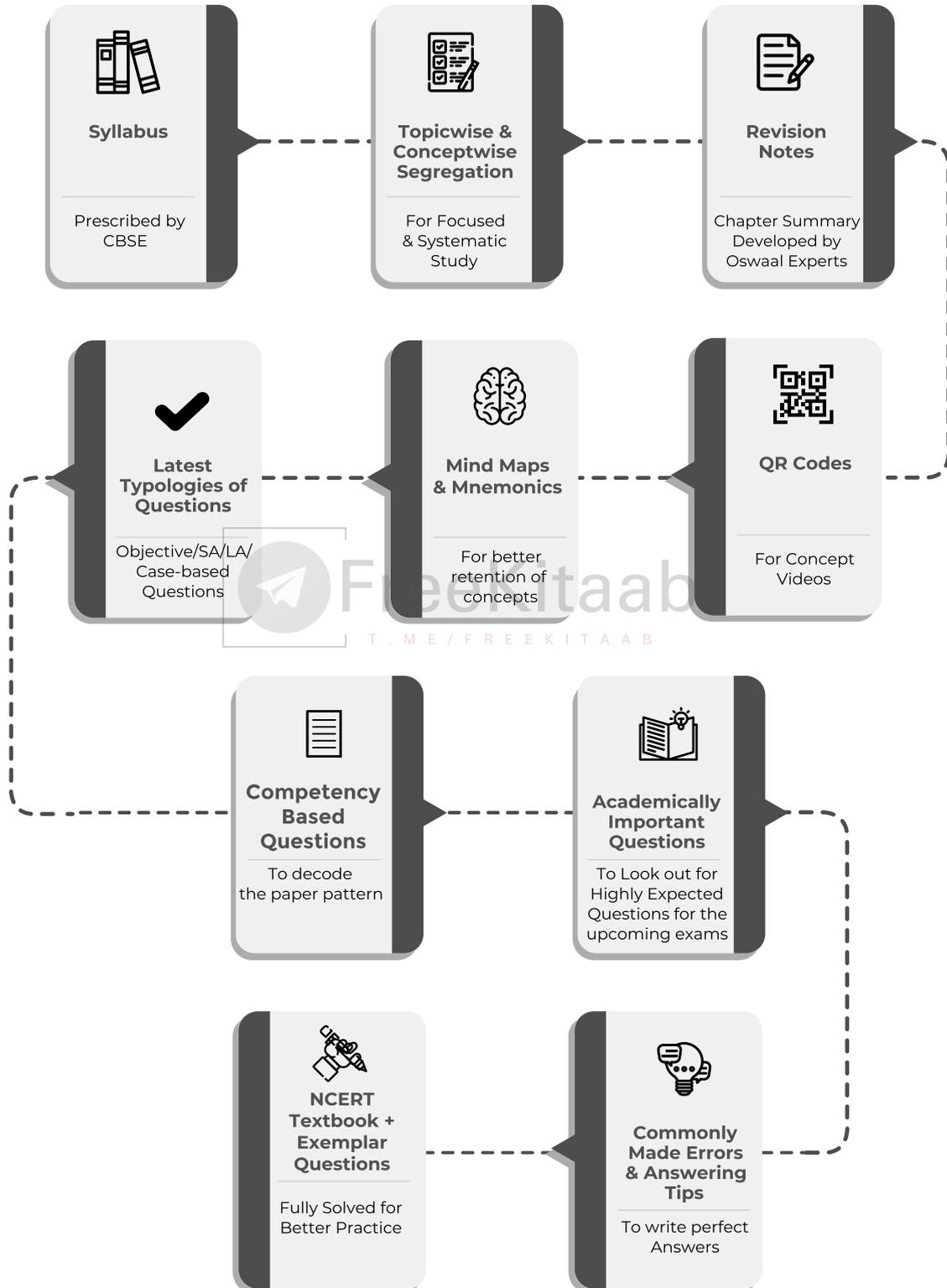






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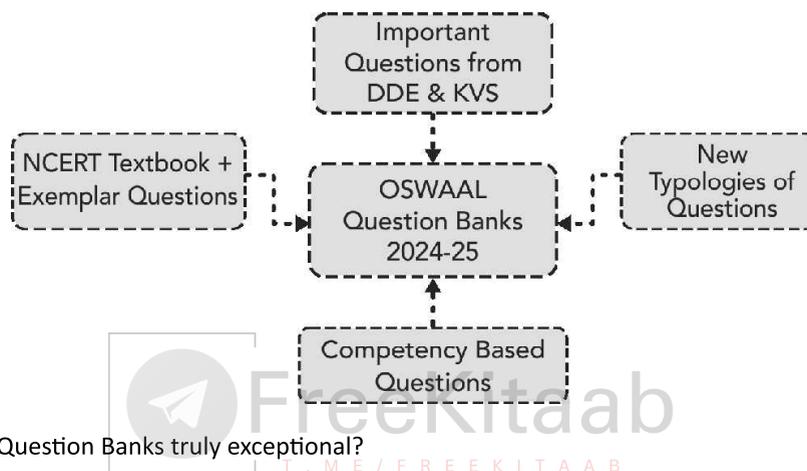


Preface

Elevate Your Performance, Surpassing the Past

Get ready for another epic journey through the academic wonders of the academic year 2024-2025 with your trustworthy companion—Oswaal Books! Remember last year’s triumphs? Well, buckle up because we are about to make this year even more awesome!

As the legendary dancer Martha Graham once said, “Practice means to perform, repeatedly in the face of all obstacles, some act of vision, of faith, of desire.” We have taken this wisdom to heart and packed it into our brand-new Question Banks for 2024-2025. They are a magical mix of CBSE Board Updates and specially crafted Questions tailored to the Latest Typologies. Oh, and did we mention the fantastic Learning Resources that come with them?



What makes these Question Banks truly exceptional?

- **100% Updated Syllabus & With Latest Questions Typologies:** We have got you covered with the latest and 100% updated curriculum
- **Timed Revision with Topic-wise Revision Notes, Smart Mind Maps & Mnemonics:** Study smart, not hard!
- **Extensive Practice with 1500+ Questions & Fully Solved NCERT+Exemplar Questions:** To give you 1500+ chances to become a champ!
- **Concept Clarity with 1000+ Concepts & 50+ Concept Videos:** For you to learn the cool way—with videos and mind-blowing concepts
- **NEP 2020 Compliance with Competency-Based Questions:** For you to be on the cutting edge of the coolest educational trends.

If you are looking to conquer every study challenge, these Question Banks are your secret weapon. It is like having a superhero ally for your exams! So, let’s kick off this exciting journey, fill those learning gaps, and rock the year with ease and confidence.

Big shoutout to our superhero team—the Oswaal Editorial Board! They’re the brains behind this incredible resource, working day and night just for you. And a massive thank you to you, our fellow Students, Parents & Teachers for your awesome inputs that make this book one-of-a-kind.

Wishing you all the best, superheroes-in-the-making! Strive for greatness! Team Oswaal Books

Syllabus

Latest Syllabus Issued by CBSE PHYSICS (Code No. 042) CLASS–XI

Time : 3 Hours

Max. Marks 70

		No. of Periods	Marks
Unit-I	Physical World and Measurement	08	23
	Chapter-2: Units and Measurements		
Unit II	Kinematics	24	
	Chapter-3: Motion in a Straight Line		
	Chapter-4: Motion in a Plane		
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	Chapter-12: Thermodynamics		
Unit IX	Behaviour of Perfect Gases and Kinetic Theory of Gases	08	
	Chapter-13: Kinetic Theory		
Unit X	Oscillations and Waves	26	
	Chapter-14: Oscillations		
	Chapter-15: Waves		
	Total	160	70

Unit I : **Physical World and Measurement**
08 Periods
Chapter-2 : Units and Measurements
Need for measurement: Units of measurement; systems of units; SI units, fundamental and derived units. significant figures. Dimensions

of physical quantities, dimensional analysis and its applications.
Unit II : **Kinematics** 24 Periods
Chapter-3 : Motion in a Straight Line
Frame of reference, Motion in a straight line, Elementary concepts of differentiation and integration for

Syllabus

describing motion, uniform and non-uniform motion, and instantaneous velocity, uniformly accelerated motion, velocity - time and position-time graphs. Relations for uniformly accelerated motion (graphical treatment).

Chapter-4 : Motion in a Plane

Scalar and vector quantities; position and displacement vectors, general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors, Unit vector; resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors.

Motion in a plane, cases of uniform velocity and uniform acceleration-projectile motion, uniform circular motion.

Unit III : Laws of Motion 14 Periods

Chapter-5 : Laws of Motion

Intuitive concept of force, Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion.

Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication.

Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on a banked road).

Unit IV : Work, Energy and Power 14 Periods

Chapter-6 : Work, Energy and Power

Work done by a constant force and a variable force; kinetic energy, work-energy theorem, power.

Notion of potential energy, potential energy of a spring, conservative forces: non-conservative forces, motion in a vertical circle; elastic and inelastic collisions in one and two dimensions.

Unit V : Motion of System of Particles and Rigid Body 18 Periods

Chapter-7: System of Particles and Rotational Motion

Centre of mass of a two-particle system, momentum conservation and Centre of mass motion. Centre of mass of a rigid body; centre of mass of a uniform rod.

Moment of a force, torque, angular momentum, law of conservation of angular momentum and its applications. Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions.

Moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects (no derivation).

Unit VI : Gravitation 12 Periods

Chapter-8 : Gravitation

Kepler's laws of planetary motion, universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential, escape velocity, orbital velocity of a satellite.

Unit VII : Properties of Bulk Matter 24 Periods

Chapter-9 : Mechanical Properties of Solids

Elasticity, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity (qualitative idea only), Poisson's ratio; elastic energy.

Chapter-10 : Mechanical Properties of Fluids

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes), effect of gravity on fluid pressure.

Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its simple applications.

Syllabus

Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

Chapter-11 : Thermal Properties of Matter

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity.

Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law .

Unit VIII : Thermodynamics 12 Periods

Chapter-12 : Thermodynamics

Thermal equilibrium and definition of temperature zeroth law of thermodynamics, heat, work and internal energy. First law of thermodynamics, Second law of thermodynamics: gaseous state of matter, change of condition of gaseous state -isothermal, adiabatic, reversible, irreversible, and cyclic processes.

Unit IX : Behaviour of Perfect Gases and Kinetic Theory of Gases 08 Periods

Chapter-13 : Kinetic Theory

Equation of state of a perfect gas, work done in compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of

freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

Unit X : Oscillations and Waves 26 Periods

Chapter-14 : Oscillations

Periodic motion - time period, frequency, displacement as a function of time, periodic functions and their application.

Simple harmonic motion (S.H.M) and its equations of motion; phase; oscillations of a loaded spring- restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Chapter-15 : Waves

Wave motion: Transverse and longitudinal waves, speed of travelling wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats.

PRACTICALS Total Periods : 60

The record, to be submitted by the students, at the time of their annual examination, has to include:

- Record of at least 8 Experiments [with 4 from each section], to be performed by the students.
- Record of at least 6 Activities [with 3 each from section A and section B], to be performed by the students.
- Report of the project carried out by the students.

EVALUATION SCHEME

Time 3 hours

Max. Marks 30

Topic	Marks
Two experiments one from each section	7 + 7
Practical record (experiment and activities)	5 Marks
One activity from any section	3 Marks
Investigatory Project	3 Marks
Viva on experiments, activities and project	5 Marks
Total	30 Marks

Syllabus

SECTION–A

Experiments

1. To measure diameter of a small spherical/cylindrical body and to measure internal diameter and depth of a given beaker/calorimeter using Vernier Callipers and hence find its volume.
2. To measure diameter of a given wire and thickness of a given sheet using screw gauge.
3. To determine volume of an irregular lamina using screw gauge.
4. To determine radius of curvature of a given spherical surface by a spherometer.
5. To determine the mass of two different objects using a beam balance.
6. To find the weight of a given body using parallelogram law of vectors.
7. Using a simple pendulum, plot its L-T² graph and use it to find the effective length of second's pendulum.
8. To study variation of time period of a simple pendulum of a given length by taking bobs of same size but different masses and interpret the result.
9. To study the relationship between force of limiting friction and normal reaction and to find the co-efficient of friction between a block and a horizontal surface.
10. To find the downward force, along an inclined plane, acting on a roller due to gravitational pull of the earth and study its relationship with the angle of inclination θ by plotting graph between force and $\text{Sin}\theta$.

Activities

1. To make a paper scale of given least count, e.g., 0.2cm, 0.5 cm.
2. To determine mass of a given body using a metre scale by principle of moments.
3. To plot a graph for a given set of data, with proper choice of scales and error bars.
4. To measure the force of limiting friction for rolling of a roller on a horizontal plane.

5. To study the variation in range of a projectile with angle of projection.
6. To study the conservation of energy of a ball rolling down on an inclined plane (using a double inclined plane).
7. To study dissipation of energy of a simple pendulum by plotting a graph between square of amplitude and time.

SECTION–B

Experiments

1. To determine Young's modulus of elasticity of the material of a given wire.
2. To find the force constant of a helical spring by plotting a graph between load and extension.
3. To study the variation in volume with pressure for a sample of air at constant temperature by plotting graphs between P and V, and between P and 1/V.
4. To determine the surface tension of water by capillary rise method.
5. To determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
6. To study the relationship between the temperature of a hot body and time by plotting a cooling curve.
7. To determine specific heat capacity of a given solid by method of mixtures.
8. To study the relation between frequency and length of a given wire under constant tension using sonometer.
9. To study the relation between the length of a given wire and tension for constant frequency using sonometer.
10. To find the speed of sound in air at room temperature using a resonance tube by two resonance positions.

Activities

1. To observe change of state and plot a cooling curve for molten wax.
2. To observe and explain the effect of heating on a bi-metallic strip.
3. To note the change in level of liquid in a container on heating and interpret the observations.

Syllabus

4. To study the effect of detergent on surface tension of water by observing capillary rise.
5. To study the factors affecting the rate of loss of heat of a liquid.
6. To study the effect of load on depression of a suitably clamped metre scale loaded at (i) its end (ii) in the middle.
7. To observe the decrease in pressure with increase in velocity of a fluid.

Practical Examination for Visually Impaired Students Class XI

Note: Same Evaluation scheme and general guidelines for visually impaired students as given for Class XII may be followed.

A. Items for Identification/Familiarity of the apparatus for assessment in practical's (All experiments)

Spherical ball, Cylindrical objects, vernier calipers, beaker, calorimeter, Screw gauge, wire, Beam balance, spring balance, weight box, gram and milligram weights, forceps, Parallelogram law of vectors apparatus, pulleys and pans used in the same 'weights' used, Bob and string used in a simple pendulum, meter scale, split cork, suspension arrangement, stop clock/stop watch, Helical spring, suspension arrangement used, weights, arrangement used for measuring extension, Sonometer, Wedges, pan and pulley used in it, 'weights' Tuning Fork, Meter scale, Beam balance, Weight box, gram and milligram weights, forceps, Resonance Tube, Tuning Fork, Meter scale, Flask/Beaker used for adding water.

B. List of Practicals

1. To measure diameter of a small spherical/cylindrical body using vernier calipers.
2. To measure the internal diameter and depth of a given beaker/calorimeter using vernier calipers and hence find its volume.
3. To measure diameter of given wire using screw gauge.
4. To measure thickness of a given sheet using screw gauge.
5. To determine the mass of a given object using a beam balance.

6. To find the weight of given body using the parallelogram law of vectors.
7. Using a simple pendulum plot L-T and L-T² graphs. Hence find the effective length of second's pendulum using appropriate length values.
8. To find the force constant of given helical spring by plotting a graph between load and extension.
9. (i) To study the relation between frequency and length of a given wire under constant tension using a sonometer.
(ii) To study the relation between the length of a given wire and tension, for constant frequency, using a sonometer.
10. To find the speed of sound in air, at room temperature, using a resonance tube, by observing the two resonance positions.

Note : The above practicals may be carried out in an experiential manner rather than recording observations.

Prescribed Books :

1. Physics Part-I, Textbook for Class XI, Published by NCERT
2. Physics Part-II, Textbook for Class XI, Published by NCERT
3. Laboratory Manual of Physics, Class XI Published by NCERT
4. The list of other related books and manuals brought out by NCERT (consider multimedia also).

Note: The content indicated in NCERT textbooks as excluded for the year 2022-23 is not to be tested by schools.



Syllabus

Practical Examination for Visually Impaired Students of Class XI Evaluation Scheme

Time 2 hours

Max. Marks: 30

Identification/Familiarity with the apparatus	5 marks
Written test (based on given/prescribed practicals)	10 marks
Practical Record	5 marks
Viva	10 marks
Total	30 marks

General Guidelines

- The practical examination will be of two-hour duration.
- A separate list of ten experiments is included here.
- The written examination in practicals for these students will be conducted at the time of practical examination of all other students.
- The written test will be of 30 minutes duration.
- The question paper given to the students should be legibly typed. It should contain a total of 15 practical skill based very short answer type questions. A student would be required to answer any 10 questions.
- A writer may be allowed to such students as per CBSE examination rules.
- All questions included in the question papers should be related to the listed practicals. Every question should require about two minutes to be answered.
- These students are also required to maintain a practical file. A student is expected to record at least five of the listed experiments as per the specific instructions for each subject. These practicals should be duly checked and signed by the internal examiner.
- The format of writing any experiment in the practical file should include aim, apparatus required, simple theory, procedure, related practical skills, precautions etc.
- Questions may be generated jointly by the external/internal examiners and used for assessment.
- The viva questions may include questions based on basic theory/principle/concept, apparatus/ materials/chemicals required, procedure, precautions, sources of error etc.

□□

Syllabus

QUESTION PAPER DESIGN

Physics (Code No. 042)

Class XI

Maximum Marks : 70

Duration : 3 Hrs.

S. No.	Typology of Questions	Total Marks	Approximate Percentage
1.	Remembering : Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. Understanding: Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	27	38%
2.	Applying: Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	22	32%
3.	Analysing : Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations Evaluating : Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. Creating: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	21	30%
	Total Marks	70	100
	Practical	30	
	Gross Total	100	

Note:

The above template is only a sample. Suitable internal variations may be made for generating similar templates keeping the overall weightage to different form of questions and typology of questions same.

□□

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VAPI	Goutam Book Sellers, 90817990813
VALSAD	Mahavir Stationers, 9429474177
NAVSARI	College Store, (ISC) NO CALL 02637-258642, 9825099121
SURAT	Shopping Point, 9824108663
VADODARA	Umakant Book Sellers & Stationer, 9624920709
ROHTAK	HARYANA Manish Traders, 9812556687, Swami Kitab Ghar, 9355611088, Babu Ram Pradeep Kumar, 9813214692, Sanjay book depot, 9255447231
REWARI	Kashi Ram Kishan Lal, 9289504004, 8920567245
BALLABGARH	Natraj Book Distributors, 9788917452
HISAR	Khurana Book Store, 9896572520
BHUNA	
JAMMU	Sahitya Sangam, 9419190177
BOKARO	JHARKHAND Bokaro Student Friends, (0654) 2233094, 7360021503, Bharati Bhawan Agencies, 9431740797
RANCHI	Crown Book Distributor & Publishers, (0651) 2213735, 9431173904, Pustak Mandir, 9431115138, Vidyarthi Pustak Bhandar, 9431310228
DUMKA	
HUBLI	KARNATAKA Renuka Book Distributor, (0836) 2244124, Vidyamandir Book Distributors, 9980773976
BANGLORE	Krishna book house, 9739847334, Hema Book Stores, 9986767000, Sapna Book House Pvt. Ltd., 9980513242, Hema Book World, (Chamrajpet) (ISC) 080-40905110, 9945731121
BELLERI	Chatinya book centre, 9886064731
ERNAKULAM	KERALA Academic Book House, (0484) 2376613, H & C Store, 9864196344, Surya Book House, 9847124217, 9847238314
KOTTAYAM	Book Centre, (0481) 2566992
TRIVANDRUM	Academic Book House, (0471) 2333349, 9447063349, Ponni Book Stall, 9037591721
CALICUT	Aman Book Stall, (0495) 2721282,
CHHINDWARA	MADHYA PRADESH Pustak Bhawan, (E & C), 8982150100
GWALIOR	Agarwal Book Depot, 9425116210

INDORE	Bhaiya Industries, 9893326853, Sushil Prakashan,(0731) 2503333, 2535892, 9425322330, Bhaiya Store, 9425318103, Arun Prakashan, 9424890785, Bhaiya Book Centre, 9424081874, Seva Suppliers, 9826451052
JABALPUR	Vinay Pustak Sadan, 8962362667, Anand Books and Stationers, 9425323508
SAGAR	Princi Book Depot, Sagar, 9977277011
KATNI	Shri Mahavir Agency, 9425363412
UJJAIN	Shreenath Book Depot, 9827544045
BHOPAL	Gupta Brother, 9644482444
PUNE	MAHARASHTRA Natraj Book Depot, (020) 24485054, 9890054092, Vikas Book House, 9921331187, Pravin Sales, 9890683475, New Saraswati Granth Bhandar, 9422323859, Akshar Books & Stationery, 7385089789, Vardhaman Educational, 9860574354, Yash Book Centre, 9890156763, Pragati Book Centre, (ISC), 9850039311, Praveen Sales, Pragati Book Centre, Pune (E & C), 9850039311
AURANGABAD	Shree Sainath Agencies, 7350294089, Maya Book Centre, (ISC), 9372360150
MUMBAI	Vidyarthi Sales Agencies, 9819776110, New Student Agencies, 7045065799, Shivam Books & Stationery, 8619805332
JALGAON	Sharma Book Depot & Stat. (ISC), 9421393040
LATUR	Yash Book House, 9637936999, Shri Ganesh Pustakalay, 9730172188
KOLHAPUR	Granth the Book World, 9922295522
NAGPUR	Laxmi Pustakalay and Stationers, (0712) 2727354, Vijay Book Depot, 9860122094
NANDED	Renuka Book distributor, 9765406133, Novelty Book Depot, 9657690220, Karamveer Book Depot, 9923966466, Arun Book & Stationers, 9423110953
NASHIK	Abhang Pustakalaya, 982347056/9175940756
DHULE	Rahul Book Centre, 9970849681, New India Book House, 9623123458
YAVATMAL	Navjeevan Book Stall, 7020525561
VASAI	Shri Ganesh Pustakalaya, 9423131275, Prime Book Centre, Vasai, 9890293662
ODISHA	A. K. Mishra Agencies, 9437025991, 9437081319
CUTTACK	M/s Pragnya, 8847888616, 9437943777, Padmalaya, 9437026922, Bidyashree, 9937017070, Books Godown, 7894281110
BHUBANESHWAR	Trimurti Book World, 9437034735
BARIPADA	Students corner, 7008435418
KEONJHAR	
PUNJAB	Bharat Book Depot, 7988455354
AMBALA	Goel Sons, 9463619978, Adarsh Enterprises, 9814347613
PATIALA	Cheap Book Store, 9872223458, 9878258592, City Book Shop, 9417440753, Subhash Book Depot, 9876453625, Paramvir Enterprises, 9878626248
JALANDHAR	Sita Ram book Depot, 9463039199, 7696141911
FEROZPUR	Amit Book, 9815807871, Gupta Brothers, 9888200206, Bhatia Book Centre, 9815277131
LUDHIANA	Mohindra Book Depot, 9814920226
CHANDIGARH	
RAJASTHAN	Laxmi General Store, Ajmer, 0145- 2428942 9460652197
AJMER	Vardhman Book Depot, 9571365020, 8003221190 Raj Traders, 9309232829
KOTA	Nakoda Book Depot, (01482) 243653, 9214983594,
BHILWARA	Alankar Book Depot, 9414707462
JAIPUR	Ravi Enterprises, 9829060694, Saraswati Book House, (0141) 2610823, 9829811155, Goyal Book Distt., 9460983939, 9414782130
UDAIPUR	Sunil Book Store, 9828682260
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AGARTALA	TRIPURA Book Corner, 8794894165, 8984657146, Book Emporium, 9089230412
COIMBATORE	TAMIL NADU Majestic Book House, (0422) 2384333, CBSC Book Shop, 9585979752
CHENNAI	Arraba Book Traders, (044) 25387868, 9841459105, M.R. Book Store (044) 25364596, Kalaimagal Store, (044) 5544072, 9940619404, Vijaya Stores, 9381037417, Bookmark It-Books & Stat. Store, 7305151653, M.K. Store, 9840030099, Tiger Books Pvt. Ltd., 9710447000, New Mylat Stationers, 9841313062, Prince Book House, Chennai, 0444-2053926, 9952068491, S K Publishers & Distributors, 9789865544, Dharmha Book Shop, 8667227171
PUDUCHERRY	Sri Lakshmi Book Seller, 7871555145
SALEM	Pattu book centre, 9894816280
TRICHY	P.R.Sons Book Seller, 9443370597, Rasi Publication, 9894816280
THENI	Maya Book Centre, 9443929274
MADURAI	Gelvi Book Shoppe, 9843057435, Jayam Book Centre, 9894658036
VELLORE	G.K.book centre and collections, 9894517994
HYDERABAD	TELANGANA Sri Balaji Book Depot, (040) 27613300, 9866355473, Shah Book House, 9849564564
	Vishal Book Distributors, 9246333166, Himalaya Book World, 7032578527

Writing Your Notes

Just in case you have forgotten today, takedown your notes!

But why is it so important?

Tools for the hands are tools for the brain writes Hetty Roessingh.

Handwritten notes are a powerful tool for encrypting embodied cognition and in turn supporting the brain's capacity for recuperation of information. If that sounds so scientific then in simple words:

Writing notes by hand help you in:

- ◆ Increasing your comprehension
- ◆ Strengthening your memory
- ◆ Igniting your creativity
- ◆ Engaging your mind
- ◆ Increasing your attention span

Are these reasons enough to get you started?

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UNIT-I

PHYSICAL WORLD AND MEASUREMENT

Study Time
Maximum Time: 3 hours
Maximum Questions: 117

CHAPTER

1

UNITS AND MEASUREMENT



Syllabus

Need for measurement : Units of measurement; systems of units; S.I. units; fundamental and derived units; significant figures. Dimensions of physical quantities; dimensional analysis and its applications.

Topic-1 Units System and Measurement

TOPIC - 1

Units System and Measurement
..... P. 1

TOPIC - 2

Dimensional Analysis
..... P. 5



Revision Notes

FreeKitaab

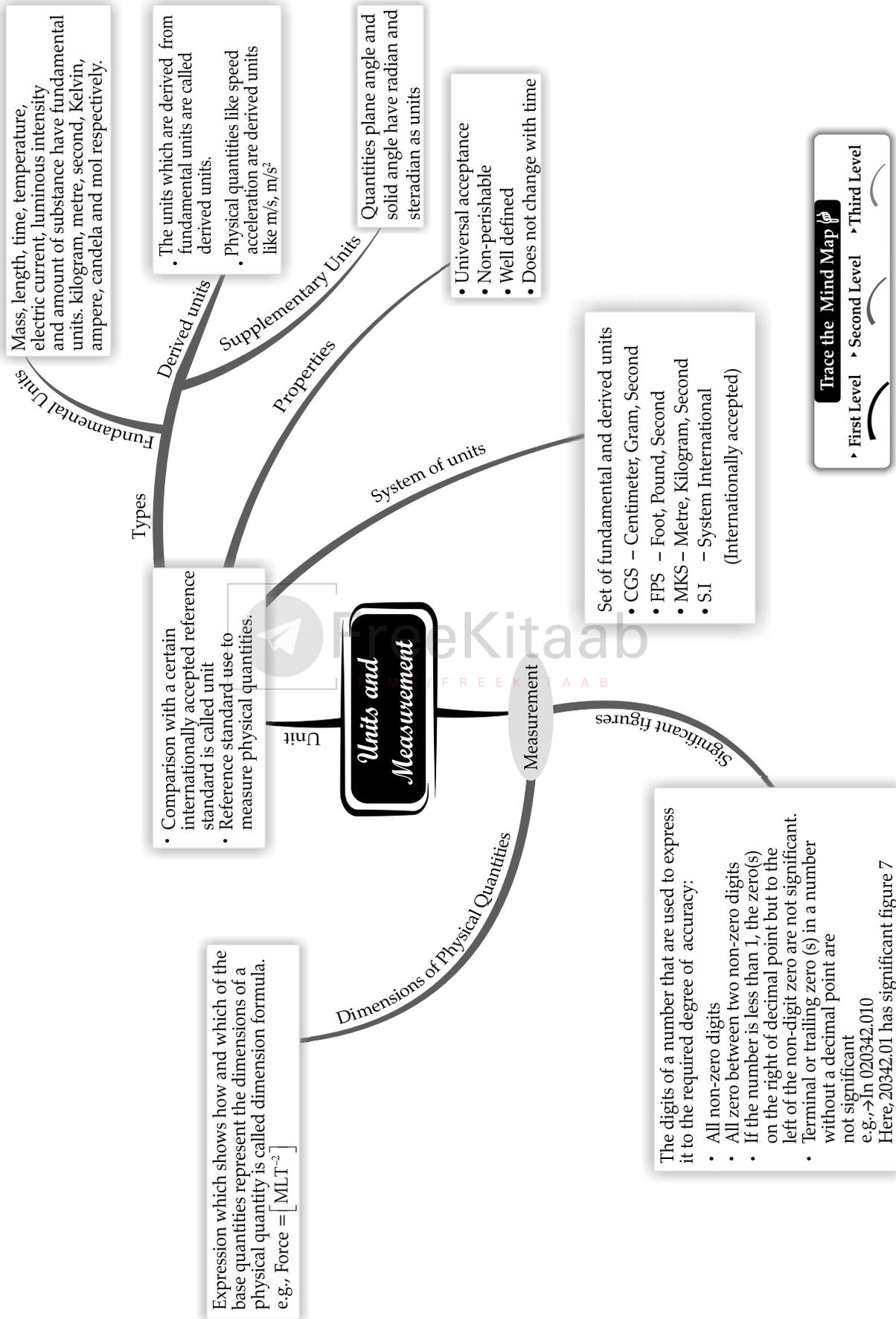
T.M.E / FREEKITAAB

- **Units** : It is the chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity.
 - **Fundamental Units** : The physical quantities which are independent of each other and which can represent remaining physical quantities are called fundamental physical quantities and their units are called fundamental units.
- Seven Fundamental physical quantities in SI system of units are :**
- (a) Mass - kg (Kilogram)
 - (b) Length - m (Meter)
 - (c) Time - s (Second)
 - (d) Temperature - K (Kelvin)
 - (e) Electric current - A (Ampere)
 - (f) Luminous Intensity - cd (Candela)
 - (g) Amount of substance - mol (Mole)
- **Derived Units** : These are the units of measurement of all other physical quantities which can be obtained from fundamental units, e.g., Velocity - (m/s), Acceleration - (m/s²), Pressure - (Pa), Force - (N) and so on.
 - **Unit system**
 - (a) **F. P. S. system** : Foot, Pound, Second.
 - (b) **C. G. S. system** : Centimetre, Gram, Second.
 - (c) **M. K. S. system** : Meter, Kilogram, Second.
 - **Significant figures** : The significant figures of a given number are those significant or important digits, which convey the meaning according to its accuracy. For example, 6.658 has four significant digits. These substantial figures provide precision to the numbers. They are also termed as significant digits.
 - **Rules for significant figure** :
 - (a) All non-zero digits are significant. 198745 contains six significant digits.
 - (b) All zeros those occur between any two non-zero digits are significant. For example, 108.0097 contains seven significant digits.
 - (c) All zeros those are on the right of a decimal point and also to the left of a non-zero digit is never significant. For example, 0.00798 contained three significant digits.
 - (d) All zeros those are on the right of a decimal point are significant, only if, a non-zero digit does not follow them. For example, 20.00 contained four significant digits.

Scan to know more about this topic



Units



Trace the Mind Map

▶ First Level ▶ Second Level ▶ Third Level

- (e) All the zeros those are on right of the last non-zero digit, after the decimal point, are significant. For example, 0.0079800 contains five significant digits.
- (f) All the zeros that are on the right of the last non-zero digit are significant if they come from a measurement. For example, 1090 m contains four significant digits.

Key Words

- **Fundamental Units** : Units of the physical quantities which are independent of each other and which can represent remaining physical quantities.
- **Derived Units** : Units of measurement of those physical quantities which can be obtained from fundamental units.

Key Formulae

- 1 AU = 1.496×10^{11} m.
- 1 ly = 9.46×10^{15} m.
- 1 parsec = 3.1×10^{16} m.
- $1 \text{ \AA} = 10^{-10}$ m; 1 nm = 10^{-9} m
 $1 \mu\text{m} = 10^{-6}$ m, 1 mm = 10^{-3} m
- 60 seconds (of arc) = 1 min (arc)
- 60 min. (of arc) = 1 degree (of arc)
- 180 degree (of arc) = π radian
- Indirect methods for long and small distances :

$$\text{Angular diameter } (\theta) = \frac{d}{D}$$

d = diameter, D = distance, radius = r

- **Magnification** :

- (a) Linear Magnification = $\frac{\text{Size of image}}{\text{Size of object}}$
- (b) Linear Magnification = $\sqrt{\text{Areal Magnification}}$



Mnemonics

Concept: The fundamental quantities of SI system:

Mnemonics: At the last **m**oment she **l**uckily caught the **t**rain.

Interpretation:

A - Amount of substance

t - Temperature (thermodynamic)

l - Length

m - Mass

lu - Luminous intensity

c - Current

t - Time



Very Short Answer Type Questions

(1 mark each)

- | | |
|---|---|
| <p>Q. 1. Define radian. [R] [NCT 2008]</p> <p>Ans. One radian is the angle subtended at the centre of the circle by an arc equal in length to the radius of the circle. 1</p> <p>Q. 2. Express one micron in metre. [A] [NCT 2010]</p> <p>Ans. 1 micron = 10^{-6} metre 1</p> <p>Q. 3. What is the number of significant figures in 0.06070 ? [U]</p> <p>Ans. 4. 1</p> <p>Q. 4. Which of the following reading is most accurate ?
 (a) 7,000 m, (b) 7×10^2 m, (c) 7×10^3 m. [A]</p> | <p>Ans. (a) 7,000 m. 1</p> <p>Q. 5. The mass of a body as measured by two students is given as 1.2 kg and 1.23 kg. Which of the two is more accurate and why ? [A]</p> <p>Ans. The second measurement is more accurate as it has more number of significant figure. 1</p> <p>Q. 6. Name the supplementary units of S.I. system. [R]</p> <p>Ans. (i) Radian (rad) for plane angle. $\frac{1}{2}$
 (ii) Steradian (sr) for solid angle. $\frac{1}{2}$</p> |
|---|---|



Short Answer Type Questions-I

(2 marks each)

Q. 1. Find the number of times the heart of a human being beats in 10 years. Assume that the heart beats once in 0.8 sec. [A] [A]

Ans. In 0.8 s, the human heart makes one beat.

∴ In 1 s the human heart makes

$$= \frac{1}{0.8} = \frac{10}{8} \text{ beats.} \quad 1$$

∴ In 10 years the human heart makes

$$= \frac{10}{8} \times 10 \times 365 \times 24 \times 60 \times 60 \text{ beats}$$

$$= 3.942 \times 10^8 \text{ beats.} \quad 1$$



Commonly Made Error

- Students commits mistake while finding beats in one second and seconds in one year.



Answering Tip

- Students should carefully find the number of beats in one second and then multiply the same with the number of seconds in an year.

Q. 2. Derive S.I. unit of Joule (J) in terms of fundamental units. [U]

Ans. Joule is a unit of work.

Using the relation,

$$\begin{aligned} \text{Work} &= \text{force} \times \text{displacement} \\ &= \text{mass} \times \text{acceleration} \times \text{displacement} \\ &= \text{mass} \times \frac{\text{velocity}}{\text{time}} \times \text{displacement} \\ &= \text{mass} \times \frac{\text{displacement}}{\text{time} \times \text{time}} \times \text{displacement} \end{aligned}$$

$$= \text{mass} \times \text{displacement}^2 \times \text{time}^{-2} \quad 1$$

Unit of work,

$$\begin{aligned} J &= \text{kg} \times \text{m}^2 \times \text{s}^{-2} \\ &= \text{kgm}^2\text{s}^{-2}. \quad 1 \end{aligned}$$

Q. 3. Give conversion of the some commonly used large units of length into metre. [U]

Ans. 1 Light year (ly) = 9.46×10^{15} m

1 Astronomical unit (AU) = 1.496×10^{11} m

1 Parallaxic second (parsec) = 3.08×10^{16} m 2

Q. 4. Why an optical microscope is not used to measure the size of a molecule ? [U]

Ans. Size of molecule ranges from 10^{-8} m to 10^{-10} m. An optical microscope uses visible light of average wavelength 6000 \AA , i.e., 6000×10^{-10} m to measure the sizes. Since, size of molecule is smaller than the wavelength of light used, so optical microscope cannot resolve a molecule. 2

Q. 5. Give conventional rules for the rounding off of uncertain digits. [U]

Ans. The conventional rules are :

(i) If the insignificant digit to be dropped is more than 5, the preceding digit is increased by 1, but if it is less than five, then preceding digits is not changed, e.g., 1.748 is rounded off to 3 significant figures as 1.75 and 1.742 as 1.74. 1

(ii) If the insignificant digit to be dropped is 5, then this digit is simply dropped if the preceding digit is even but if odd, then the preceding digit is increased by 1.

e.g., the number 1.845 rounded off to three significant digits is 1.84 but for number 1.875 it is 1.88. 1



Short Answer Type Questions-II

(3 marks each)

Q. 1. What is system of units ? Mention some of them. [R]

Ans. In general, a complete set of base units as well as derived units is called system of units.

But, it was a practice to name the system of units after three fundamental units of length, mass and time only, e.g.,

(a) **FPS system** : In this system length is measured in foot (f), mass in pound (p) and time in second (s). So, this system is called FPS system. This system is also known as British Engineering system of units or simply British system of units.

(b) **C.G.S. system** : In this system length is measured in centimeter (cm), mass in gram (g) and time in second (s), so this system is called C.G.S. system.

(c) **M.K.S. system** : In this system length is measured in metre (m), mass in Kilograms (kg) and time in second (s), so this system is called M.K.S. system or metric system. (1 mark each)

Q. 2. What is a prefix ? Give some common prefixes for multiples and submultiples. [R]

Ans. Prefix is used to increase or decrease the value of a fundamental or derived unit as per practical requirements.

Multiples

$$\text{Exa (E)} = 10^{18}$$

$$\text{Peta (P)} = 10^{15}$$

$$\text{Tera (T)} = 10^{12}$$

$$\text{Giga (G)} = 10^9$$

Submultiples

$$\text{atto (a)} = 10^{-18}$$

$$\text{femto (f)} = 10^{-15}$$

$$\text{pico (p)} = 10^{-12}$$

$$\text{nano (n)} = 10^{-9}$$

Mega (M) = 10^6	micro (μ) = 10^{-6}
Kilo (k) = 10^3	milli (m) = 10^{-3}
Hecto (h) = 10^2	centi (c) = 10^{-2}
Deca (da) = 10^1	deci (d) = 10^{-1}

3



Commonly Made Error

- ▶ Students generally replace some multiples with submultiples and vice versa.



Answering Tip

- ▶ Students should ensure the correctness of multiples and submultiples along with their prefixes used.

Q. 3. List the S.I. base quantities and find their units with symbols. R

Ans.

S. No.	Base quantity	S.I. unit	Symbol
(i)	Length	metre	m
(ii)	Mass	kilogram	kg
(iii)	Time	second	s
(iv)	Electric current	ampere	A
(v)	Temperature	kelvin	K
(vi)	Amount of substance	mole	mol
(vii)	Luminous intensity	candela	cd

(½ mark each for any six points)



Long Answer Type Questions

(5 marks each)

Q. 1. Write the S. I. units of the following physical quantities—

- | | |
|--------------------------|-----------------------|
| (a) Luminous intensity, | (b) Temperature, |
| (c) Plane angle, | (d) Electric current, |
| (e) Amount of substance, | (f) Solid angle, |
| (g) Pressure. | |

R

- Ans. (a) candela (cd) (b) kelvin (K)
- (c) radian (rad) (d) ampere (A)
- (e) mole (mol) (f) steradian (Sr)
- (g) N/m^2 = pascal (pa) 5

Q. 2. What do you understand by the following :

- (a) Century, (b) Shake, (c) Lunar month, (d) Leap year, (e) Tropical year. R

Ans. (a) It is the largest unit of time.

1 century = 100 years.

(b) It is the smallest unit of time, 1 shake = 10^{-8} s.

(c) It is the time taken by the Moon to complete one revolution around the Earth. 1 lunar month = 29.53 days.

(d) A year which is divisible by 4 and in which the month of February is of 29 days is called leap year. Also a year is leap year if it is visible by 100 and also by 400. If the year is divisible by 100 but not by 400, then the year is not a leap year.

(e) The year in which total solar eclipse takes place is called tropical year. (1 mark each)

Topic-2 Dimensional Analysis



Revision Notes

- ▶ Dimensional analysis is the study of relationship between physical quantities and the fundamental quantities.
- ▶ Dimensional equation is the equation expressing of the relationship between physical quantities and the fundamental quantities.
- ▶ **Principle of homogeneity:** Dimensions of each term of a dimensional equation on both side should be same.
- ▶ Conversion of units from one system to another:
 n_1 and n_2 be the magnitudes of a physical quantity in two systems respectively. General dimensions of the physical quantity be $[M^a L^b T^c]$. If dimensions in two system be $u_1 = [M_1^a L_1^b T_1^c]$ and $u_2 = [M_2^a L_2^b T_2^c]$ respectively then $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$
- ▶ Dimensional analysis used to derive formula of a physical quantity.
- ▶ Correctness of formula of a physical quantity may be checked using principle of homogeneity.

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more about
this topic



Units

➤ **Limitations of dimensional analysis:**

- Cannot provide information about dimensional constant.
- Formula containing trigonometric functions, logarithmic functions, and exponential function cannot be derived.
- Cannot provide information whether the quantity is a vector or a scalar.
- If the physical quantity depends on more than three quantities then it becomes difficult to derive the formula.
- It is not possible to find the formula correctly when the physical quantity is not related by multiplication of the other quantities.



Key Words

- **Dimensions of physical quantity** are the powers to which the symbols of fundamental quantities are raised to represent a derived unit of that quantity.
- **Dimensional formula of the given physical quantity** is the expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity.
- **Dimensional constants** are the quantities whose values are constant and they possess dimensions e.g., universal gravitational constant G etc.
- **Dimensional variables** are the quantities whose values are variable and they possess dimensions e.g., area, volume, etc.
- **Dimensionalless constants** are the quantities whose value are constant but they do not possess dimensions e.g., mathematical constants– π , e and numbers.
- **Dimensionalless variables** are the quantities whose values are variable and they do not have dimensions e.g., angle, strain, etc.



Key Formulae

- Conversion of one system of units into another E / F R E E K I T A A B

$$n_2 = \frac{n_1 u_1}{u_2} = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c$$



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is understood by dimensions ? [R]

Ans. Dimensions in respect to a physical quantity are the exponents (say powers) to which the fundamental physical quantities are raised to represent that physical quantity. 1

Q. 2. What is dimensional formula ? [R]

Ans. Dimensional formula is a formula in which fundamental physical quantities of a quantity are expressed with their powers, e.g., dimensional formula of speed is given by distance/time i.e., $[LT^{-1}]$. 1

Q. 3. What is dimensional equation ? [R]

Ans. A dimensional equation consists of a physical

quantity equated to its dimensional formula.

e.g., Velocity = $[LT^{-1}]$

Q. 4. State the principle of homogeneity of dimensions ? [R]

Ans. It states that the dimensions of each term on both sides of an equation are same. 1

Q. 5. Will the dimensions of a physical quantity be same, whatever be the units in which it is measured? Why ? [U]

Ans. Yes, the dimensions don't depend on the system of units chosen. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. Check whether the given equation

$$F.s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

is dimensionally correct, or not

where, m is mass of the body, v is its final velocity, u is its initial velocity, F is applied force and s is distance covered. [U] [NCT 2008]

Ans. The dimension of L.H.S.

$$F.s = [MLT^{-2}].[L]$$

$= [ML^2T^{-2}]$ ½

The dimensions of R.H.S.

$$\frac{1}{2}mv^2 \text{ or } \frac{1}{2}mu^2 = [M][LT^{-1}]^2$$

$$= [ML^2T^{-2}]$$
 ½

So, dimensions of L.H.S. = R.H.S.

So, given equation is dimensionally correct. 1



Commonly Made Error

- ▶ Students may commit mistake while calculating the dimension of the given quantities.



Answering Tip

- ▶ Apply principle of homogeneity in solving such problems.

Q. 2. Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments :

(i) $\frac{3}{16}mv^2$

(ii) $\frac{1}{2}mv^2 + ma$ U [KVS 2008]

Ans. Dimensions of K.E. $= [ML^2T^{-2}]$ ½

(i) Dimensions of $\frac{3}{16}mv^2 = [M][LT^{-1}]^2$

$$= [ML^2T^{-2}]$$
 ½

(ii) Dimensions of $\frac{1}{2}mv^2 = [ML^2T^{-2}]$

Dimensions of $ma = [MLT^{-2}]$

Dimensions of $\frac{1}{2}mv^2 + ma = [ML^2T^{-2}] + [MLT^{-2}]$ ½

So, (ii) formula is incorrect as two physical quantities of different dimensions are added which is not possible according to principle homogeneity. ½



Short Answer Type Questions-II

(3 marks each)

Q. 1. A satellite is observed from two points A and B at a distance 1.3×10^7 m apart on Earth. If angle subtended at the satellite is $1^\circ 30'$, find distance of the satellite from Earth.

Ans. We have, $\theta = 1^\circ 30' = (60' + 30') = 90'$

$$= (90 \times 60)''$$

$$= 5400''$$
 1

$$= (5400) (4.85 \times 10^{-6}) \text{ rad}$$

$[\because 1'' = 4.85 \times 10^{-6} \text{ rad}]$

and $b = 1.3 \times 10^7$ m.

using, $D = \frac{b}{\theta}$, we get 1

$$D = \frac{1.3 \times 10^7}{5400 \times 4.85 \times 10^{-6}}$$

$$= \frac{1.3 \times 10^{11}}{54 \times 4.85}$$

$$= 4.96 \times 10^8 \text{ m.}$$
 1

Q. 2. Distance of a star from Earth is 1.5×10^{11} m. If the angular size of the star is $2000''$. Find the diameter of the star. AI

Q. 3. Deduce the dimensional formula for R, using ideal gas equation $PV = nRT$. U [NCT 2010]

Ans. Dimensions of $R = \frac{[P][V]}{[n][T]}$

$$= \frac{[ML^{-1}T^{-2}][L^3]}{[\text{mol}][K]}$$

$$= [ML^2T^{-2}K^{-1}\text{mol}^{-1}]$$
 2


Commonly Made Error

- ▶ Some students treat 'n' as a constant and keep it dimensionless.



Answering Tip

- ▶ Students should carefully use the dimension of all the quantities and then deduce the result.

Q. 4. Mention some physical quantities having same dimensional formulae but different units.

Ans. (a) (i) Linear momentum $[MLT^{-1}]$ kg ms⁻¹

(ii) Impulse $[MLT^{-1}]$ Ns ½

(b) (i) Work and energy $[ML^2T^{-2}]$ J

(ii) Torque $[ML^2T^{-2}]$ Nm ½

(c) (i) Surface tension $[ML^0T^{-2}]$ Nm⁻¹

(ii) Surface energy $[ML^0T^{-2}]$ Jm⁻² ½

(d) (i) Angular velocity $[M^0L^0T^{-1}]$ rad s⁻¹

(ii) Velocity gradient $[M^0L^0T^{-1}]$ s⁻¹

(iii) Frequency $[M^0L^0T^{-1}]$ Hz ½

Q. 5. Give some physical quantities having same dimensional formulae as well as same units. R

Ans. (a) Work/potential energy/K.E./M.E./heat

Dimensions	Unit	1
$[ML^2T^{-2}]$	Joule	1

(b) Pressure/stress/modulus of elasticity

Dimensions	Unit	1
$[ML^2T^{-2}]$	Nm ⁻²	1

Ans. We have, $\alpha = 2000''$
 $= 2000 \times 4.85 \times 10^{-6} \text{ rad.}$
 $[\because 1'' = 4.85 \times 10^{-6} \text{ rad}]$ **1**
 and $D = 1.5 \times 10^{11}$
 Using $d = \alpha D$, we get **1**
 $d = 2000 \times 4.85 \times 10^{-6} \times 1.5 \times 10^{11}$
 $= 2 \times 4.85 \times 1.5 \times 10^8$
 $= 1.455 \times 10^9 \text{ m.}$ **1**

Q. 3. Mankind has existed for about 10^6 years whereas the universe is about 10^{10} year old. If the age

of universe is taken to be one day, how many seconds has the mankind existed ?

Ans. Since, 10^{10} years = 1 day **1**
 10^6 years = $\frac{1}{10^{10}} \times 10^6$ days
 $= 10^{-4}$ days **1**
 $= 10^{-4} \times 24 \times 60 \times 60 \text{ s}$
 $= 86400 \times 10^{-4} \text{ s}$
 $= 8.64 \text{ sec.}$ **1**



Objective Type Questions

(1 marks each)

A Multiple Choice Questions

Q. 1. Which of the followings is the proper representation of a 10 cm long scale?

- (A) The scale of the length is 10 cm.
- (B) This is a 10 cm. long scale.
- (C) This a 10 c.m. long scale.
- (D) The scale of the length is 10 c.m.

Ans. Option (A) is correct.

Explanation: While units are symbolically represented, there should not be any dots. But if it appears at the ends of a sentence, then a dot may appear as a full-stop.

Q. 2. Which one of the following representation of temperature is not correct?

- (A) 50 K (B) 100°K
- (C) 225°C (D) 125°F

Ans. Option (B) is correct.

Explanation: While representing temperature in Kelvin scale, °K should not be written.

Q. 3. Which one of the following is correct representation of current?

- (A) 5 a (B) 5 Ampere
- (C) 5 ampere (D) 5 amperes

Ans. Option (C) is correct.

Explanation: When symbolically written, the first letter of the unit in the name of scientist should be a capital letter. But when the unit is written in complete word form then it should be written in small letters.

Q. 4. Number of significant figures in 0.0024500 is

- (A) 3 (B) 5
- (C) 8 (D) 7

Ans. Option (B) is correct.

Explanation: All the zeros on the right immediate after the decimal are not significant.

All the zeros those are on right of the last non-zero digit, after the decimal point, are significant.

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is true

Q. 1. Assertion (A): $1 \text{ kg} = 10^9 \mu\text{g}$

Reason (R): $1 \text{ kg} = 10^3 \text{ g}$ and $1 \text{ g} = 10^6 \mu\text{g}$.

Ans. Option (A) is correct.

Explanation:

$$1 \text{ kg} = 10^3 \text{ g}$$

Or, $1 \text{ kg} = 10^3 \times 10^6 \mu\text{g}$

$\therefore 1 \text{ kg} = 10^9 \mu\text{g}$

So, assertion and reason both are true. Reason explains the assertion.

Q. 2. Assertion (A): Angle and solid angle both are dimensionless quantities.

Reason (R): Dimensionless quantities cannot have unit.

Ans. Option (C) is correct.

Explanation:

$$\text{Angle} = \frac{\text{Arc}}{\text{radius}}$$

$$\text{Solid angle} = \frac{\text{Arc}}{(\text{radius})^2}$$

So, both are dimensionless. Hence, the assertion is true.

In SI system the unit of angle is radian and the unit of solid angle is steradian. Hence, the reason is false.

Q. 1. Assertion (A): The time period of a pendulum is

given by $T = k \sqrt{\left(\frac{l}{g}\right)}$.

Reason (R): The value of the constant k can be determined by dimensional analysis.

Ans. Option (C) is correct.

Explanation: Time period of a pendulum is given

$$\text{by } T = k \sqrt{\left(\frac{l}{g}\right)} = 2\pi \sqrt{\left(\frac{l}{g}\right)}.$$

Hence, the assertion is true.

The value of the constant k cannot be determined by dimensional analysis. This is a limitation of dimensional analysis.



NCERT Corner

Q. 1. Fill in the blanks :

- (a) The volume of a cube of side 1 cm is equal to m^3 .
- (b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to mm^2 .
- (c) A vehicle moving with a speed of 18 kmh^{-1} covers m in 1 s.
- (d) The relative density of lead is 11.3. Its density is g cm^{-3} or kg m^{-3} .

Ans. (a) 10^{-6} (b) 15072
(c) 5 (d) 11.3, 11300 (½ mark each)

Q. 2. Fill in the blanks by suitable conversion of units :

- (a) $1 \text{ kg m}^2\text{s}^{-2} = \dots \text{g cm}^2\text{s}^{-2}$
- (b) $1 \text{ m} = \dots \text{ly}$ (light year)
- (c) $3.0 \text{ ms}^{-2} = \dots \text{kmh}^{-2}$
- (d) $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} = \dots \text{cm}^3 \text{ s}^{-2} \text{ g}^{-1}$

Ans. (a) 10^7 (b) 1.053×10^{-16}
(c) 3.888×10^4 (d) 6.67×10^{-8}

Q. 3. A calorie is a unit of heat or energy and it equals about 4.2 J, where $1 \text{ J} = 1 \text{ kgm}^2\text{s}^{-2}$. Suppose we employ a system of units in which the unit of mass equals $\alpha \text{ kg}$, the unit of length $\beta \text{ m}$, the unit of time is $\gamma \text{ s}$. Show that a calorie has a magnitude $4.2 \alpha^{-1} \beta^{-2} \gamma^2$ in terms of new units. AI

Ans. Given : $1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kgm}^2\text{s}^{-2}$

$$\text{So, } a = 1, b = 2, c = -2$$

$$\text{In S.I. system, } n_1 = 4.2, M_1 = 1 \text{ kg, } L_1 = 1 \text{ m,}$$

$$T_1 = 1 \text{ s, } n_2 = ?, M_2 = \alpha \text{ kg, } L_2 = \beta \text{ m, } T_2 = \gamma \text{ s}$$

From equations,

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$n_2 = 4.2 \left[\frac{1 \text{ kg}}{\alpha \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{\beta \text{ m}} \right]^2 \left[\frac{1 \text{ s}}{\gamma \text{ s}} \right]^{-2}$$

$$\therefore n_2 = 4.2 \alpha^{-1} \beta^{-2} \gamma^2 \text{ in new system.}$$

Hence, proved.

Q. 4. Explain the statement clearly :

“To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison.” In view of this, re-frame the following statements wherever necessary :

- (a) Atoms are very small objects.
- (b) A jet plane moves with great speed.
- (c) The mass of Jupiter is very large.

- (d) The air inside this room contains a large number of molecules.
- (e) A proton is much more massive than an electron.
- (f) The speed of sound is much smaller than the speed of light.

Ans. The physical quantities are called large or small in comparison to some standard units of measurement. So, statement is correct :

- (a) As size of an atom is smaller than the sharp tip of pin.
- (b) As a jet plane moves faster than a superfast train.
- (c) As mass of Jupiter is very large as compared to the mass of Earth.
- (d) As air inside the room contains more number of molecules than in one mole of air.
- (e) Statement is true.
- (f) Statement is true. 2

Q. 5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?

Ans. Distance between Sun and Earth is
 $= 3 \times 10^8 \text{ m/s} \times 500 \text{ s}$
 $= 500 \times 3 \times 10^8 \text{ m}$

As the speed of light in vacuum = 1 ms^{-1} (in new system). So, distance between Sun and Earth.
 $= 500 \text{ new units.}$

Q. 6. Which of the following is the most precise device for measuring length?

- (a) A vernier callipers with 20 division on the sliding scale.
- (b) A screw gauge of pitch 1 mm and 100 divisions on the circular scale.
- (c) An optical instrument that can measure length to within a wavelength of light.

Ans. (a) Least count of vernier callipers

$$= \frac{1}{20} = 0.05 \text{ mm} = 5 \times 10^{-5} \text{ m}$$

(b) Least count of screw gauge

$$= \frac{1 \times 10^{-3}}{100} = 1 \times 10^{-5} \text{ m}$$

(c) Least count of optical instrument

$$= 5 \times 10^{-7} \text{ m.}$$

\therefore Optical instrument is most precise instrument.

Q. 7. A student measures the thickness of a human hair with the help of a microscope of magnification 100. He takes 20 readings and finds average width of the hair in the field of view of microscope is 3.5 mm. What is the thickness of the hair ?

Ans. Magnification,

$$\begin{aligned} \text{Real width} &= \frac{\text{Observed width (y)}}{\text{magnification}} \\ &= \frac{3.5}{100} = 0.035 \text{ mm} \end{aligned}$$

Q. 8. Answer the following :

- You are given a thread and a metre scale. How will you estimate the diameter of the thread ?
- A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?
- The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

Ans. (a) By winding the thread in close turns on a pencil and using the formula i.e.,

$$\text{Diameter} = \frac{\text{Average length}}{\text{No. of turns}}$$

- Yes, by increasing the number of divisions on circular scale as

$$\text{Least count} = \frac{\text{Pitch}}{\text{No. of divisions on circular scale}}$$

- Random error is reduced on taking large number of observations.

Q. 9. The photograph of a house occupies an area of 1.75 cm² on a 35 mm slide. The slide is projected on to a screen and the area of the house on the screen is 1.55 m². What is the linear magnification of the projector, screen arrangement ?

Ans. Here size of an object = Area of object

$$\begin{aligned} &= 1.75 \text{ cm}^2 \\ &= 1.75 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Size of the image} &= \text{Area of the image} \\ &= 1.55 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Areal magnification} &= \frac{\text{Area of image}}{\text{Area of object}} \\ &= \frac{1.55}{1.75 \times 10^{-4}} = 8.857 \times 10^3 \end{aligned}$$

$$\text{Linear magnification} = \sqrt{8857} = 94.1$$

Q. 10. State the number of significant figure in :

- | | |
|-------------------------------|------------------------------|
| (a) 0.007 m ² | (b) 2.64 × 10 ²⁴ |
| (c) 0.2370 g cm ⁻³ | (d) 6.320 J |
| (e) 6.032 Nm ⁻² | (f) 0.0006032 m ² |

- Ans.** (a) 1 (b) 3
 (c) 4 (d) 4
 (e) 4 (f) 4

Q. 11. Length, breadth and thickness of a metal piece are 4.234 m, 1.005 m and 2.01 cm respectively. Find its area and volume to correct significant figures.

Ans. $A = 2(l \times b + b \times t + t \times l)$
 $= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \text{ m}^2$
 $= 2(4.25517 + 0.0202005 + 0.0851034) \text{ m}^2$
 $= 2 \times 4.3604739 \text{ m}^2 = 8.7209478 \text{ m}^2$
 $= 8.72 \text{ m}^2$ (approximated to 3 significant figures)

$$\begin{aligned} \text{Volume} &= 4.234 \times 1.005 \times (2.01 \times 10^{-2}) \text{ m}^3 \\ &= 8.55289 \times 10^{-2} \text{ m}^3 \\ &= 0.0855 \text{ m}^3. \end{aligned}$$

Q. 12. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the mass of the pieces to corrects significant figures ?

Ans. Here mass of the box,

$$m = 2.30 \text{ kg}$$

Mass of one gold piece,

$$\begin{aligned} m_1 &= 20.15 \text{ g} \\ &= 0.02015 \text{ kg} \end{aligned}$$

Mass of second gold piece,

$$\begin{aligned} m_2 &= 20.17 \text{ g} \\ &= 0.02017 \text{ kg} \end{aligned}$$

(a) Total mass = $m + m_1 + m_2$
 $= (2.300 + 0.02015 + 0.02017) \text{ kg}$
 $= 2.34032 \text{ kg}.$

As the least no. of significant figures in the mass of box is 2, so, maximum no. of significant figures in the result can be 2.

$$\text{Total mass} = 2.3 \text{ kg}.$$

(b) Difference in masses,

$$\begin{aligned} m &= m_2 - m_1 \\ &= 20.17 - 20.15 = 0.02 \text{ g} \end{aligned}$$

Since, there are two significant figures, so, the difference in masses to the correct significant figures is 0.02 g.

Q. 13. A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and speed of light c . (This relation first arose as a consequence of special theory of relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c . He writes

$$m = \frac{m_0}{(1 - v^2)^{1/2}}. \text{ Guess where to put the missing } c.$$

[NCERT TB Q. No. 15]

Ans. From principle of homogeneity of dimensions, power of M, L and T on either side of the formula must be equal. In the given formula, on R.H.S., the denominator $(1 - v^2)^{1/2}$ must be dimensionless.

Hence, the correct formula is $\frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$.

Q. 14. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å. $1 \text{ Å} = 10^{-10} \text{ m}$. The size of the hydrogen atom is about 0.5 Å . What is the total atomic volume in m^3 of a mole of hydrogen atoms ?

[NCERT TB Q. No. 16]

Ans. Here, $r = 0.5 \text{ Å}$
 $= 0.5 \times 10^{-10} \text{ m}$
 $V_1 =$ Volume of each hydrogen atom
 $= \frac{4}{3} \pi r^3$
 $= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$
 $= 5.236 \times 10^{-31} \text{ m}^3$

According to Avogadro's hypothesis, one mole of hydrogen contains

$$N = 6.023 \times 10^{23} \text{ atoms}$$

\therefore Atomic volume of 1 mole of hydrogen atoms,

$$\begin{aligned} V &= NV_1 \\ V &= 6.023 \times 10^{23} \times 5.236 \times 10^{-31} \\ &= 3.154 \times 10^{-7} \text{ m}^3 \\ &= 3.154 \times 10^{-7} \text{ m}^3. \end{aligned}$$

Q. 15. One mole of an ideal gas at NTP and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?

[NCERT TB Q. No. 17]

Ans. Atomic volume $= \frac{4}{3} \pi R^3 \times N$
 $= \frac{4}{3} \pi (0.5 \times 10^{-10})^3 \times 6.023 \times 10^{23}$
 $= 3.154 \times 10^{-7} \text{ m}^3$.

Molar volume $= 22.4 \text{ L}$

$$= 22.4 \times 10^{-3} \text{ m}^3$$

$$\begin{aligned} \frac{\text{Molar volume}}{\text{Atomic volume}} &= \frac{22.4 \times 10^{-3}}{3.154 \times 10^{-7}} \\ &= 7.1 \times 10^4 \end{aligned}$$

The large value of the ratio shows that the inter molecular separation in a gas is much larger than the size of a molecule.

Q. 16. Explain this common observation clearly. If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly

in a direction opposite to the train's motion, but the distant, objects (hill tops, the Moon, the stars etc.) seem to be stationary (In fact, since you are aware that you are moving, these distant objects seem to move with you). [NCERT TB Q. No. 18]

Ans. The line joining the object to the eye is line of sight. If we move rapidly, the line of sight nearby tree changes its direction of motion rapidly. It means near objects make greater angle than distant objects. Thus, the trees appear to run in opposite direction. But in case of Moon, stars etc., this change is negligible, so they appear stationary.

Q. 17. The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K and its outer surface at a temperature of about 6000 K . At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : **A1** [NCERT TB Q. No. 23]

Mass of Sun $= 2.0 \times 10^{30} \text{ kg}$,

Radius of the Sun $= 7.0 \times 10^8 \text{ m}$

Ans. Here, $M = 2 \times 10^{30} \text{ kg}$
 $R = 7.0 \times 10^8 \text{ m}$

\therefore Mass density of Sun

$$\begin{aligned} &= \frac{\text{Mass of sun}}{\text{Volume of sun}} = \frac{M}{V} \\ &= \frac{M}{\frac{4}{3} \pi R^3} \\ &= \frac{2 \times 10^{30}}{\frac{4}{3} \times 3.14 \times (7 \times 10^8)^3} \\ &= \frac{3 \times 2 \times 10^{30}}{4 \times 3.14 \times 343 \times 10^{24}} \text{ kg m}^{-3} \\ &= 1.39 \times 10^3 \text{ kg m}^{-3} \end{aligned}$$

This density is the range of density of solids and not gases.

Explanation—The temperature of the inner core of the Sun exceeds 10^7 K while the temperature of the outer surface of the Sun is nearly 6000 K . At such extremely high temperature, no substance can exist either in a solid or in a liquid phase. So, the Sun is made of ionized matter, i.e., hot plasma. The high density of the plasma is due to the inward gravitational attraction on outer layers due to the inner layers of the Sun.



NCERT Exemplar (Only Important Questions)

A Multiple Choice Questions-I

Q. 1. The number of significant figures in 0.06900 is

- (A) 5 (B) 4
(C) 2 (D) 3

Ans. Option (B) is correct.

Q. 2. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is

- (A) 663.821 (B) 664
(C) 663.8 (D) 663.82

Ans. Option (C) is correct.

Q. 3. The mass and volume of a body are 4.237 g and 2.5 cm³, respectively. The density of the material of the body in correct significant figures is

- (A) 1.6048 g cm⁻³ (B) 1.69 g cm⁻³
(C) 1.7 g cm⁻³ (D) 1.695 g cm⁻³

Ans. Option (C) is correct.

Explanation:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{4.237\text{g}}{2.5\text{cm}^3} = 1.6948\text{gcm}^{-3}$$

Q. 4. The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give

- (A) 2.75 and 2.74 (B) 2.74 and 2.73
(C) 2.75 and 2.73 (D) 2.74 and 2.74

Ans. Option (D) is correct.

Q. 5. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is

- (A) 164 ± 3 cm² (B) 163.62 ± 2.6 cm²
(C) 163.6 ± 2.6 cm² (D) 163.62 ± 3 cm²

Ans. Option (A) is correct.

Explanation: $l = 16.2 \pm 0.1\text{cm} = 16.2\text{cm} \pm .6\%$

$$b = 10.1 \pm 0.1\text{cm} = 10.1\text{cm} \pm 1\%$$

$$A = l \times b = 163.62 \pm 1.6\%$$

$$= 163.62 \pm 2.6$$

$$= 163.62 \pm 3\text{cm}^2$$

Q. 6. Which of the following pairs of physical quantities does not have same dimensional formula? AI

- (A) Work and torque.
(B) Angular momentum and Planck's constant.
(C) Tension and surface tension.
(D) Impulse and linear momentum.

Ans. Option (C) is correct.

Explanation:

$$\text{Tension} = \text{Force} = [\text{MLT}^{-2}]$$

$$\text{Surface Tension} = \frac{\text{Force}}{\text{length}} = \frac{[\text{MLT}^{-2}]}{[\text{L}]} \\ = [\text{ML}^0\text{T}^{-2}]$$

Q. 7. Measure of two quantities along with the precision of respective measuring instrument is

$$A = 2.5 \text{ m s}^{-1} \pm 0.5 \text{ m s}^{-1}$$

$$B = 0.10 \text{ s} \pm 0.01 \text{ s}$$

The value of AB will be

- (A) (0.25 ± 0.08) m (B) (0.25 ± 0.5) m
(C) (0.25 ± 0.05) m (D) (0.25 ± 0.135) m

Ans. Option (A) is correct.

Explanation:

$$AB = (2.5 \times 0.10)\text{m} \\ = 0.25\text{m}$$

$$\frac{\Delta AB}{AB} = \pm \left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) = \left(\frac{0.5}{2.5} + \frac{0.01}{0.10} \right) \\ = 0.3$$

$$\Delta AB = 0.3 \times 0.25\text{m} = 0.075\text{m} \\ = \pm 0.08\text{m}$$

$$\therefore AB = 0.25 \pm 0.08\text{m}$$

Q. 8. You measure two quantities as $A = 1.0 \text{ m} \pm 0.2 \text{ m}$, $B = 2.0 \text{ m} \pm 0.2 \text{ m}$. We should report correct value for \sqrt{AB} as AI

- (A) 1.4 m ± 0.4 m (B) 1.41 m ± 0.15 m
(C) 1.4 m ± 0.3 m (D) 1.4 m ± 0.2 m

Ans. Option (D) is correct.

Explanation:

$$AB = 1 \times 2 = 2\text{m}^2$$

$$\sqrt{AB} = \sqrt{2} \text{ m} = 1.414\text{m} = 1.4\text{m}$$

$$\frac{\Delta \sqrt{AB}}{\sqrt{AB}} = \frac{1}{2} \left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) = \frac{1}{2} \left(\frac{0.2}{1.0} + \frac{0.2}{2.0} \right) = \frac{0.3}{2}$$

$$\Delta \sqrt{AB} = \frac{0.3}{2} \times \sqrt{AB} = \frac{0.3}{2} \times 1.414 = 0.212\text{m}$$

$$\Delta \sqrt{AB} = 0.2$$

$$\therefore \sqrt{AB} = 1.4\text{m} \pm 0.2\text{m}$$

Q. 9. Which of the following measurements is most precise?

- (A) 5.00 mm (B) 5.00 cm
(C) 5.00 m (D) 5.00 km.

Ans. Option (A) is correct.

Q. 10. The mean length of an object is 5 cm. Which of the following measurements is most accurate?

- (A) 4.9 cm (B) 4.805 cm
(C) 5.25 cm (D) 5.4 cm

Ans. Option (A) is correct.

Q. 11. Young's modulus of steel is $1.9 \times 10^{11} \text{ N/m}^2$. When expressed in CGS units of dyne/cm², it will be equal to ($1 \text{ N} = 10^5 \text{ dyne}$, $1 \text{ m}^2 = 10^4 \text{ cm}^2$)

- (A) 1.9×10^{10} (B) 1.9×10^{11}
(C) 1.9×10^{12} (D) 1.9×10^{13}

Ans. Option (C) is correct.

Explanation:

$$1.9 \times 10^{11} \left(\frac{10^5 \text{ dyne}}{10^4 \text{ cm}^2} \right) = 1.9 \times 10^{12} \text{ dyne cm}^{-2}$$

Q. 12. If momentum (p), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula. AI

- (A) $(p^1 A^{-1} T^1)$ (B) $(p^2 A^1 T^1)$
(C) $(p^1 A^{-1/2} T^1)$ (D) $(p^1 A^{1/2} T^{-1})$

Ans. Option (D) is correct.

Explanation:

$$E = K p^a A^b T^c \text{ \{Here K is constant\} ... (i)}$$

$$E = [ML^2 T^{-2}] \dots \text{(ii)}$$

$$[ML^2 T^{-2}] = p^a A^b T^c \quad \text{\{by (i) \& (ii)\}}$$

$$= [MLT^{-1}]^a [M^0 L^2 T^0]^b [M^0 L^0 T]^c$$

$$\therefore a = 1, a + 2b = 2, -a + c = -2$$

On solving this we get

$$a = 1, b = \frac{1}{2}, c = -1$$

$$\therefore E = [p^1 A^{1/2} T^{-1}]$$

B Multiple Choice Questions-II

Q. 13. On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct:

(A) $y = a \sin \pi t/T$

(B) $y = a \sin vt$

(C) $y = \frac{a}{t} \sin \left(\frac{t}{a} \right)$

(D) $y = a\sqrt{2} \left(\sin \frac{2\pi T}{T} - \cos \frac{2\pi T}{T} \right)$

Ans. Option (B, C) is correct.

Explanation: Because trigonometric functions are dimensionless.

Q. 14. If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

- (A) $(P - Q)/R$ (B) $PQ - R$
(C) PQ/R (D) $(PR - Q^2)/R$
(E) $(R + Q)/P$

Ans. Option (A, E) is correct.

Q. 15. Photon is quantum of radiation with energy $E = h\nu$ where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of

- (A) Linear impulse
(B) Angular impulse
(C) Linear momentum
(D) Angular momentum

Ans. Option (B, D) is correct.

Q. 16. If Planck's constant (h) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?

- (A) Mass of electron (m_e)
(B) Universal gravitational constant (G)
(C) Charge of electron (e)
(D) Mass of proton (m_p)

Ans. Option (A, B, D) is correct.

Q. 17. Which of the following ratios express pressure?

- (A) Force/Area (B) Energy/Volume
(C) Energy/Area (D) Force / Volume

Ans. Option (A, B) is correct.

Q. 18. Which of the following are not a unit of time? AI

- (A) Second (B) Parsec
(C) Year (D) Light year

Ans. Option (B, D) is correct.



Very Short Answer Type Questions

Q. 19. Why do we have different units for same physical quantity?

Ans. The value of any given physical quantity may vary over a wide range.

Because, bodies differ in order of magnitude significantly in respect to the same physical quantity. For example, interatomic distances are of the order of angstroms, Inter-city distances are of the order of km and interstellar distances are of the order of light year.

Q. 20. The radius of atom is of the order of 1 \AA and radius of nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus? AI

Ans. Given : Radius of atom, $r_a = 1 \text{ \AA} = 10^{-10} \text{ m}$
Radius of nucleus, $r_n = 1 \text{ fermi} = 10^{-15} \text{ m}$

$$\text{Volume of atom; } V_1 = \frac{4}{3} \pi r_a^3$$

$$\text{Volume of nucleus, } V_2 = \frac{4}{3} \pi r_n^3$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3} \pi r_a^3}{\frac{4}{3} \pi r_n^3} = \left(\frac{r_a}{r_n} \right)^3 = \left(\frac{10^{-10}}{10^{-15}} \right)^3$$

$$\frac{V_1}{V_2} = 10^{15}$$

Q. 21. Name the device used for measuring the mass of atom and molecules.

Ans. Mass spectrograph.

Q. 22. Express unified atomic mass unit in kg.

Ans. 1 atomic Mass unit = $\frac{1}{12}$ of mass of ${}_6\text{C}^{12}$ atom

Mass of 1 mole of ${}_6\text{C}^{12}$ atom = 12 g

No. of atom in 1 mole = Avogadro number

Mass of ${}_6\text{C}^{12}$ atom = $\frac{12}{6.023 \times 10^{23}}$ g

1 atomic mass unit = $\frac{1}{12} \times \frac{12}{6.023 \times 10^{23}}$ g
= 1.67×10^{-27} kg

Q. 23. A function $f(\theta)$ is defined as :

$$f(\theta) = 1 - \theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} \dots$$

Why is it necessary for $f(\theta)$ to be a dimensionless quantity ?

Ans. Since, $f(\theta)$ is a sum of different powers of θ , it has to be dimensionless.

According to homogeneity principle, if RHS is dimensionless, then LHS should also be dimensionless.

Q. 24. Why length, mass and time are chosen as base quantities in mechanics?

Ans. Because all other quantities of mechanics can be expressed in terms of length, mass and time through simple relations.

And length, mass & time cannot be derived from one another, i.e. they are independent quantities.



Short Answer Type Questions

Q. 25. Which of the following time measuring devices is most precise?

- (a) A wall clock.
- (b) A stop watch.
- (c) A digital watch.
- (d) An atomic clock.

Give reason for your answer.

[NCERT Exemplar Q. No. 26]

Ans. An atomic clock is the most precise time measuring device because atomic oscillations are repeated with a precision of 1s in 10^{13} s.

Q. 26. The distance of a galaxy is of the order of 10^{25} m. Calculate the order of magnitude of time taken by light to reach us from the galaxy.

[NCERT Exemplar Q. No. 27]

Ans. Given: Distance of galaxy = 10^{25} m

Speed of light = 3×10^8 m/s.

Time taken to reach us from galaxy by light,

$$\begin{aligned} T &= \frac{\text{Distance}}{\text{speed}} \\ &= \frac{10^{25}}{3 \times 10^8} \\ &= 3.33 \times 10^{16} \text{ s} \\ &\approx 3 \times 10^{16} \text{ sec} \end{aligned}$$

Q. 27. During a total solar eclipse the Moon almost entirely covers the sphere of the Sun. Write the relation between the distances and sizes of the Sun and Moon. [NCERT Exemplar Q. No. 29]

Ans. R_{me} = distance of Moon from Earth

R_{se} = distance of Sun from Earth

Let θ be angle made by Sun and Moon and

A_s = area of Sun, A_m = area of Moon

$$\begin{aligned} \Rightarrow \theta &= \frac{A_s}{R_{se}^2} = \frac{A_m}{R_{me}^2} \\ \Rightarrow \theta &= \frac{\pi R_s^2}{R_{se}^2} = \frac{\pi R_m^2}{R_{me}^2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \left(\frac{R_s}{R_{se}}\right)^2 &= \left(\frac{R_m}{R_{me}}\right)^2 \\ \Rightarrow \frac{R_s}{R_m} &= \frac{R_{se}}{R_{me}} \end{aligned}$$

Q. 28. If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s, what is the unit of mass in this system of units?

[A1] [NCERT Exemplar Q. No. 30]

Ans. Given:

Dimensions of Force, $F = [\text{MLT}^{-2}]$
= 100N

Time, $T = 100$ s

Length, $L = 10$ m

Substituting eqⁿ (2) & eqⁿ (3) in eqⁿ (1)

$M \times 10 \times (100)^{-2} = 100$

$M = 100 \times 1000$

$M = 10^5$ kg

Q. 29. Give an example of

- (a) a physical quantity which has a unit but no dimensions.
- (b) a physical quantity which has neither unit nor dimensions.
- (c) a constant which has a unit.
- (d) a constant which has no unit.

[NCERT Exemplar Q. No. 31]

Ans. (a) Angle or solid angle.

(b) Relative density, etc.

(c) Planck's constant, universal gravitational constant, etc.

(d) Reynold's number.

Q. 30. Calculate the length of the arc of a circle of radius 31.0 cm which subtends an angle of $\frac{\pi}{6}$ at the centre. [NCERT Exemplar Q. No. 32]

Ans. Angle, $\theta = \frac{l}{r}$ radian

From question, $\theta = \frac{\pi}{6} = \frac{l}{31}$

$$\begin{aligned} \Rightarrow \text{length, } l &= 31 \times \frac{\pi}{6} \text{ cm} \\ &= \frac{31 \times 3.14}{6} \\ &= 16.22 \text{ cm} \\ &\approx 16.2 \text{ cm} \end{aligned}$$

Q. 31. Calculate the solid angle subtended by the periphery of an area of 1 cm² at a point situated symmetrically at a distance of 5 cm from the area. **[AI]** [NCERT Exemplar Q. No. 33]

Ans. As solid angle $\Omega = \frac{\text{Area}}{(\text{distance})^2}$

[Area = 1cm², distance = 5cm]

$$\begin{aligned} &= \frac{1 \text{ cm}^2}{(5 \text{ cm})^2} \\ &= \frac{1}{25} = 4 \times 10^{-2} \text{ steradian.} \end{aligned}$$

Solid angle, $\Omega = 4 \times 10^{-2}$ steradian

Q. 32. The displacement of a progressive wave is represented by $y = A \sin(\omega t - kx)$, where x is

distance and t is time. Write the dimensional formula of (i) ω and (ii) k .

[NCERT Exemplar Q. No. 34]

Ans. (i) Dimensional formula of $\omega = T^{-1}$.

(ii) Dimensional formula of $k = L^{-1}$.



Long Answer Type Questions

Q. 33. A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5 J measure in this new system? [NCERT Exemplar Q. No. 36]

Ans. Dimension of energy = $[ML^2T^{-2}]$,

$$n_1 u_2 = n_2 u_1$$

$$\Rightarrow n_2 = n_1 \frac{u_1}{u_2}$$

$$= n_1 \left[\frac{M_1}{M_2} \right]^1 \left[\frac{L_1}{L_2} \right]^2 \left[\frac{T_1}{T_2} \right]^{-2}$$

Given:

$$n_1 = 5 \text{ J}$$

$$M_2 = \alpha \text{ kg,}$$

$$M_1 = 1 \text{ kg,}$$

$$L_2 = \beta \text{ m, } T_1 = 1 \text{ sec,}$$

$$T_2 = \gamma \text{ s, } L_1 = 1 \text{ m,}$$

$$\therefore n_2 = 5 \text{ J} \left[\frac{1 \text{ kg}}{\alpha \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{\beta \text{ m}} \right]^2 \left[\frac{1 \text{ sec}}{\gamma \text{ sec}} \right]^{-2}$$

$$n_2 = 5 \text{ J} [\alpha^{-1} \beta^{-2} \gamma^2]$$

$$\text{New system} = \frac{5\gamma^2}{\alpha\beta^2} \text{ J}$$

Q. 34. The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as –

$$V = \frac{\pi Pr^4}{8 \eta l}$$

where P is the pressure difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $[ML^{-1}T^{-1}]$.

Check whether the equation is dimensionally correct. [NCERT Exemplar Q. No. 37]

Ans. The dimensional part in the expression is $\frac{Pr^4}{\eta l}$.

Therefore, the dimensions of the right hand side

$$\text{comes out to be } \frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L][T]} = \frac{[L^3]}{[T]}, \text{ which is}$$

volume upon time. As $[V] = \frac{L^3}{T}$, the formula is

dimensional correct, because LHS = RHS.

Q. 35. In the expression $P = EL^2 m^{-5} G^{-2}$, Here, E , m , L and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

[AI] [NCERT Exemplar Q. No. 39]

Ans. Since, E , L and G have dimensional formulas:

$$E = [ML^2T^{-2}]$$

$$L = [ML^2T^{-1}]$$

$$G = [L^3M^{-1}T^{-2}]$$

Hence, $P = E L^2 m^{-5} G^{-2}$ will have dimensions:

$$\begin{aligned} [P] &= \frac{[ML^2T^{-2}][M^2L^4T^{-2}]}{[M^5][L^6M^{-2}T^{-4}]} \\ &= M^0L^0T^0 \end{aligned}$$

Thus, P is dimensionless quantity.

Q. 36. If velocity of light c , Planck's constant h and gravitational constant G are taken as fundamental quantities then express mass, length and time in terms of dimensions of these quantities. [NCERT Exemplar Q. No. 40]

Ans. (i) Let $m \propto c^x h^y G^z$

$$m = K c^x h^y G^z \tag{A}$$

$$h = [ML^2T^{-1}], c = [LT^{-1}], G = [M^{-1}L^3T^{-2}]$$

(K = dimensionless)

$$\begin{aligned} \text{or } [ML^0T^0] &= [LT^{-1}]^x [ML^2T^{-1}]^y [M^{-1}L^3T^{-2}]^z \\ &= [M^{y-z} L^{x+2y+3z} T^{-x-y-2z}] \end{aligned}$$

Comparing powers-

$$y - z = 1$$

(1)

$$x + 2y + 3z = 0 \quad (2)$$

$$-x - y - 2z = 0 \quad (3)$$

Adding above all three equations-

$$2y = 1$$

$$\Rightarrow y = \frac{1}{2}$$

$$\text{So, } K = -\frac{1}{2}, x = \frac{1}{2}$$

Putting in equation (A)-

$$m = kc^{\frac{1}{2}}h^{\frac{1}{2}}G^{-\frac{1}{2}}$$

$$m = K\sqrt{\frac{ch}{G}}$$

(ii) Let $L \propto c^x h^y G^z$

$$L = Kc^x h^y G^z \quad (B)$$

Substituting in B

$$[M^0 L T^0] = [L T^{-1}]^x [M L^2 T^{-1}]^y [M^{-1} L^3 T^{-2}]^z$$

$$= [M^{y-z} L^{x+2y+3z} T^{-x-y-2z}]$$

Comparing powers-

$$y - z = 0 \quad (a)$$

$$x + 2y + 3z = 1 \quad (b)$$

$$-x - y - 2z = 0 \quad (c)$$

Adding (a), (b), (c), we get-

$$y = \frac{1}{2}, z = \frac{1}{2}, x = -\frac{3}{2}$$

Putting in (B)-

$$L = Kc^{-\frac{3}{2}}h^{\frac{1}{2}}G^{\frac{1}{2}}$$

$$L = K\sqrt{\frac{hG}{c^3}}$$

(iii) Let $T \propto c^x h^y G^z$

$$T = Kc^x h^y G^z \quad (C)$$

Substituting dimensions-

$$[M^0 L^0 T^1] = [L T^{-1}]^x [M L^2 T^{-1}]^y [M^{-1} L^3 T^{-2}]^z$$

$$= [M^{y-z} L^{x+2y+3z} T^{-x-y-2z}]$$

Comparing powers-

$$y - z = 0 \quad (1)$$

$$x + 2y + 3z = 0 \quad (2)$$

$$-x - y - 2z = 1 \quad (3)$$

Adding (1), (2), (3), we get-

$$y = \frac{1}{2}, z = \frac{1}{2}, x = -\frac{5}{2}$$

Substituting in (C)

$$T = Kc^{-\frac{5}{2}}h^{\frac{1}{2}}G^{\frac{1}{2}}$$

$$T = K\sqrt{\frac{hG}{c^5}}$$

Q. 37. An artificial satellite is revolving around a planet of mass M and radius R , in a circular orbit of radius r . From Kepler's Third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r . Show using dimensional analysis, that

$$T = \frac{K}{R} \sqrt{\frac{r^3}{g}}$$

Where K is dimensionless constant and g is acceleration due to gravity.

[A1] [NCERT Exemplar Q. No. 41]

Ans. Given From Kepler's III law, $T \propto r^{3/2}$.

T is also function of g and R .

$$\Rightarrow T \propto r^{3/2} R^y g^x = k r^{3/2} R^y g^x$$

[k = dimension less constant of proportionality]

Substituting dimensionless in each term-

$$\therefore [L^0 M^0 T^1] = k [L^{3/2} M^0 T^0] [L^1 M^0 T^{-2}]^y [L]^x$$

$$= k [L^{x+y+3/2} T^{-2y}]$$

$$\text{For } L, 0 = \frac{3}{2} + x + y$$

$$\text{For } T, 1 = 0 - 2y \Rightarrow y = -\frac{1}{2}$$

$$\text{Therefore, } 0 = \frac{3}{2} - \frac{1}{2} + y \Rightarrow y = -1$$

$$\text{Thus, } T = k r^{3/2} g^{-1/2} R^{-1} = \frac{k}{R} \sqrt{\frac{r^3}{g}}$$

Q. 38. (a) How many astronomical units (A. U.) make 1 parsec?

(b) Consider a Sun like star at a distance of 2 parsecs. When it is seen through a telescope with 100 magnifications, what should be the angular size of the star? Sun appears to be $(1/2)^\circ$ from the Earth. Due to atmospheric fluctuations, eye can't resolve objects smaller than 1 arc minute.

(c) Mars has approximately half of the Earth's diameter. When it is closest to the Earth it is at about 1/2 A. U. from the Earth. Calculate what size it will appear when seen through the same telescope.

(Comment : This is to illustrate why a telescope can magnify planets but not stars.)

[NCERT Exemplar Q. No. 43]

Ans. (a) By definition of parsec

$$\therefore 1 \text{ parsec} = \left(\frac{1 \text{ AU}}{1 \text{ arcsec}} \right)$$

$$1 \text{ deg} = 3600 \text{ arc sec}$$

$$\therefore 1 \text{ parsec} = \frac{\pi}{13600 \times 180} \text{ radians}$$

$$\therefore 1 \text{ parsec} = \frac{13600 \times 180}{\pi} \text{ AU} = 206265 \text{ AU} \approx 2 \times 10^5 \text{ AU}$$

(b) At 1 AU distance, Sun is $(1/2)$ degree in diameter.

Therefore, at 1 parsec, star is $\frac{1/2}{2 \times 10^5}$ degree in

diameter = 0.25×10^{-5} arc min

With 100 magnification, it should look 0.25×10^{-3} arc min. However, due to atmospheric fluctuations, it will still look of about 1 arc min.

\therefore It can't be magnified using telescope.

$$(c) \quad \frac{D_{mars}}{D_{earth}} = \frac{1}{2}, \quad \frac{D_{earth}}{D_{sun}} = \frac{1}{100} \quad [\text{Here, } D = \text{diameter}]$$

$$\frac{D_{sun}}{D_{moon}} = 400, \quad \frac{D_{earth}}{D_{moon}} = 4 \Rightarrow \frac{D_{mars}}{D_{sun}} = \frac{1}{2} \times \frac{1}{100}$$

At 1 AU Sun is seen as $1/2$ degree in diameter, and Mars will be seen as $1/400$ degree in diameter. i.e.,

$$\text{Mars diameter} = \frac{1}{2} \times \frac{1}{200} = \frac{1}{400} \text{ at } 1/2 \text{ AU,}$$

$$\text{Mars diameter} = \frac{1}{400} \times 2^\circ = \left(\frac{1}{200}\right)^\circ$$

At $1/2$ AU Mars will be seen as $1/400$ degree in diameter. With 100 magnification Mars will be seen

$$\text{Mars diameter} = \frac{1^\circ}{200} \times 100 = \left(\frac{1}{2}\right)^\circ = 30.$$

This is larger than resolution limit due to atmospheric fluctuations. Hence, it looks magnified.

- Q. 39. Einstein's mass energy relation emerging out of his famous theory of relativity relates mass (m) to energy (E) as $E = mc^2$, where c is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear**

level is usually measured in MeV, where $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$; the masses are measured in unified atomic mass unit (u) where $1u = 1.67 \times 10^{-27} \text{ kg}$.

(a) Show that the energy equivalent of $1 u$ is 931.5 MeV .

(b) A student writes the relation as $1 u = 931.5 \text{ MeV}$. The teacher points out that the relation is dimensionally incorrect. Write the correct relation. [NCERT Exemplar Q. No. 44]

Ans. (a) We can apply Einstein's mass energy relation in this problem, $E = mc^2$, to calculate the energy equivalent of the given mass.

Here,

$$1 \text{ amu} = 1u = 1.67 \times 10^{-27} \text{ kg}$$

Applying $E = mc^2$,

$$E = (1.67 \times 10^{-27})(3 \times 10^8)^2 \text{ J}$$

$$= 1.67 \times 9 \times 10^{-11} \text{ J}$$

$$\text{or } E = \frac{1.67 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV}$$

$$= 939.3 \text{ MeV}$$

$$\approx 931.5 \text{ MeV}$$

$$(b) \text{ As } E = mc^2 \Rightarrow m = \frac{E}{c^2}$$

$$\text{According to this } 1u = \frac{931.5 \text{ MeV}}{c^2}$$

Hence, the dimensionally correct relation is

$$1 \text{ amu} \times c^2 = 1u \times c^2$$

$$= 931.5 \text{ MeV}$$



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

- I. Read the following text and answer any 4 of the following questions on the basis of the same:**

Student "A" recorded 5 reading of measurement of length of a rod. The average value obtained was 2.6250 m. He rounded off it to 2.63 m. His teacher commented that the result was not correct. So, he repeated the experiment. This time he obtained the average value 2.6350 m. He rounded off it to 2.63 m. This time also his teacher commented that the result was not correct.

"A" had a peep into the readings of his friend "B". "B" obtained an average value 2.6750 m and rounded it off to 2.68 m. Teacher remarked that B's result was perfect.

- Q. 1. Mass of an object is 11.25 g and its volume is 2 cm^3 . What should be the most significant representation of its density?**

- (A) 5.625 g/cm^3 (B) 5.63 g/cm^3
(C) 5 g/cm^3 (D) 5.62 g/cm^3

Ans. Option (C) is correct.

Explanation: During multiplication and division the significant digit in the result should be equal to the lowest significant digit of the operands.

In this problem, significant digit of volume is 1 and it is the lowest. So, the most significant result should have 1 significant digit.

- Q. 2. What should be the correct result of student "A" after proper rounding off to two decimal places when he obtained an average value 2.6250?**

- (A) 2.62 (B) 2.63
(C) 2.60 (D) None of these

Ans. Option (A) is correct.

Explanation: If the digit from where the number is truncated is 5, the digit after it is 0 and the digit previous to it is even then the digit should remain unchanged.

In the problem, 2.6250 the number is to be truncated at 5. The digit after it is 0. The digit just before it is 2, which is an even digit. So, as per rule there should not be any change. The result after rounding off should be 2.62.

Q. 3. What should be the correct result of student "A" after proper rounding off to two decimal places when he obtained an average value 2.6350?

- (A) 2.60 (B) 2.63
(C) 2.64 (D) None of these

Ans. Option (C) is correct.

Explanation: If the digit from where the number is truncated is 5, digit after it is 0 and the digit previous to it is odd then the digit is to be increased by 1.

In the problem, 2.6350 the number is to be truncated at 5. The digit after it is 0. The digit just before it is 3, which is an odd digit. So, as per rule the digit 3 is to be increased by 1. The result after rounding off should be 2.64.

Q. 4. 3.6453 and 3.6554 are rounded off to two decimal places. The results will be

- (A) 3.64, 3.65 (B) 3.65, 3.66
(C) 3.65, 3.65 (D) 3.6, 3.6

Ans. Option (B) is correct.

Explanation: If the digit from where the number is truncated is 5. The digit after it is 3 in one number and 4 in other number i.e., both are non-zero digits. So, as per rule the digit before 5 is to be increased by 1.

Q. 5. 355.0 kg = 355000 g. Significant figures in the two numbers are

- (A) 4, 6 (B) 3, 6
(C) 3, 3 (D) 4, 4

Ans. Option (D) is correct.

Explanation: Number of significant figures in 355.0 kg is 4.

Decimal is removed and additional zeros have been put to convert it to gram.

In such case the significant figures do not increase due to zeros. It remains 4.

II. Read the following text and answer any 4 of the following questions on the basis of the same:

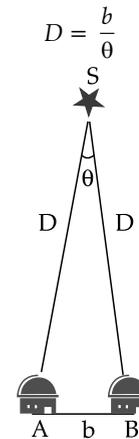
Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method.

When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax. The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes. To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance $AB = b$ at the same time as shown in Fig.

We measure the angle between the two directions along which the planet is viewed at these two points. The $\angle ASB$ in figure represented by symbol θ is called the parallax angle or parallactic angle.

As the planet is very far away, $\frac{b}{D} \ll 1$ and

therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius $AS = BS$ so that $AB = b = D\theta$ where θ is in radians, then



Q. 1. In parallax method the distance between two observation points is called

- (A) Base (B) Basis
(C) Distance (D) Basic

Ans. Option (B) is correct.

Explanation: The distance between the two observatories is the "basis".

Q. 2. A star at a distance 8×10^8 km is being observed from Vainu Bappu Observatory, Kavalur and ARIES Observatory, Nainital. The distance between the observatories is 2500 km. What is the "basis" of this measurement?

- (A) 2500 km
(B) 1250 km
(C) 5000 km
(D) Data is insufficient to calculate

Ans. Option (A) is correct.

Q. 3. A star at a distance 2.5×10^8 km is being observed from Vainu Bappu Observatory, Kavalur and ARIES Observatory, Nainital. The distance between the observatories is 2500 km.

What is the value of parallax angle?

- (A) 625×10^8 radian (B) 10^{-5} radian
(C) 10^5 radian (D) 10^{-5} degree

Ans. Option (B) is correct.

Explanation: $b = D\theta$

$b = 2500$ km, $D = 2.5 \times 10^8$ km

So, $\theta = \frac{b}{D} = \frac{2500}{2.5} \times 10^8 = 10^{-5}$ radian

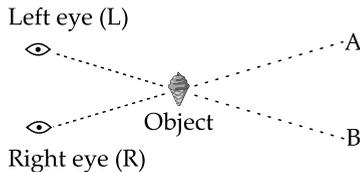
Q. 4. Under which condition $D = \frac{b}{\theta}$ relation is valid? ($D =$ Distance of star or planet from earth, $\theta =$ parallax angle, $b =$ basis)

- (A) $\frac{b}{D} \ll 1$ (B) $\frac{b}{D} \gg 1$
 (C) $\frac{D}{b} \ll 1$ (D) $\frac{D}{b} = 1$

Ans. Option (A) is correct.

Q. 5. When you hold an ice-cream cone in front of you against some specific point on the background (a wall) and look at it first through your left eye (closing the right eye), it seems to be at position B. Now if you look at it through your right eye (closing the left eye), you will notice that now it is at position A.

What is the "basis" of this observation?



- (A) AB (B) LR
 (C) $\frac{(LR + AB)}{2}$ (D) LR + AB

Ans. Option (B) is correct.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

In earlier time scientists of different countries were using different systems of units for measurement. Three such systems, the CGS, the FPS (or British) system and the MKS system were

in use extensively till recently. The system of units which is at present internationally accepted for measurement is SI system.

In SI, there are seven base units. Units of some physical quantities can be derived from these seven base units. Some common SI prefixes for multiples and sub-multiples are Giga, mega, kilo, pico, nano, micro, milli, centi etc.

Q. 1. Name the seven base quantities of SI system.

Ans. Seven base units of SI system are: Length, Mass, Time, Electric current, thermodynamic temperature, Amount of substance, Luminous intensity.

Q. 2. What are the base units of CGS and FPS system?

Ans. The base units for length, mass and time:

In CGS system centimetre, gram and second respectively.

In FPS system foot, pound and second respectively

Q. 3. Express $1\mu\text{m}$ and 1fm in meter.

Ans. $1\mu\text{m} = 10^{-6}\text{m}$
 $1\text{fm} = 10^{-15}\text{m}$

Q. 4. Define A.U. and light year.

Ans. A.U. is average distance of the Sun from the Earth.

Light year is the distance that light travels with velocity of $3 \times 10^8 \text{ m s}^{-1}$ in 1 year.

Q. 5. Mention three physical quantities having same dimensional formulae but different units.

Ans. Following quantities have same dimensional formula: $[M^0L^0T^{-1}]$. But they have different units:

- (i) Angular velocity (unit: rad s^{-1})
- (ii) Velocity gradient (Unit: s^{-1})
- (iii) Frequency (Unit: Hz)

UNIT-II KINEMATICS

Study Time
Maximum Time: 3 hours
Maximum Questions: 84

CHAPTER

2

MOTION IN A STRAIGHT LINE



Syllabus

Frame of reference. Motion in a straight line.

Elementary concepts of differentiation and integration for describing motion.

Uniform and non-uniform motion, average speed and instantaneous velocity.

Uniformly accelerated motion, velocity-time, position-time graphs.

Relations for uniformly accelerated motion (graphical treatment).

Topic-1 Motion & Velocity

TOPIC - 1

Motion & Velocity

.... P. 20

TOPIC - 2

Uniformly Accelerated Motion

.... P. 24



Revision Notes

➤ **Rest** : An object or a particle is said to be in the state of rest when it does not change its position with time w.r.t. same reference point.

Depending upon the position of observer, the state of rest of a particle is of two types :

- (a) Absolute state of rest,
- (b) Relative state of rest.

➤ **Motion** : An object or a particle is said to be in the state of motion when it changes its position with time w.r.t. same reference point.

The motion of an object can be either linear or curvilinear, circular or in a plane or in a space.

(a) Linear or Rectilinear or Translatory motion :

- (i) It is the motion in which a particle moves along a straight line with respect to a point of reference.
- (ii) A body is said to be in linear motion if every constituent particle of the body move along parallel straight line and covers same distance in the given time.

(b) Circular or Rotatory Motion :

- (i) A motion in which a particle or a point mass body is moving in a circle.
- (ii) In rotatory motion all its constituent particles move simultaneously along concentric circles.

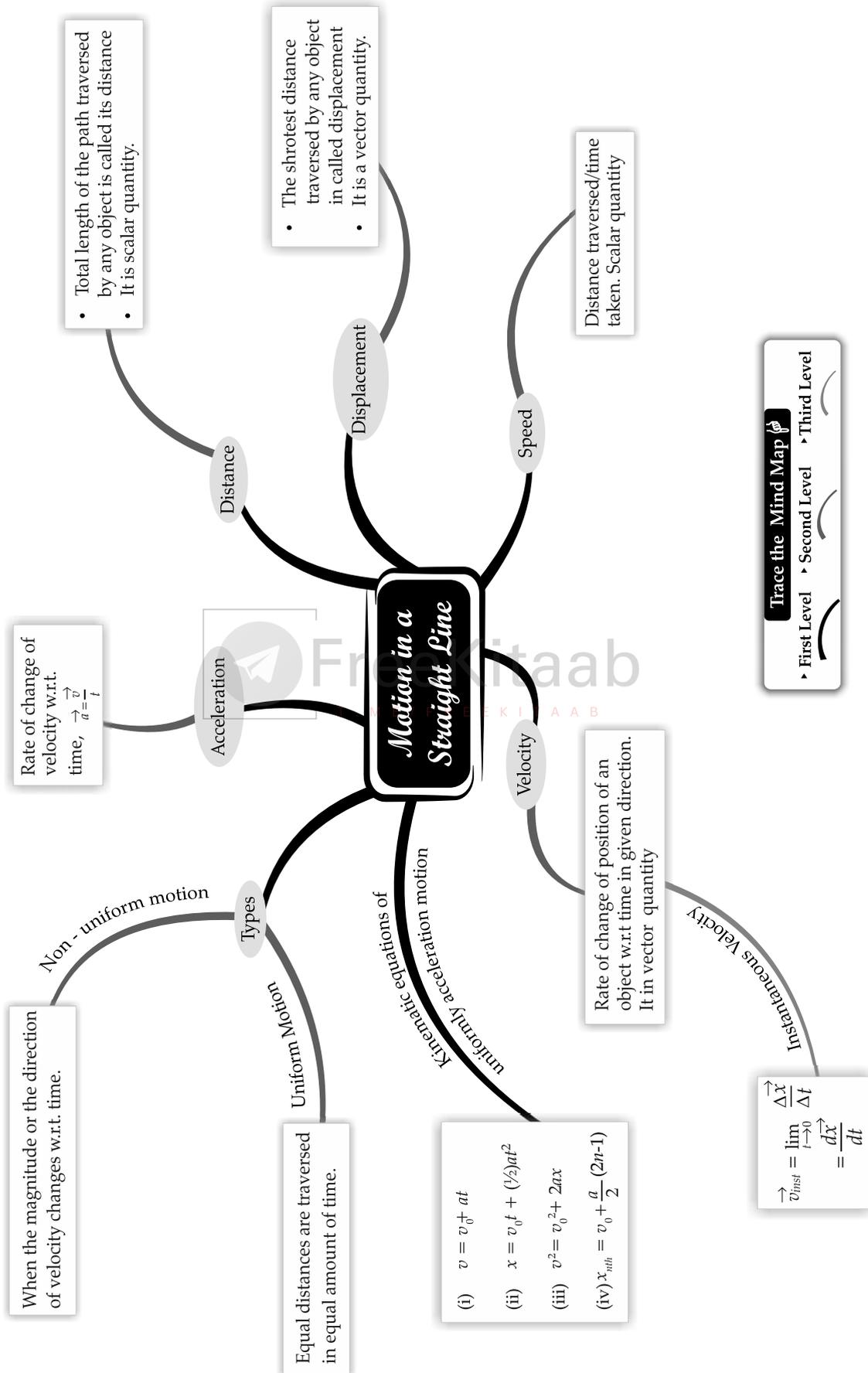
(c) Oscillatory or Vibratory Motion :

- (i) In oscillatory motion the body moves back & forth repeatedly in definite interval of time about a fixed point.
- (ii) If the amplitude of oscillating body is very small, the motion is called vibratory motion.

➤ **Dimensional Motion**

(a) Motion in 1-D :

- (i) It is that motion in which a particle moves in one particular direction with respect to a point of reference.
- (ii) In 1-D, the particle or a body moves along a straight line or a well defined path. Therefore, one dimensional motion is sometimes known as linear motion.



Trace the Mind Map

→ First Level → Second Level → Third Level

(b) Motion in 2-D

(i) If two out of three coordinates specifying the position of the object change with respect to time, the motion is called 2-D motion.

(c) Motion in 3-D

(i) If all the three coordinates specifying the position of the object change with respect to time, the motion is called 3-D motion.

➤ **Path Length or Distance :**

(a) Path Length is defined as the actual path traversed by body during motion in a given interval of time.

(b) Distance is a scalar quantity.

(c) The S.I. unit of distance is metre and C.G.S. unit is centimeter.

(d) The value of distance traversed by a moving body can never be zero or negative.

➤ **Displacement :**

(a) Displacement of a body in a given time is defined as the change in the position of the body in a particular direction during that time. It may also defined as the shortest distance between initial and final position of the object.

(b) Displacement is a vector quantity as it possesses both magnitude and direction.

(c) The unit of displacement is same as that of path length.

(d) The value of displacement can be positive, zero or negative.

(e) The value of displacement can never be greater than the distance travelled.

(f) When a moving body returns to its starting point, then its effective displacement is zero.

➤ **Difference between Distance & Displacement :**

S. No.	Distance	Displacement
1.	Actual path traversed by object in given time.	Shortest distance between initial & final positions of object in given time.
2.	Scalar quantity.	Vector quantity.
3.	It cannot be zero or negative, it will be always positive.	It can be positive, negative or zero.
4.	It is either equal or greater than displacement but never less than displacement.	It is either equal or less than distance but never greater than distance.
5.	It can have many values depending upon path followed between two positions.	It has unique value.
6.	Between two positions of an object, it tells type of path followed.	It does not tell type of path followed.

➤ **Instantaneous velocity :** If a body is moving with a variable velocity, then the velocity of the body at a given instant of time is called its instantaneous velocity.



Key Words

- Uniform motion is said to be in an object when velocity is uniform i.e., it undergoes equal displacements in equal intervals of time, howsoever small these intervals may be.
- Non-uniform motion is said to be in an object when it undergoes equal displacements in unequal intervals of time, howsoever small these intervals may be.

Mnemonics

Concept: Motion In A Straight Line
Mnemonics: Delhi to Vadodara via Tundla
Agra.

Interpretation: Displacement/time = Velocity
 Velocity / time = Acceleration

Key Formulae

- Path length or distance, $D = \text{Speed} \times \text{Time}$
- Displacement = Velocity \times Time



Very Short Answer Type Questions

(1 mark each)

Q. 1. Can you think of a situation where a body falling under the gravity has constant velocity? Give example. AI A

Ans. Yes, terminal velocity of a body. 1

Q. 2. Give an example of a motion which even though is accelerated motion yet it is called uniform motion. A

Ans. Uniform circular motion, in this body move constant speed but continuous changing direction. 1

Q. 3. How many dimensions of motion does the following have? A

- (a) Train moving fast on its track.
- (b) A lizard moving on a wall in a room.
- (c) Kite flying in the sky.
- (d) Bee flying in a closed room.

Ans. (a) One dimensional motion

(b) Two dimensional motion

(c) Three dimensional motion

(d) Three dimensional motion. 1

Q. 4. Why speed of an object cannot be negative? U

Ans. Speed of an object cannot be negative because the distance can never be negative. 1/2

Or as speed is a scalar quantity, so it can never be negative. 1/2

Q. 5. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero? AI U

Ans. No, it will not be zero because at highest point of vertical upward motion under gravity the instantaneous velocity is zero but its instantaneous acceleration is not zero. 1



Short Answer Type Questions-I

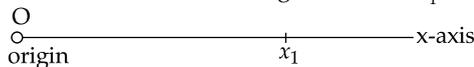
(2 marks each)

Q. 1. What is translational motion or one dimensional motion. AI R

Ans. In such motions the position given by only one of the three cartesian co-ordinates i.e., x or y or z changes with time, e.g.,

- (i) A fruit falling from a tree. 1/2
- (ii) A train running on a straight railway line. 1/2
- (iii) A car running on a narrow straight road etc. 1/2

In the given figure a particle moves from origin along x -axis so its co-ordinate changes from O to x_1 . 1/2

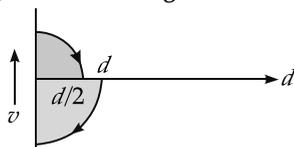


Q. 2. A man standing on the edge of a cliff throws a stone straight up with initial speed u and then throw another stone straight down with same initial speed u and from the same position. Find ratios of speeds, two stones would attain, when they hit ground at the base of the cliff. A

Ans. The stone thrown upward reaches back to the thrower with speed u . 1

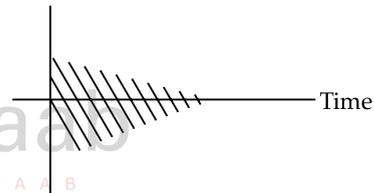
Thus both the stones fall under the influence of gravity with same initial velocity u so the two stones will hit the ground with same speed. Hence ratio of their speed when they hit ground is 1. 1

Q. 3. What can be represented by the graph given below, where d is height and v is velocity? U



Ans. This graph can be for a ball dropped vertically from a height d . It hits the ground with some downward velocity and bounces up to height $d/2$ where its upward velocity becomes zero. 2

Q. 4. Suggest a suitable physical situation for the following graph. A



Ans. Suitable physical situation – A ball is thrown up with some initial velocity. It is rebounding from the floor with reduced speed after each hit. 2

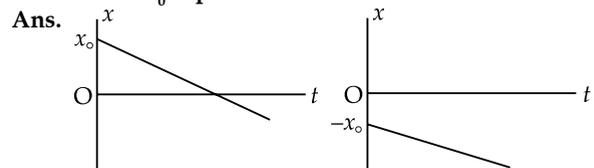
Q. 5. An object is in uniform motion along a straight line, what will be position time graph for the motion of object, if U

(i) $x_0 = \text{positive}$, $v = |\vec{v}|$ negative is constant.

(ii) both x_0 and v are negative $|\vec{v}|$ is constant.

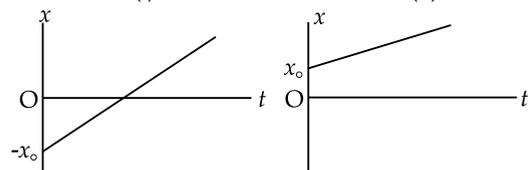
(iii) $x_0 = \text{negative}$, $v = \text{positive}$ $|\vec{v}|$ is constant.

(iv) both x_0 and v are positive $|\vec{v}|$ is constant. where x_0 is position at $t = 0$.



(i)

(ii)



(iii)

(iv)



Short Answer Type Questions-II

(3 marks each)

Q. 1. An object moving on a straight line covers first half of the distance at speed v and second half of the distance at speed $2v$. Find (i) average speed, (ii) mean speed. [A] [NCT 2011]

Ans. Let total distance be x .

$$\text{Distance of first half} = \frac{x}{2}$$

$$\text{speed} = v$$

$$\text{Time taken, } t_1 = \frac{\frac{x}{2}}{v} = \frac{x}{2v}$$

$$\text{Distance of second half} = \frac{x}{2}$$

$$\text{speed} = 2v$$

$$\text{time taken, } t_2 = \frac{\frac{x}{2}}{2v} = \frac{x}{4v} \quad 1$$

(i) Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{x}{\frac{x}{2v} + \frac{x}{4v}} = \frac{4v}{3} \quad 1$$

(ii) Mean speed = $\frac{v+2v}{2} = \frac{3v}{2} \quad 1$



Commonly Made Error

► Students may commit mistake while calculating the total time taken in the given problem.



Answering Tip

► Students should assume the distance appropriately and then calculate the required quantities by taking LCM correctly.



Long Answer Type Questions

(5 marks each)

Q. 1. Define displacement. What are its characteristics? [R]

Ans. Refer to 'Revision notes'. 5

Q. 2. Define the following terms : (a) Speed, (b) Uniform speed, (c) Variable speed, (d) Average speed, (e) Instantaneous speed, (f) Velocity, (g) Uniform velocity, (h) Variable velocity, (i) Uniform motion, (j) Instantaneous velocity. [R]

Ans. Refer to 'Revision notes'. 5

Q. 3. What are the important points about the uniform motion? [R]

Ans. Following are some important points about the uniform motion :

(i) The velocity in uniform motion does not depend upon the time interval $(t_2 - t_1)$. 1

(ii) The velocity in uniform motion is independent of the choice of origin. 1

(iii) No force acts on the object having uniform motion. 1

(iv) Velocity is taken to be positive when the object moves towards right of the origin and it is taken negative if the object moves towards left of the origin. 2

Topic-2 Uniformly Accelerated Motion



Revision Notes

► **Accelerated motion** : When an object is moving in non-uniform motion, the velocity is different at different instants i.e., the velocity keeps on changing with time. This motion is an accelerated motion.

► **Acceleration** : It is defined as the ratio of change in velocity & the corresponding time taken by the mirror object;

(a) It is vector quantity.

(b) It is either positive or negative.

(c) Negative acceleration is called retardation.

(d) Unit- m/s^2 in SI & cm/s^2 in CGS system.

- (e) Dimensional formula– [LT⁻²].
- (i) **Uniform acceleration** : An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.
- (ii) **Variable acceleration** :
- (a) An object is said to be moving with a variable acceleration when its velocity changes by unequal amounts in equal intervals of time.
- (b) The velocity time graph of a body having variable acceleration is represented by a curve.
- (iii) **Average acceleration** : When an object is moving with a variable acceleration, then the average acceleration of the body is defined as the ratio of the total change in velocity during motion to the total time taken, i.e.,

$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

(iv) **Instantaneous acceleration** :

- (a) If a body is moving with a variable acceleration, then the acceleration of a body at the given instant of time is called instantaneous acceleration.
- (b) If at an instant t , a body while moving with a variable acceleration shows a change in velocity $\Delta \vec{v}$ in a small interval of time Δt , so that $\Delta t \rightarrow 0$, then

$$\text{Instantaneous acceleration} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

where, $\frac{d\vec{v}}{dt}$ is the derivative of velocity (\vec{v}) w.r.t. time.



Key Terms

- Total displacement of the body in the given time is equal to the area which velocity time-graph encloses with time axis.
- Uniformly accelerated object in a given time-interval is represented by the slope on the velocity-time graph for a given time-interval.
- Acceleration of object is the slope of velocity-time graph of uniformly accelerated motion.



Key Formula

- Suppose
- u = initial velocity of body,
 a = uniform acceleration of the body,
 v = velocity of the body after time t ,
 s = distance travelled by body in time t ,
 s_n = distance travelled by body in n^{th} second.
- (a) **The equations of motion for accelerated body are :**

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2} at^2$

(iii) $v^2 = u^2 + 2as$

(iv) $s_n = u + \frac{a}{2} (2n - 1)$

- (b) **The equations of motion for retarded body (here, a is negative) are :**

(i) $v = u - at$

(ii) $s = ut - \frac{1}{2} at^2$

(iii) $v^2 = u^2 - 2as$

(iv) $s_n = u - \frac{a}{2} (2n - 1)$

- (c) **The equations of motion for a body falling down under gravity (here, $a = +g$, $s = h$) are :**

(i) $v = u + gt$

(ii) $h = ut + \frac{1}{2} gt^2$

(iii) $v^2 = u^2 + 2gh$

(iv) $h_n = u + \frac{g}{2} (2n - 1)$

Scan to know more about this topic



Uniformly accelerated motion

(d) The equations of motion for a body going up under gravity (here $a = -g$, $s = h$) are :

(i) $v = u - gt$ (ii) $h = ut - \frac{1}{2}gt^2$
 (iii) $v^2 = u^2 - 2gh$ (iv) $h_n = u - \frac{g}{2}(2n - 1)$

(e) The maximum height attained by a body thrown vertically upwards with initial velocity u is

$$h_{max} = \frac{u^2}{2g}$$

(f) Time taken to reach the maximum height is

$$t = \frac{u}{g}$$

(g) Total time taken by body in going up and coming down,

$$T = 2t = \frac{2u}{g}$$

(h) The initial velocity of body in order to attain height h is,

$$u = \sqrt{2gh}$$



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is de-acceleration or retardation ? R

Ans. Negative acceleration is called retardation or de-acceleration, i.e., when velocity of an object is steadily decreasing then the object is said to have retardation. 1

Q. 2. What is represented by the slope of $v-t$ graph ? U

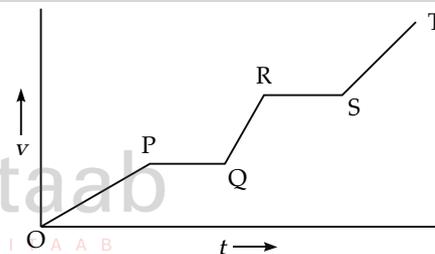
Ans. Slope of $v-t$ graph represent acceleration. 1

Q. 3. A stone is thrown vertically upwards from the surface of earth. What is the direction of the velocity and acceleration of the stone (a) on its upward motion, (b) on its downward motion ? A

Ans. (a) Velocity is vertically upward and acceleration is vertically downward. ½

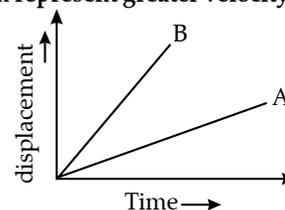
(b) Both velocity and acceleration are vertically downward. ½

Q. 4. Give the interval in which magnitude of maximum acceleration is available. U



Ans. Slope of $v-t$ graph gives acceleration. So, QR is the interval which gives maximum slope. Thus QR is the interval which gives maximum acceleration. 1

Q. 5. Which represent greater velocity. A [KVS 2013]



Ans. B, represent greater velocity because it has greater slope. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. What are positive and negative acceleration in straight line motion? R

Ans. Acceleration is positive when speed of an object increases with time.
 Acceleration is negative when speed of an object decreases with time. 2

Q. 2. Can a body have zero velocity and still be accelerating? If yes, give any situation. A I U

Ans. Yes, at the highest point of vertical upward motion under gravity. 2

Q. 3. The displacement of a body is proportional to t^3 , where t is time elapsed. What is the nature of acceleration-time graph of the body? U

Ans. Acceleration is directly proportional to time in the above situation, so a-t graph is a straight line. 2

Q. 4 What are the different methods to derive the equation for uniform acceleration motion? R

Ans. (i) From definitions of average velocity and acceleration.

(ii) From velocity time graphs.

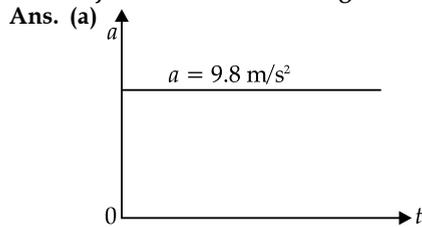
(iii) By using calculus. 2



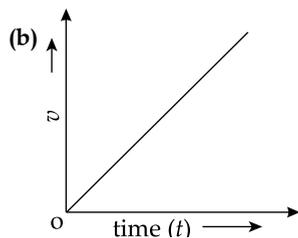
Short Answer Type Questions-II

(3 marks each)

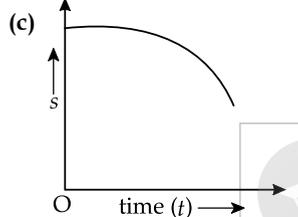
Q. 1. Draw (a) acceleration - time (b) velocity - time (c) position - time graphs representing motion of an object under free fall. Neglect air resistance. [A]



1

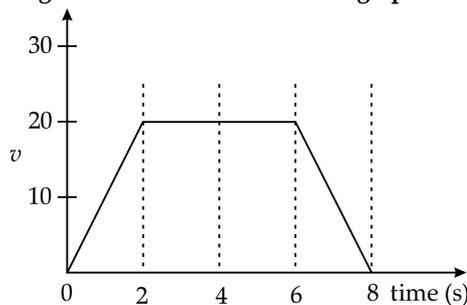


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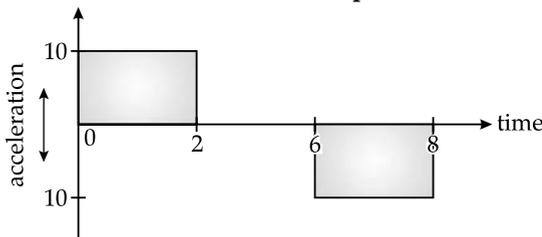


1

Q. 2. The velocity time graph for a particle is shown in figure. Draw acceleration time graph from it. [U]



Ans. Acceleration-Time Graph



3



Commonly Made Error

► Students sometimes draw a linear line for the time between 2 s to 6 s, above the time axis.

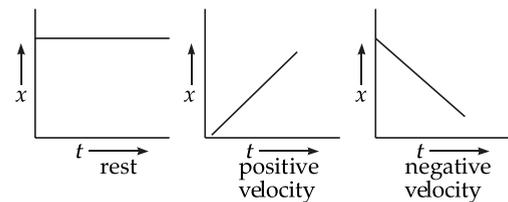


Answering Tip

► Students should first divide the graph into three portions, i.e., 0 to 2s, 2s to 6s and 6s to 8s. Then think about the change in the velocity as given in these intervals and finally, plot the acceleration-time graph for the same. For zero acceleration the graph should merge with the time axis.

Q. 3. Give position-time graphs for one object moving with negative velocity, moving with positive velocity and at rest. [A]

Ans.



(a)

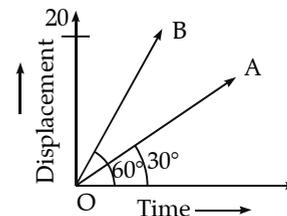
(b)

(c)

(1 mark each)

Q. 4. The two straight rays OA and OB on the same displacement-time graph make angle 30° and 60° with time axis respectively as shown in figure: [A]

- Which ray represents greater velocity?
- What is the ratio of two velocity represented by OA and OB?



Ans. (i) Since, the slope of the displacement-time graph of uniform motion in one dimension represents the velocity of the object, hence the line showing greater slope in graph corresponds to greater velocity of the object. Therefore OB represents greater velocity. 2

(ii) Ratio of two velocities

$$\frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

1



Long Answer Type Questions

(5 marks each)

Q. 1. Derive second equation of motion with the use of $v-t$ graph. [KVS 2013]

Ans. Graphical derivation of 2nd equation of motion :

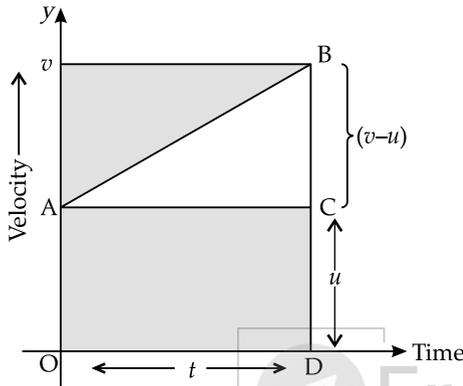
Distance travelled 's' = Area of trapezium $ABDO$ 1

Area of rectangle $ACDO$ + Area of ΔABC

$$s = OD \times OA + \frac{1}{2} \times BC \times AC$$

$$= t \times u + \frac{1}{2} (v - u)t \quad 1$$

$$= ut + \frac{1}{2} (v - u)t. \quad 1$$



According to 1st eqn. of motion

$$v = u + at$$

$$\therefore v - u = at$$

$$\therefore s = ut + \frac{1}{2} at^2 \quad 1$$



Commonly Made Error

- ▶ Students generally commit mistakes while using the formulas for the covered areas.



Answering Tip

- ▶ Students should first know how to derive the second equation of motion graphically, then use the formulas carefully to get the desired result.

Q. 2. Derive the relations :

(i) $v^2 - u^2 = 2as$ (ii) $v = u + at$

(iii) $s = ut + \frac{1}{2} at^2$ [MSE Chandigarh, 2009] 5

Ans. (i) Let

x_1, v_1 = position and velocity of the object at time t_1 respectively.

x_2, v_2 = position and velocity of the object at time t_2 .

a = uniform acceleration of the object.

$$v^2 - u^2 = 2as$$

Derivation—Acceleration is given by

$$a = \frac{v_2 - v_1}{t_2 - t_1}, \quad \frac{1}{2}$$

or $t_2 - t_1 = \frac{v_2 - v_1}{a} \quad \dots(i)$

Since, $x_2 - x_1 = v_1(t_2 - t_1) + \frac{1}{2} a(t_2 - t_1)^2 \dots(ii)$

From (i) and (ii), we get

$$\begin{aligned} x_2 - x_1 &= v_1 \frac{v_2 - v_1}{a} + \frac{1}{2} a \left[\frac{v_2 - v_1}{a} \right]^2 \\ &= \frac{v_1 v_2 - v_1^2}{a} + \frac{v_2^2 + v_1^2 - 2v_1 v_2}{2a} \\ &= \frac{2v_1 v_2 - 2v_1^2 + v_2^2 + v_1^2 - 2v_1 v_2}{2a} \end{aligned}$$

$$x_2 - x_1 = \frac{v_2^2 - v_1^2}{2a} \quad \dots(iii) \quad 1$$

or $v_2^2 - v_1^2 = 2a(x_2 - x_1) \quad \dots(iv)$

Now if $v_1 = u$ at $t_1 = 0$

$v_2 = v$ at $t_2 = t \quad \dots(v)$

$$x_2 - x_1 = s$$

Then from (iv) and (v), we get

$$v^2 - u^2 = 2as \quad \dots(vi) \quad \frac{1}{2}$$

$$v = u + at$$

Derivation—By definition of acceleration,

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

or $v_2 - v_1 = a(t_2 - t_1)$

or $v_2 = v_1 + a(t_2 - t_1) \quad \dots(i)$

where v_1 and v_2 are the velocities of an object at times t_1 and t_2 respectively.

If $v_1 = u$ (initial velocity of the object)

at $t_1 = 0$

$v_2 = v$ (final velocity of the object)

at $t_2 = t$.

Then (i) reduces to $v = u + at \quad 1$

(iii) $s = ut + \frac{1}{2} at^2$

Derivation—

Let v_{av} = average velocity in $t_2 - t_1$ interval.

\therefore By definition

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} \quad 1$$

or $x_2 - x_1 = v_{av}(t_2 - t_1) \quad \dots(i)$

Since, $v_{av} = \frac{v_1 + v_2}{2} \quad \dots(ii)$

\therefore From eqns. (i) and (ii),

$$x_2 - x_1 = \frac{v_1 + v_2}{2} (t_2 - t_1) \quad \dots(iii)$$

Also we know that

$$v_2 = v_1 + a(t_2 - t_1) \quad \dots(iv)$$

(d) a tumbling beaker slipped off the edge of a table.

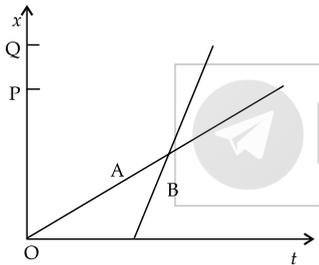
Ans. (a) Since, the motion of the train between two distant stations is smooth throughout, so keeping in view the long distance covered between the two stations in reasonable duration of time, the size of the train is neglected and it is considered as a point object.

(b) The distance covered by the monkey in reasonable duration of time is more so the monkey is considered as a point object. (Since its size is smaller.)

(c) Since, the turning of the ball is not smooth but sharp so the distance covered by it in reasonable duration of time is not large so this ball cannot be treated as a point object.

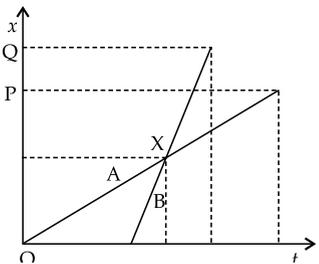
(d) Since, the beaker is tumbling and then it slips off, so the distance covered by it in reasonable duration of time is not large. Thus, it is not treated as a point object.

Q. 2 The position time ($x-t$) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose the correct entries in the brackets below:



- (a) (A/B) lives closer to the school than (B/A)
- (b) (A/B) starts from the school earlier than (B/A)
- (c) A and B reach home at the (same/different) time.
- (d) (A/B) walks faster than (B/A)
- (e) (A/B) overtakes (B/A) on the road (once/twice).

Ans.



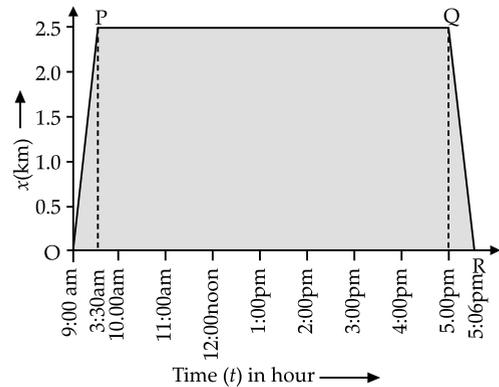
- (a) A lives closer to the school than B, because B has to cover longer distance [$OP < OQ$].
- (b) A starts from the school earlier than B because $t = 0$ for A but B has some finite value of time.
- (c) A and B reach home at different time.
- (d) B walks faster than A since, the slope of ($x-t$) curve for B is greater than that of A.
- (e) B overtakes A on the road once (at X, i.e., the point of intersection).

Q. 3. A woman starts from her home at 9.00 a.m., walks with a speed of 5 kmh^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 p.m. and returns home by an auto with a speed of 25 kmh^{-1} . Choose suitable scales and plot the $x-t$ graph of the motion.

Ans. $x-t$ graph of the motion of woman is shown in given figure.

Let $v_1 =$ speed of woman while going to office = 5 kmh^{-1}

$$x = \text{Distance covered by her} \\ = 2.5 \text{ km}$$



If $t_1 =$ time taken to reach office, then it can be calculated by using the formula

$$x = v_1 t_1$$

$$t_1 = \frac{x}{v_1}$$

Hence,
$$t_1 = \frac{2.5}{5} = \frac{1}{2} \text{ h} = 30 \text{ minutes.}$$

When O is regarded as origin for both time and distance, then at $t = 9.00 \text{ a.m.}$, $x = 0$, and at $t = 9.30 \text{ a.m.}$, $x = 2.5 \text{ km}$ and she reaches in her office. So, OP represents $x-t$ graph of the motion when the woman walks from her home to office.

When she stays at her office from 9.30 a.m. to 5.00 p.m., she is at rest and her stay is represented by the straight line PQ in the graph.

On return, speed of auto, $v_2 = 25 \text{ km/h.}$

Let, $t_2 =$ time taken by her, i.e., by auto from office to her home, then

$$t_2 = \frac{x}{v_2} = \frac{2.5}{25} = \frac{1}{10} \text{ hour} \\ = 6 \text{ minutes.}$$

So, she reaches back to her home at 5.06 p.m.

Her motion on the return journey is shown by QR part of the graph.

Scale taken

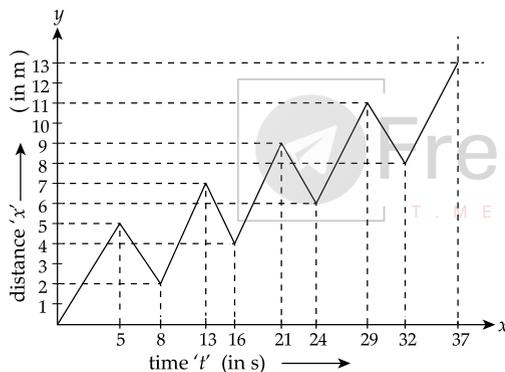
Time on x -axis, 1 division = 1 hour.

Distance on y -axis, 1 division = 0.5 km.

Q. 4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed

again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the $x-t$ graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Ans. Distance covered with 1 step = 1 m
 Time taken = 1 s
 Time taken to move first 5 m forward = 5 s
 Time taken to move 3 m backward = 3 s
 Net distance covered = 5 - 3 = 2 m
 Net time taken to cover 2 m = 8 s
 Drunkard covers 2 m in 8 s.
 Drunkard covers 4 m in 16 s.
 Drunkard covers 6 m in 24 s.
 Drunkard covers 8 m in 32 s.
 In the next 5 s, the drunkard covers a distance of 5 m and a total distance of 13 m and falls into the pit.
 Net time taken by the drunkard to cover 13m = 32 + 5 = 37 s
 The $x-t$ graph of the drunkard's motion can be shown as:



Q. 5. A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop? [NCERT TB Q. No. 6]

Ans. Initial velocity of car,
 $u = 126 \text{ kmh}^{-1}$
 $= 126 \times \frac{5}{18} \text{ ms}^{-1}$
 $= 35 \text{ ms}^{-1}$... (i)

Since, the car finally comes to rest, $v = 0$
 Distance covered, $s = 200 \text{ m}$, $a = ?$, $t = ?$

From equation of motion
 $v^2 = u^2 - 2as$
 or, $a = \frac{v^2 - u^2}{2s}$... (ii)

Substituting the values from eq. (i) in eq. (ii)

$$a = \frac{0 - (35)^2}{2 \times 200} = - \frac{35 \times 35}{400}$$

$$= - \frac{49}{16} \text{ ms}^{-2}$$

$$= - 3.06 \text{ ms}^{-2}$$

Negative sign shows that acceleration is negative which is called retardation, i.e., car is uniformly retarded at $-a = 3.06 \text{ ms}^{-2}$.

To find t , let us use the relation

$$v = u + at$$

$$\therefore t = \frac{v - u}{a}$$

Here, $a = - 3.06 \text{ ms}^{-2}$, $v = 0$, $u = 35 \text{ ms}^{-1}$.

$$\therefore t = \frac{v - u}{a} = \frac{0 - 35}{-3.06} = 11.44 \text{ s}$$

$$\therefore t = 11.44 \text{ sec.}$$

Q. 6. A player throws a ball upwards with an initial speed of 29.4 ms^{-1} .

- What is the direction of acceleration during the upward motion of the ball ?
- What are the velocity and acceleration of the ball at the highest point of its motion ?
- Choose the $x = 0 \text{ m}$ and $t = 0 \text{ s}$ to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x -axis, and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.
- To what height does the ball rise and after how long does the ball return to the player's hands ? (Take $g = 9.8 \text{ ms}^{-2}$ and neglect air resistance.)

[NCERT TB Q. No. 10]

Ans. (a) As the ball is moving under the effect of gravity, the direction of acceleration due to gravity remains vertically downwards.

(b) If the ball is at the highest point of its motion, its velocity becomes zero and the acceleration is equal to the acceleration due to gravity = 9.8 ms^{-2} in vertically downward direction.

(c) The highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of x -axis.

For upward motion, sign of position is positive, sign of velocity is negative and the sign of acceleration is positive, i.e., $v < 0$, $a > 0$.

For downward motion, sign of position is positive, sign of velocity is positive and the sign of acceleration is also positive, i.e., $v > 0$, $a > 0$.

(d) Suppose, $t =$ time taken by the ball to reach the highest point.

$H =$ height of the highest point from the ground.
 During vertically upward motion of the ball,

\therefore Initial velocity,

$$u = 29.4 \text{ ms}^{-2},$$

$$g = 9.8 \text{ ms}^{-2},$$

Final velocity $v = 0, h = H = ?, t = ?$

Applying the relation,

$$v^2 = u^2 - 2gh$$

$$v^2 - u^2 = -2gh$$

$$0^2 - (29.4)^2 = -2 \times 9.8 H$$

or
$$H = \frac{29.4 \times 29.4}{2 \times 9.8} = 44.1 \text{ m}$$

During upward motion,

$$v = u - gt$$

$$0 = 29.4 + 9.8 \times t$$

$\therefore t = \frac{29.4}{9.8} = 3 \text{ s.}$

i.e., time of ascent = 3 s.

It is also known that when the object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

\therefore Total time after which the ball returns to the player's hand = $2t = 2 \times 3 = 6 \text{ s.}$

Q. 7. Read each statement below carefully and state with reasons and examples, if it is correct or wrong. A particle in one-dimensional motion.

- (a) with zero speed at an instant may have non-zero acceleration at that instant.
- (b) with zero speed may have non-zero velocity.
- (c) with constant speed must have zero acceleration.
- (d) with positive value of acceleration must be speeding up.

[NCERT TB Q. No. 11]

Ans. (a) Correct. Example, if a ball is in vertically upward motion, i.e., thrown vertically upward, then it will be of zero speed at the highest point and an acceleration of 9.8 ms^{-2} in downward direction.

- (b) Incorrect, because speed is the magnitude of velocity.
- (c) Incorrect change in direction of velocity also produces acceleration.
- (d) Incorrect

Example, if we take the positive direction of acceleration due to gravity downwards, a ball dropped from a tower will gain speed continuously.

Q. 8 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed – time graph of its motion between $t = 0$ to 12 s. ($g = 10 \text{ ms}^{-2}$)

[NCERT TB Q. No. 12]

Ans. Given, $h = 90 \text{ m}, u = 0, g = 10 \text{ m/s}^2,$

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 90}{10}} = 3\sqrt{2} = 4.24 \text{ s.}$$

$$v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 90} = 30\sqrt{2} \text{ m/s} = 42.43 \text{ m/s.}$$

In each collision, ball loses one tenth of its speed,

$$\begin{aligned} \therefore u_2 &= \frac{9}{10} v_1 = \frac{9}{10} \times 30\sqrt{2} \\ &= 27\sqrt{2} \\ &= 38.2 \text{ m/s} \end{aligned}$$

Time taken to reach highest point

$$t_2 = \frac{u_2}{g} = \frac{27\sqrt{2}}{10} = 3.82 \text{ s.}$$

It will take same time to reach the ground i.e.,

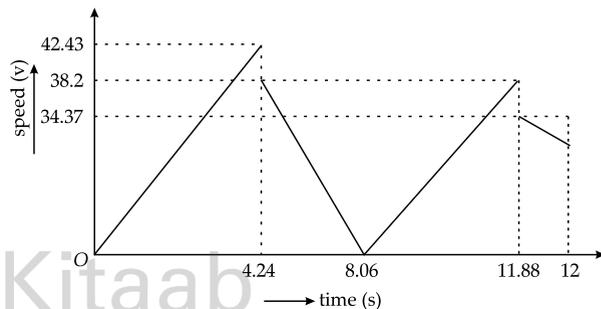
$$t_3 = 3.82 \text{ s.}$$

Final velocity will be same as the initial velocity i.e., 38.2 m/s.

Now,

$$\begin{aligned} u_3 &= \frac{9}{10} u_2 \\ &= \frac{9}{10} \times 38.2 \\ &= 34.37 \text{ m/s.} \end{aligned}$$

$$\begin{aligned} \text{Total time lapsed} &= t_1 + t_2 + t_3 \\ &= 4.24 + 3.82 + 3.82 \\ &= 11.88 \text{ s} \end{aligned}$$



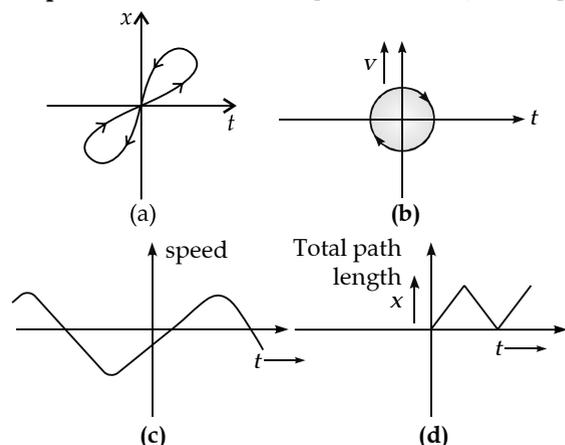
Q. 9. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

[NCERT TB Q. No. 15]

Ans. Instantaneous velocity is given by the first derivative of distance with respect to time i.e., here the time interval ' dt ' is so small that it is assumed that an object does not change its direction of motion. As a result both the total path length and magnitude of displacement become equal in this interval of time. Therefore instantaneous speed is always equal to instantaneous velocity.

Q. 10. Look at the graphs (a) to (d) (fig.) carefully and state with reasons which of the following cannot possibly represent one dimensional motion of a particle.

[NCERT TB Q. No. 16]



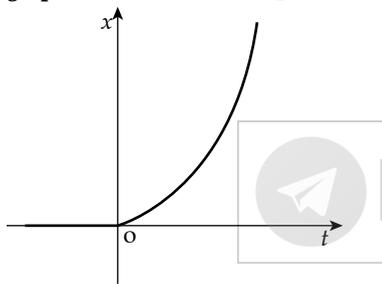
Ans. Figure (a) does not represent one dimensional motion of particle because the particle has two different positions at the same instant which is not the case of one dimensional motion. Figure

Graph (b) does not represent one dimensional motion because at the same instant a particle cannot have positive and negative velocity if the motion is one dimensional.

Graph (c) does not represent one dimensional motion because speed cannot be negative as shown in graph.

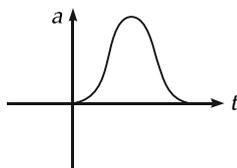
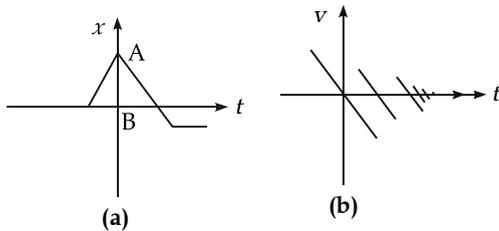
Graph (d) also does not represent one dimensional motion of the particle because here the total path length is shown to decrease with time which is not possible in one dimensional motion.

Q. 11. Figure shows the $x-t$ plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph. [NCERT TB Q. No. 17]



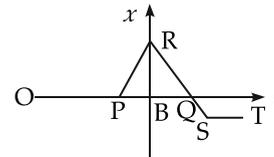
Ans. No, because the $x-t$ graph does not represent the trajectory of the path followed by a particles. From the graph, a suitable physical context can be the particle dropped from the top of the tower at $t = 0$ and $x = 0$

Q. 12. Suggest a suitable physical situation for each of the following graphs : [NCERT TB Q. No. 19]



Ans. (a) A ball at rest on a smooth floor is kicked, OP, shows it at rest position, at R the ball rebounds from the wall. RQ represents the bounced ball with reduced speed because the slope of RQ is less than that of PR. (The slope of $x-t$ graph gives

the speed of the moving body), ST shows the rest position of the ball.

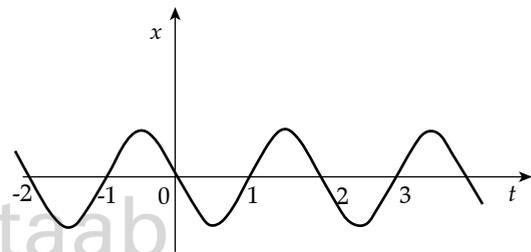


(b) The graph shows the case of a ball thrown up with some initial velocity and rebounding from the floor with reduced speed after each hit every time on the floor, i.e., after every bounce.

(c) It is the acceleration-time graph. The graph represents a uniformly moving cricket ball turned back after hitting the bat for a very short time interval.

Q. 13. Figure gives the $x-t$ plot of a particle executing one-dimensional simple harmonic motion.

(You will learn about this motion in more detail in Chapter 14). Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3$ s, 1.2 s, -1.2 s. [NCERT TB Q. No. 20]



Ans. Negative, Negative, Positive
Positive, Positive, Negative
Negative, Positive, Positive

For simple harmonic motion (SHM) of a particle, the relation of acceleration (a) with angular frequency (ω) as

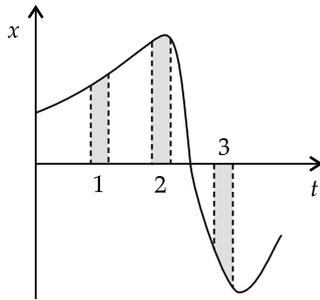
$$a \propto -\omega^2 x \quad \dots(i)$$

(i) At $t = 0.3$ s time interval, x is negative. Thus, the slope of the $x-t$ plot will be positive. Therefore, both position and velocity are negative. However, using equation (i), acceleration of the particle will be positive.

(ii) At $t = 1.2$ s time interval, x is positive. Thus, the slope of the $x-t$ plot will also be positive. Therefore, both position and velocity are positive. However, using equation (i), acceleration of the particle comes to be negative.

(iii) At $t = -1.2$ s time interval, x is negative. Thus, the slope of the $x-t$ plot will also be negative. Since both x and t are negative, the velocity comes to be positive. From equation (i), it can be inferred that the acceleration of the particle will be positive.

Q. 14. Figure gives $x-t$ plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval. [NCERT TB Q. No. 21]



Ans. Average speed in small interval of time is equal to slope of x - t graph in same interval.

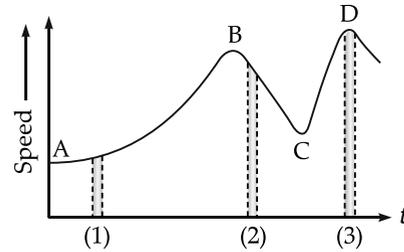
Average speed is greatest in interval 3 because slope is greatest in this interval.

Average speed is least in interval 2 because slope is least in this interval.

Average speed is positive in intervals 1 and 2 and negative in interval 3.

- Q. 15.** Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. (i) In which interval the average acceleration greatest in magnitude? (ii) In which interval is the average speed greatest? (iii) Choosing the positive direction as the constant direction of motions give the signs of v and a in the three intervals. (iv) What are the accelerations at the points A, B, C and D?

[NCERT TB Q. No. 22]



Ans. (i) The magnitude of the average acceleration is given by $a_{av} = \frac{\text{Change in speed}}{\text{Time interval}}$

i.e., average acceleration in a small interval of time is equal to the slope of v - t graph in that time interval.

As the slope of v - t graph is maximum in the interval 2 as compared to intervals 1 and 3, hence the magnitude of average acceleration is greatest in interval 2.

(ii) The average speed is greatest in the interval 3 as peak D is at maximum on speed axis.

(iii) $v > 0$, i.e., positive in all the three intervals. The slope is positive in intervals 1 and 3, so ' a ', i.e. acceleration is positive in these intervals while the slope is negative in interval 2, so acceleration is negative in it.

$a > 0$, i.e., positive in interval 1 and 3. and, $a < 0$, i.e., negative in interval 2.

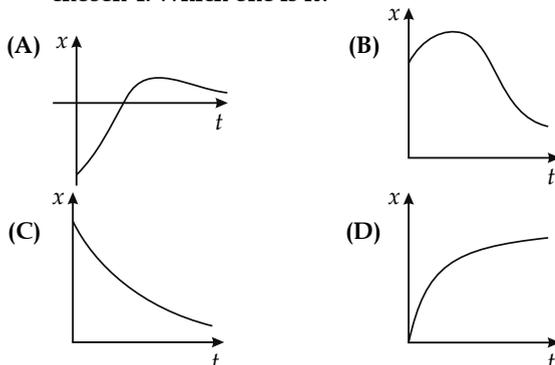
(iv) As speed is constant at points A, B, C and D, hence the acceleration is zero at all the four points.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

- Q. 1.** Among the four graphs shown below, there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for a suitably chosen T . Which one is it?



Ans. Option (B) is correct.

Explanation: In (B) for the value of displacement, two timings are there. Therefore for one time, the average velocity is positive and for other time is equal but negative. Due to this average velocity can vanish.

- Q. 2.** A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and

positive direction upwards for all quantities, which one of the following is correct?

- (A) $x < 0, v < 0, a > 0$
(B) $x > 0, v < 0, a < 0$
(C) $x > 0, v < 0, a > 0$
(D) $x > 0, v > 0, a < 0$

Ans. Option (A) is correct.

Explanation: Lift is coming downwards and about to stop at 4th floor, so it's retarding. Thus, a is acting downwards, so $a > 0$ and the value of x becomes less here negative, i.e., $x < 0$, velocity is downwards (i.e., negative) so $v < 0$.

- Q. 3.** In one dimensional motion, instantaneous speed v satisfies $0 \leq v < v_0$

- (A) The displacement in time T must always take non-negative values.
(B) The displacement x in time T satisfies $-v_0 T < x < v_0 T$.
(C) The acceleration is always a non-negative number.
(D) The motion has no turning points.

Ans. Option (B) is correct.

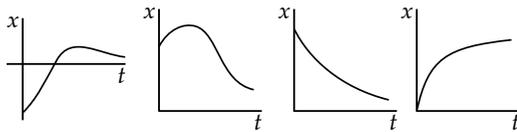
Explanation: Since, Maximum distance covered in time $T = v_0 T$. That's why, for the object having one dimensional motion, the displacement x in time T satisfies $-v_0 T < x < v_0 T$.



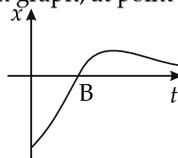
Very Short Answer Type Questions

Q. 4. Refer to the graphs given below. Match the following [NCERT TB Q. No. 12]

Graph	Characteristics
(a)	(i) has $v > 0$ and $a < 0$ throughout.
(b)	(ii) has $x > 0$ throughout and has a point with $v = 0$ and a point with $a = 0$.
(c)	(iii) has a point with zero displacement for $t > 0$.
(d)	(iv) has $v < 0$ and $a > 0$.



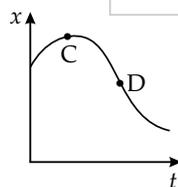
Ans. (a) (iii) From graph, at point B, $x = 0$ for $t > 0$.



(b) (ii) From graph, throughout the motion $x > 0$.

At point C, $\frac{dx}{dt} = v = 0$

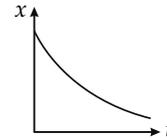
Since, at D, curvature changes, hence $a = 0$



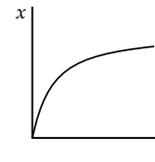
(c) (iv) From graph, $\frac{dx}{dt} =$ negative ; So, $v < 0$;

$a = \frac{d^2x}{dt^2}$ is positive.

Since, rate of change of negative velocity decreases. So, $a > 0$

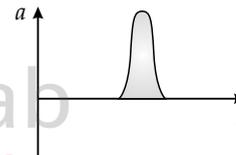


(d) (i) This graph is reverse of the graph above $v > 0$ but $a < 0$.



Q. 5. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in the backward direction as positive). [NCERT TB Q. No. 13]

Ans. Variation of ball's acceleration with time-



Q. 6. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where $g =$ gravitational acceleration and b is constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed? [NCERT TB Q. No. 16]

Ans. When $a = 0$, velocity of the particle becomes constant i.e., $v = \text{constant} = v_c$

Then from given equation $a = g - bv$,

$$0 = g - bv_c \Rightarrow v_c = g/b.$$



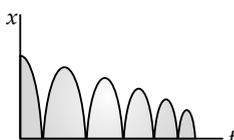
Short Answer Type Questions

Q. 7. A ball is dropped and its displacement vs time graph is as shown in figure (displacement x is from ground and all quantities are +ve upwards).

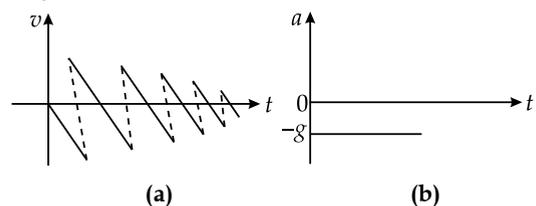
(a) Plot qualitatively velocity vs time graph.

(b) Plot qualitatively acceleration vs time graph.

[NCERT Exemplar Q. No. 17]



Ans. Velocity will go on reducing but the acceleration ($-g$) will remain same.



Q. 8. A particle executes the motion described by $x(t) = x_0(1 - e^{-t})$; $t \geq 0, x_0 > 0$.

- (a) Where does the particles start and with what velocity?
 (b) Find maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increase with time and $v(t)$ decreases with time.

[NCERT Exemplar Q. No. 18]

Ans. As $x(t) = x_0(1 - e^{-\gamma t})$
 For velocity,

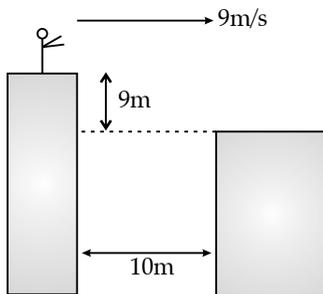
$$v(t) = \frac{dx(t)}{dt} = +x_0\gamma e^{-\gamma t}$$

For acceleration,

$$a(t) = \frac{dv(t)}{dt} = -x_0\gamma^2 e^{-\gamma t}$$

- (a) When $t = 0$; $x(t) = x_0(1 - e^{-0}) = x_0(1 - 1) = 0$
 $v(t = 0) = x_0\gamma e^{-0} = x_0\gamma(1) = \gamma x_0$
- (b) (i) $x(t)$ is maximum, When $t = -\infty$, $x(t) = x_0$
 $x(t)$ is minimum, when $t = 0$, $x(t) = 0$
- (ii) $v(t)$ is maximum, when $t = 0$, $v(0) = x_0\gamma$
 $v(t)$ is minimum, when $t = \infty$, $v(t) = 0$
- (iii) $a(t)$ is maximum, when $t = \infty$, $a(t) = 0$
 $a(t)$ is minimum, when $t = 0$, $a(t) = -x_0\gamma^2$

Q. 9. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of at lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building? (take $g = 10 \text{ m/s}^2$) [NCERT Exemplar Q. No. 20]



Ans. Horizontal speed of man, $u_x = 9 \text{ m/s}$
 Horizontal distance between two buildings, $x = 10 \text{ m}$
 Height difference between heights of two buildings, $h = 9 \text{ m}$
 $g = 10 \text{ m/s}^2$
 Suppose t is the time taken by the man to fall vertically downward by a height h
 $h = u_y t + \frac{1}{2} g t^2$
 $9 = 0 \times t + \frac{1}{2} \times 10 \times t^2 \Rightarrow 5t^2 = 9$
 $t = \sqrt{\frac{9}{5}} = 1.346 \text{ s}$

If the distance covered by the man along x -axis during this time t is x' , then

$$x' = u_x t = 9 \times 1.34 = 12.06 \text{ m}$$

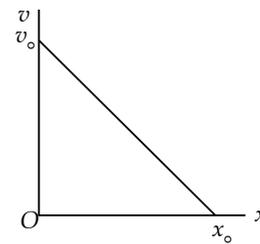
Here, $x' > x$

So, man will land successfully from building A to building B.

Q. 10. The velocity-displacement graph of a particle is shown in figure.

- (a) Write the relation between v and x .
 (b) Obtain the relation between acceleration and displacement and plot it.

[A I] [NCERT Exemplar Q. No. 22]



Ans. (a) Initial velocity = v_0
 Let the distance travelled in time $t = x_0$

From graph, $\tan \theta = \frac{v_0 - v}{x_0 - x}$

$$v = (-v_0/x_0) x + v_0$$

(b) $a = \frac{dv}{dt}$

or, $a = \frac{d}{dt} \left(-\frac{v_0}{x_0} x + v_0 \right)$

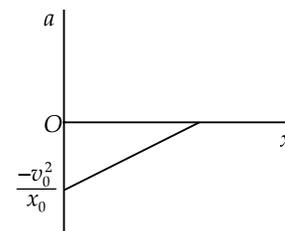
or, $a = \left(-\frac{v_0}{x_0} \frac{dx}{dt} + 0 \right)$

or, $a = -\frac{v_0}{x_0} v$

or, $a = -\frac{v_0}{x_0} \left(-\frac{v_0}{x_0} x + v_0 \right)$

$\therefore a = \frac{v_0^2}{x_0^2} x - \frac{v_0^2}{x_0}$

The variation of a with x is shown in the figure. It is a straight line with a positive slope and a negative intercept.





Long Answer Type Questions

Q. 11. It is a common observation that rain clouds can be at about a kilometre altitude above the grounds.

- If a rain drop falls from such a height freely under gravity, what will be its speed? Also calculate in km/h. ($g = 10 \text{ m/s}^2$)
- A typical rain drop is about 4 mm diameter. Momentum is mass \times speed in magnitude. Estimate its momentum when it hits ground.
- Estimate the time required to flatten the drop.
- Rate of change of momentum is force. Estimate how much force such a drop would exert on you.
- Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.

(Assume that umbrella is circular and has a diameter of 1 m and cloth is not pierced through !!) [NCERT Exemplar Q. No. 23]

Ans. (a) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1000} = 141 \text{ m/s}$
 $= 507.6 \text{ km/h.}$

(b) $m = \frac{4\pi}{3} r^3 \rho = \frac{4\pi}{3} (2 \times 10^{-3})^3 (10^3)$
 $= 3.4 \times 10^{-5} \text{ kg.}$
 $= 3.4 \times 10^{-3} \times 141$

$p = mv \approx 4.7 \times 10^{-3} \text{ kg m/s} \approx 5 \times 10^{-3} \text{ kg m/s.}$

(c) Diameter $\approx 4 \text{ mm}$

$\Delta t \approx d/v = 28 \mu\text{s} \approx 30 \mu\text{s}$

(d) $F = \frac{\Delta p}{\Delta t} = \frac{4.7 \times 10^{-3}}{28 \times 10^{-6}} \approx 168 \text{ N} = \pi(1)^2/4$

(e) Area of cross-section $= \pi d^2/4 = \pi(1)^2/4.$

With average separation of 5 cm, no. of drops that will fall almost simultaneously is $\frac{0.8 \text{ m}^2}{(5 \times 10^{-2})^2} \approx 320.$

Net force $= 320 \times 160 \approx 54000 \text{ N}$

Q. 12. A motor car moving at a speed of 72 km/h can not come to a stop in less than 3.0 s while for a truck this time interval is 5.0 s. On a highway the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump into (collide with) the truck. Human response time is 0.5 s.

(Comment : This is to illustrate why vehicles carry the message on the rear side "Keep safe Distance") [NCERT Exemplar Q. No. 24]

Ans.



A is the position of car

B is the position of the truck.

Initial velocity of car $= u_C = \frac{72 \text{ km}}{\text{h}} = \frac{20 \text{ m}}{\text{s}}$

Initial velocity of truck $= u_T = \frac{72 \text{ km}}{\text{h}} = \frac{20 \text{ m}}{\text{s}}$

Truck stops in 5s.

So, its acceleration $= \frac{(v_T - u_T)}{t}$

Or, $a_T = \frac{(0 - 20)}{5}$

$\therefore a_T = -\frac{4 \text{ m}}{\text{s}}$

Car stops in 3s.

So, its acceleration $= \frac{(v_C - u_C)}{t}$

Or, $a_C = \frac{(0 - 20)}{3}$

$\therefore a_C = -\frac{20 \text{ m}}{3 \text{ s}}$

Say after t s, after applying the brake by the truck, the velocities of car and truck become equal at point C to avoid collision.

Velocity of truck after t s is $20 - 4t$

In case of car, the brake is applied after 0.5s. So,

velocity of car after t s is $20 - \left(\frac{20}{3}\right)(t - 0.5)$

Equating these two velocities,

$20 - 4t = 20 - \left(\frac{20}{3}\right)(t - 0.5)$

Or $3t = 5t - 5 \times 0.5$

$\therefore t = 1.25 \text{ s}$

Distance travelled by truck, after applying brake,
 $BC = u_C t + \frac{1}{2} a_T t^2$

Or, $BC = 20 \times 1.25 - \frac{1}{2} \times 4 \times (1.25)^2$

$\therefore BC = 21.875 \text{ m}$

Distance travelled by car, after applying brake,
 $DC = u_C(t - 0.5) + \frac{1}{2} a_C(t - 0.5)^2$ {since, brake is applied after 0.5 s that of truck}

Or, $DC = 20 \times (1.25 - 0.5) - \frac{1}{2} \times \left(\frac{20}{3}\right)$

$\times (1.25 - 0.5)^2$

$\therefore DC = 13.125 \text{ m}$

Distance moved by car due to human response time $= AD = 20 \times 0.5 = 10 \text{ m}$

$\therefore AC = AD + DC = 10 + 13.125 = 23.125 \text{ m}$

And $BC = 21.875 \text{ m}$

So, Minimum distance between the car and the truck $= AB = AC - BC = 23.125 - 21.875 = 1.25 \text{ m}$

Q. 13. A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time t is given by $v(t) = 2t(3 - t)$; $0 < t < 3$ and $v(t) = -(t - 3)(6 - t)$ for $3 < t < 6$ s in m/s. It repeats this cycle till it reaches the height of 20 m.

- (a) At what time is its velocity maximum?
 (b) At what time is its average velocity maximum?
 (c) At what time is its acceleration maximum in magnitude?
 (d) How many cycles (counting fractions) are required to reach the top?

[NCERT Exemplar Q. No. 25]

Ans. Given, $v(t) = 2t(3 - t) = 6t - 2t^2$

- (a) For maximum velocity,

$$\frac{dv(t)}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}(6t - 2t^2) = 0$$

$$\Rightarrow 6 - 4t = 0 \Rightarrow t = \frac{6}{4} = \frac{3}{2} \text{ s} = 1.5 \text{ s}$$

- (b) $\frac{ds}{dt} = 6t - 2t^2$

$$ds = (6t - 2t^2) dt \quad [s = \text{displacement}]$$

\therefore Distance travelled in time interval 0 to 3s.

$$s = \int_0^3 (6t - 2t^2) dt$$

$$= \left[\frac{6t^2}{2} - \frac{2t^3}{3} \right]_0^3 = \left[3t^2 - \frac{2}{3}t^3 \right]_0^3$$

$$= 3 \times 9 - \frac{2}{3} \times 3 \times 3 \times 3$$

$$= 27 - 18 = 9 \text{ m.}$$

$$\text{Average velocity} = \frac{9}{3} = 3 \text{ m/s.}$$

As,

$$v_{\text{avg}} = 6t - 2t^2 \Rightarrow 3 = 6t - 2t^2 \Rightarrow 2t^2 - 6t + 3 = 0$$

$$\Rightarrow t = \frac{6 \pm \sqrt{6^2 - 4 \times 2 \times 3}}{2 \times 2} = \frac{6 \pm \sqrt{36 - 24}}{4}$$

$$= \frac{6 \pm \sqrt{12}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

$$\text{Taking '+'ve sign we get } t_1 = \frac{3 + \sqrt{3}}{2} = 2.36 \text{ sec.}$$

$$\text{Taking '-'ve sign we get } t_2 = \frac{3 - \sqrt{3}}{2} = 0.633 \text{ sec}$$

which is less than the least of the clock so it cannot be measured,

\therefore rejecting t_2 , the average velocity is maximum at 2.36 sec.

- (c) Acceleration will be maximum when velocity is zero

$$\therefore (6t - 2t^2) = 0$$

$$t(6 - 2t) = 0$$

$$t = 0, 3 \text{ sec}$$

$$(d) s_1 = \int_0^3 (6t - 2t^2) dt$$

$$= \left[\frac{6t^2}{2} - \frac{2t^3}{3} \right]_0^3$$

$$= 9 \text{ m}$$

$$s_2 = \int_3^6 (-(t - 3)(6 - t)) dt$$

$$= \int_3^6 -(6t - t^2 - 18 + 3t) dt$$

$$= \int_3^6 (-t^2 + 9t - 18) dt$$

$$= \left[-\frac{t^3}{3} + \frac{9t^2}{2} + 18t \right]_3^6$$

$$= -4.5 \text{ m}$$

Total distance travelled in one cycle is
 $= 9 - 4.5 \text{ m} = 4.5 \text{ m}$

$$\text{No. of cycles} = \frac{20}{4.5} = 4.44 \text{ cycles} \approx 5 \text{ cycles}$$

Q. 14. A man is standing on top of a building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second balls is +15 m at $t = 2$ s. The gap is found to remain constant. Calculate the velocity with which the balls were thrown and the exact time interval between their throw.

[NCERT Exemplar Q. No. 26]

Ans. Let the speeds of two balls (1 & 2) be v_1 and v_2 .

$$\text{Given, } v_1 = 2v, v_2 = v$$

If y_1 and y_2 the maximum height covered by the balls 1 and 2, respectively, then

$$y_1 = \frac{v_1^2}{2g} = \frac{4v^2}{2g} \text{ and } y_2 = \frac{v_2^2}{2g} = \frac{v^2}{2g}$$

$$\text{since, } y_1 - y_2 = 15 \text{ m,}$$

$$\frac{4v^2}{2g} - \frac{v^2}{2g} = 15 \text{ m}$$

$$\text{or } \frac{3v^2}{2g} = 15 \text{ m}$$

$$v^2 = \sqrt{5 \text{ m} \times (2 \times 10)}$$

$$v = 10 \text{ m/s}$$

$$\text{clearly, } v_1 = 20 \text{ m/s, } v_2 = 10 \text{ m/s.}$$

For ball 1,

$$v_1 = u_1 - gt_1$$

$$\text{or, } 0 = 20 - 10 t_1$$

$$\therefore t_1 = \text{time to reach maximum height} = 2 \text{ s}$$

For ball 2,

$$v_2 = u_2 - gt_2$$

$$\text{or, } 0 = 10 - 10 t_2$$

$$\therefore t_2 = \text{time to reach maximum height} = 1 \text{ s}$$

$$\therefore \text{Time interval} = 1 \text{ s}$$



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

An object is dropped from the top of a tower, it is accelerated downward under the influence of the force of gravity. The acceleration developed in the body is equal to the acceleration due to gravity. If air resistance is neglected, the object is said to be in free fall and in such situation time of fall is irrespective of the dimension of the body, its mass and density. Free fall is thus a case of motion with uniform acceleration.

At the time of drop, the velocity of the body is 0. As the body comes down the velocity increases and it becomes maximum just before touching the earth.

Q. 1. What is the acceleration of a freely falling body?

- (A) 9.8 m/s²
- (B) Less than 9.8 m/s²
- (C) More than 9.8 m/s²
- (D) Depends on the mass of the body

Ans. Option (A) is correct.

Explanation: For a freely falling body, the acceleration developed in the body is equal to the acceleration due to gravity of the earth.

Q. 2. Under which consideration, the acceleration developed in an object is taken to be constant?

- (A) Height through which the object falls is greater than earth's radius.
- (B) Height through which the object falls is smaller than earth's radius.
- (C) The mass of the object is negligible.
- (D) Both (a) and (d)

Ans. Option (B) is correct.

Explanation: If the height through which the object falls is small compared to the earth's radius, the acceleration developed in the object i.e., g can be taken to be constant.

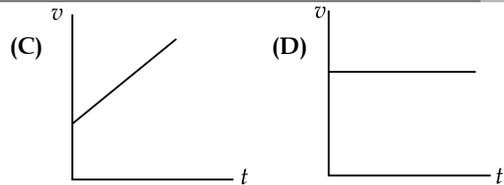
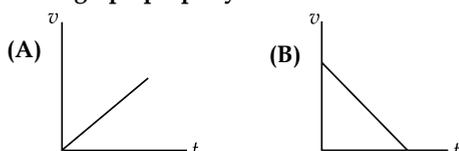
Q. 3. Which of the following equation of motion is correct for free fall?

- (A) Velocity = $v = 9.8t$
- (B) Height travelled = $h = 9.8t$
- (C) Velocity = $v = \sqrt{(19.6h)}$
- (D) Both (A) and (C)

Ans. Option (D) is correct.

Explanation: In the equation of motion $v = u + at$, if we put $u = 0$ and $a = g = 9.8$ for free fall, then $v = 9.8t$
In the equation of motion $v^2 = u^2 + 2as$, if we put $u = 0$ and $a = g = 9.8$ for free fall, then $v = \sqrt{(19.6h)}$

Q. 4. For a free fall which one of the following represents the $v-t$ graph properly?



Ans. Option (A) is correct.

Explanation: Equation of motion:

$$v = u + at$$

putting $u = 0, a = g = 9.8$

$$v = 9.8t$$

So, it is an equation of a straight time passing through origin.

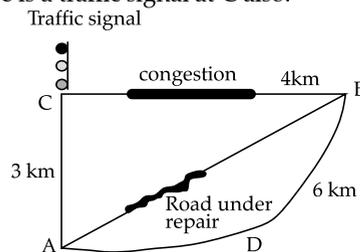
Q. 5. Which of the following statements is true?

- (A) During free fall air resistance is neglected.
- (B) Free fall is a motion with uniform acceleration.
- (C) Acceleration remains constant throughout the motion.
- (D) All of the above,

Ans. Option (D) is correct.

II. Read the following text and answer any 4 of the following questions on the basis of the same:

Tabu lives at A. He was supposed to go to his uncle's house at B. A and B is connected by a straight road 5 km long. But that day the road was under repair. So, all the buses were following a diversion via C. A to B via C is 7 km. Moreover this route is congested. There is a traffic signal at C also.



Tabu got a seat just behind the driver. Once he noticed that the reading of the speedometer was 15 km/h. But ultimately the bus took 1 hour to reach B. He could not understand the fallacy.

Q. 1. What is the distance and displacement of Tabu?

- (A) 7 km, 5 km
- (B) 5 km, 7 km
- (C) 5 km, 5 km
- (D) 7 km, 7 km

Ans. Option (A) is correct.

Explanation: Distance is the actual path covered i.e., $3 + 4 = 7$ km. Displacement is the straight line distance from A to B i.e., $\sqrt{3^2 + 4^2} = 5$ km

Q. 2. Why the speedometer reading was 15 km/h, but actual time required to cover 7 km was 1 hour?

- (A) Speedometer was erratic.
- (B) Actual distance was more than 7 km.
- (C) Halt timing at the traffic signal, slow speed at the congested areas and halt-timing at the bus stops are also to be taken into account.

(D) Both (A) and (B).

Ans. Option (C) is correct.

Explanation: The bus might have run at minimum speed 15 km/h for sometime. But halt timings at the traffic signal, slow speed at the congested areas and halt-timing at the bus stops increase the actual time taken.

Q. 3. Speedometer measures

- (A) Average speed (B) Instantaneous speed
(C) Distance traversed (D) None of these

Ans. Option (B) is correct.

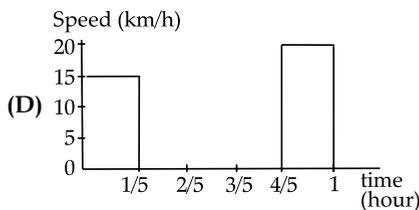
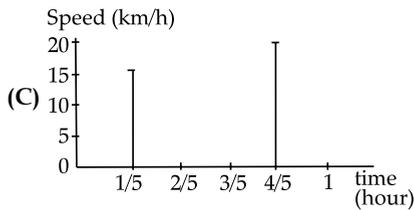
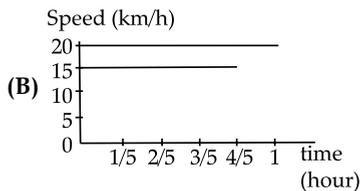
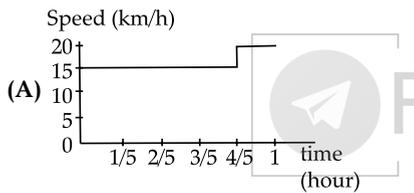
Explanation: A speedometer is a gauge that measures and displays the instantaneous speed of a vehicle.

Q. 4. Which instrument is used the distance covered by a vehicle?

- (A) Odometer (B) Speedometer
(C) Synchrometer (D) Barometer

Ans. Option (A) is correct.

Q. 5. Which of the following graphs represents the motion of the bus if it covers AC distance at a speed of 15 km/h, CB distance at a speed 20 km/h and total distance is covered in 1 hour including halt at traffic signal?



Ans. Option (D) is correct.

Explanation: It is a time-speed graph.

For AC: speed = 15 km/h, AC = 3 km,

$$\text{so time taken} = \frac{3}{15} = \frac{1}{5} \text{ h}$$

For CB, speed = 20 km/h, CB = 4 km,

$$\text{so time taken} = \frac{4}{20} = \frac{1}{5} \text{ h}$$

Total time taken = 1 h

So, halt time at traffic signal

$$= 1 - \left(\frac{1}{5} + \frac{1}{5} \right) = \frac{3}{5} \text{ h}$$

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

The velocity of an object, in general, changes during its course of motion. Should it be described as the rate of change in velocity with distance or with time? This was a problem even in Galileo's time. It was first thought that this change could be described by the rate of change of velocity with distance. But, through his studies of motion of freely falling objects and motion of objects on an inclined plane, Galileo concluded that the rate of change of velocity with time is a constant of motion for all objects in free fall. On the other hand, the change in velocity with distance is not constant – it decreases with the increasing distance of fall.

This led to the concept of acceleration as the rate of change of velocity with time.

Q. 1. How did Galileo concluded that the change of velocity with time is a constant for all objects in free fall?

Ans. Galileo studied of motion of freely falling objects and motion of objects on an inclined plane and concluded that the rate of change of velocity with time is a constant of motion for all objects in free fall.

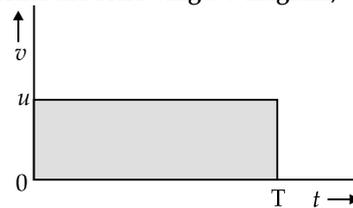
Q. 2. How the acceleration can be determined from a $v-t$ graph?

Ans. The acceleration of an object can be determined from the slope of the graph.

Q. 3. In many $v-t$ and $a-t$ graphs sharp kinks at some points are observed. Are these realistic situations? Explain.

Ans. These are not realistic situations. At these points the functions are not differentiable. This means acceleration and velocity cannot change abruptly at any instant. Changes are always continuous.

Q. 4. Consider the following $v-t$ diagram,



Find the value of acceleration and the distance covered.

Ans. Since, the velocity is not changing. The acceleration is 0.

The distance covered is the area in between the graph and the time axis i.e., uT .

Q. 5. Use differential calculus to express the acceleration in terms of velocity(v), distance(x) and time(t).

$$\text{Ans. } a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Study Time
Maximum Time: 3 hours
Maximum Questions: 91

CHAPTER

3

MOTION IN A PLANE



Syllabus

Scalar and vector quantities; position and displacement vectors; general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors; Unit vector; resolution of a vector in a plane; rectangular components; scalar and vector products of vectors; motion in a plane; cases of uniform velocity and uniform acceleration; projectile motion; uniform circular motion.

Topic-1 Scalar and Vector Quantities

TOPIC - 1

Scalar and Vector Quantities

.... P. 42

TOPIC - 2

Projectile Motion

.... P. 50



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Revision Notes

Scalar : A physical quantity which has only magnitude and no direction is called a scalar quantity or scalar.

Vector : A physical quantity which constitutes of magnitude as well as direction is called a vector quantity or vector. If it follows law of vector addition too.

➤ Unit vector :

(i) A unit vector of \vec{A} is written as \hat{A} and is given by $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

(ii) A unit vector is dimensionless quantity of magnitude equal to unity. Its magnitude is 1 and it can have any direction.

(iii) In cartesian co-ordinates, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along x, y and z -axes, respectively.

➤ **Polar vectors :** These are those vectors which have a linear directional effect. For example, force, linear momentum, linear velocity etc.

➤ **Axial vectors or rotational vectors :** These vectors represent rotational effect. They are always directed along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque, angular momentum etc. are few examples of axial vectors.

➤ Some vector laws :

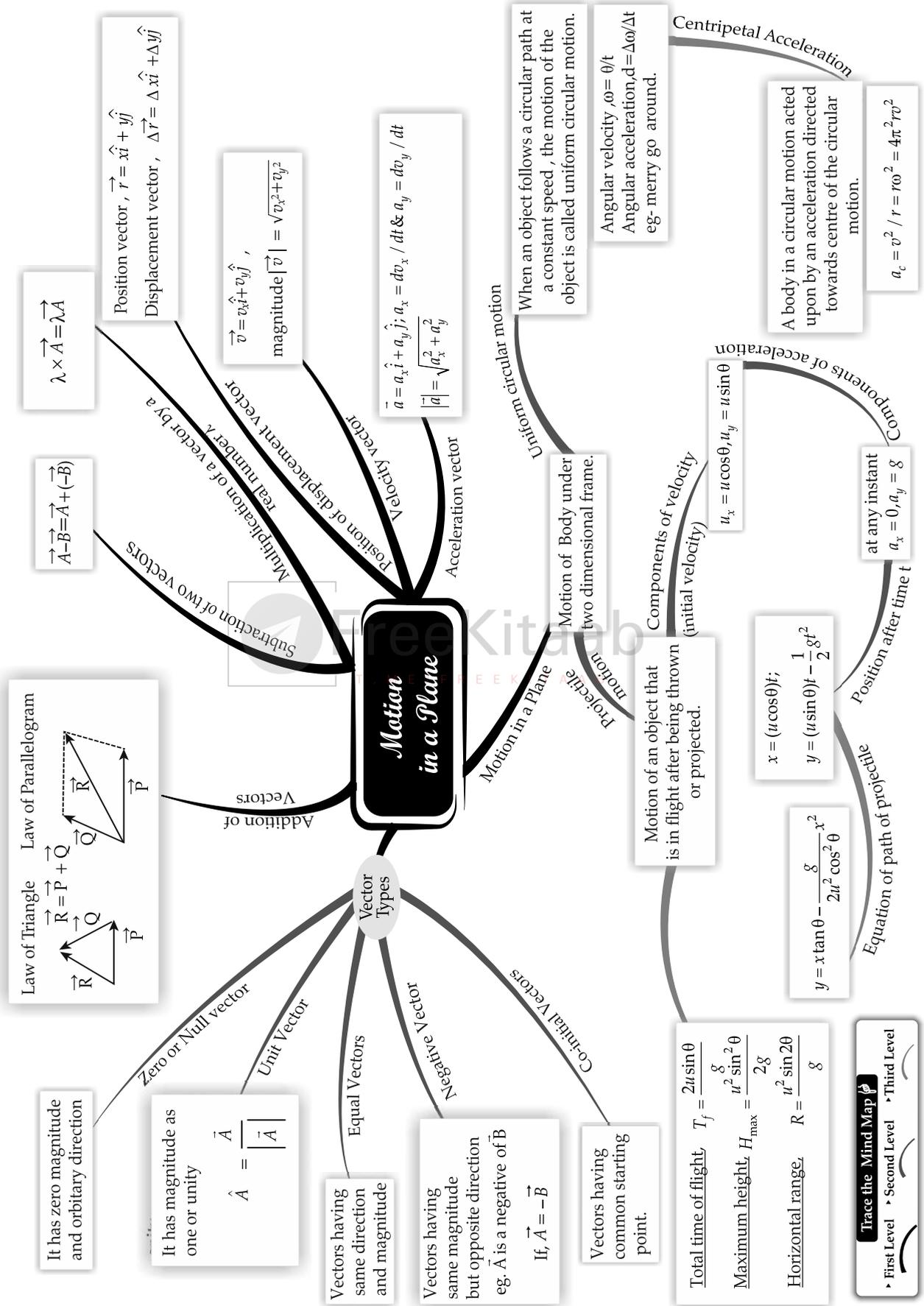
(1) **General law for addition of vector :** It states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with tail of third vector and so on then resultant vector is represented in magnitude and direction by the line starting from tail of first vector to head of last vector.

(2) **Triangle Law :** It states that if two vectors acting on a particle at the same time are represented in magnitude and direction by the two sides of a triangle taken in one order, their resultant vector is represented in magnitude and direction by the third side of triangle taken in opposite order.

Scan to know more about this topic



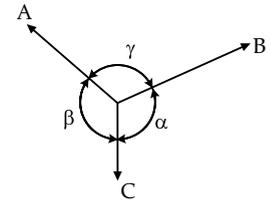
Vector and scalar



(3) **Parallelogram Law** : It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.

➤ **Lami's Theorem** : It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces, i.e.,

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$



Key Words

- **Modulus of vector** is the magnitude of vector.
- **Equal vectors** are those vectors which have equal magnitude and same direction.
- **Negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of given vector.
- **Co-initial vectors** are those vectors which have common initial point.
- **Collinear vectors** are those vectors which are having equal or unequal magnitudes and are acting along parallel straight lines.
- **Coplanar vectors** are those vectors which are acting in the same plane.
- **Localized vector** is that vector whose initial point is fixed.
- **Non-Localized vector** is that vector whose initial point is not fixed.
- **Zero or Null vector** is that vector which has zero magnitude and an arbitrary direction and represented by $\vec{0}$.
- **Displacement vector** is that vector which tells how much and in which direction an object has changed its position in a given interval of time.
- **Resultant vector** is defined as that single vector which produces the same effect as is produced by two or more individual vectors together.
- **Equilibrate vector** is a single vector which balances two or more vectors acting on a body at the same time.

Key Formulae

- $\vec{R} = \vec{A} + \vec{B}$
 $R = \sqrt{A^2 + B^2 + 2AB\cos \theta}$
 $\tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$; $\beta =$ angle of \vec{R} with \vec{A} .
- $\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
 $R = \sqrt{A^2 + B^2 - 2AB\cos \theta}$
 $\tan \beta = \frac{B \sin(180^\circ)}{A + B \cos(180^\circ)} = \frac{B \sin \theta}{A - B \cos \theta}$
- $\vec{A} = A_x \hat{i} + A_y \hat{j}$ and $A_x = A \cos \theta, A_y = A \sin \theta$ (in 2D)
- $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$, $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ (in 3D)
- $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$, $|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$
- $\vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} + (A_z + B_z) \hat{k}$
- **Unit Vector or \hat{A} is**
 $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x \hat{i} + A_y \hat{j} + A_z \hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$

➤ $\vec{A} \cdot \vec{B} = AB \cos \theta$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

➤ If two vectors are parallel to each other i.e., $\theta = 0^\circ$

$\vec{A} \cdot \vec{B} = AB \cos 0^\circ = AB$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

➤ If two vectors are perpendicular to each other i.e., $\theta = 90^\circ$.

$\vec{A} \cdot \vec{B} = AB \cos 90^\circ = 0$

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

➤ If two vectors are parallel to each other i.e., $\theta = 0^\circ$

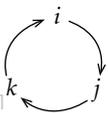
➤ $\vec{A} \times \vec{B} = AB \sin 0^\circ \hat{n} = \vec{0}$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

➤ If two vectors are perpendicular to each other i.e., $\theta = 90^\circ$

$\therefore \vec{A} \times \vec{B} = AB \sin 90^\circ = AB$

➤ Trick to remember Cross product



$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$
and $\hat{i} \times \hat{k} = -\hat{j}, \hat{k} \times \hat{j} = -\hat{i}, \hat{j} \times \hat{i} = -\hat{k}$

➤ **Area of triangle** = $\frac{1}{2} |\vec{A} \times \vec{B}|$

➤ **Area of parallelogram** = $|\vec{A} \times \vec{B}|$

➤ **Unit vector perpendicular** to \vec{A} & \vec{B}

$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

where, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

➤ If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$

then $\vec{A} \times \vec{B} = \vec{B} \times \vec{C} = \vec{C} \times \vec{A}$

• $\sin \theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}| |\vec{B}|}$

• $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$

• $\tan \theta = \frac{\vec{A} \times \vec{B}}{\vec{A} \cdot \vec{B}}$





Mnemonics

Concept: Cross and dot product of two vectors.

Mnemonics: **A** and **B** crossed **S**ikkim and **d**rove to **C**alcutta.

Interpretation:

A - \vec{A}

B - \vec{B}

c - Cross product

s - $\sin \theta$

d - dot product

c - $\cos \theta$

$$\vec{A} \times \vec{B} = AB \sin \theta$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$



Very Short Answer Type Questions

(1 mark each)

Q. 1. A vector is multiplied by a scalar. Is it necessary that its units will never change? [U]

Ans. No, unit of a vector multiplied by a scalar may change, e.g., velocity (ms^{-1}) multiplied by mass (kg) gives momentum (kgms^{-1}). 1

Q. 2. Can a vector be zero if any of the component of this vector is not zero? [U]

Ans. No, because a vector having any component zero may have non-zero components along other two axes. 1

Q. 3. Forty vectors each of magnitude 20 units are completely represented by the sides of a polygon taken in the same order. What will be the resultant? [A]

Ans. Their resultant will be zero. This is because the vector sum of all the vectors represented by the sides of a closed polygon taken in the same order is zero. 1

Q. 4. Find the angle between two vectors.

$$\vec{A} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \vec{B} = -4\hat{i} + \hat{j} - 2\hat{k}.$$

[A] [KVS 2013]

Ans. $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$

$$\therefore \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\vec{A} \cdot \vec{B} = (\hat{i} + 2\hat{j} - \hat{k}) \cdot (-4\hat{i} + \hat{j} - 2\hat{k})$$

$$= (-4 + 2 + 2) = -4 + 4 = 0$$

$$\therefore \cos \theta = 0$$

$$\theta = \cos^{-1} 0$$

hence, $\theta = 90^\circ$. 1

Q. 5. A vector \vec{A} is expressed as $\vec{A} = A_x \hat{i} + A_y \hat{j}$, where A_x and A_y are its components along x -axis and y -axis respectively. If vector \vec{A} makes an angle θ with x -axis, then θ is given by which expression? [A] [R] [NCT 2008]

Ans. Here,

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

$$\tan \theta = \frac{A_y}{A_x}$$

or

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right) \quad 1$$



Commonly Made Error

Students find it difficult to use trigonometric and inverse trigonometric functions to calculate the expression for the angle.



Answering Tip

Students should exercise the basic trigonometric and inverse trigonometric functions before attempting these types of questions.

Q. 6. Two bodies of mass 2g and 10g have position vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 3\hat{k}$ respectively. Find the position vector of center of mass. [A] [KVS 2013]

Ans. $M_1 = 2\text{g}, r_1 = 3\hat{i} + 2\hat{j} - \hat{k}$

$$M_2 = 10\text{g}, r_2 = 3\hat{i} - \hat{j} + 3\hat{k}$$

$$\text{Position vector of centre of mass} = \frac{M_1 r_1 + M_2 r_2}{M_1 + M_2}$$

$$= \frac{2(3\hat{i} + 2\hat{j} - \hat{k}) + 10(3\hat{i} - \hat{j} + 3\hat{k})}{10 + 2}$$

$$= 3\hat{i} - \frac{1}{2}\hat{j} + \frac{7}{3}\hat{k}$$

1



Short Answer Type Questions-I

(2 marks each)

Q. 1. What is the differences between the following two data ? [R]

(i) 85 km/h east, (ii) (8 h) (5 km/h east).

Ans. (i) It is the product of a pure number and a vector (velocity). Hence, the unit of product is the same as that of vector, i.e., the product is a velocity of 85 km/h, towards east. 1

(ii) It is the product of a scalar (time) and a vector (velocity). Hence, the unit of the product will be $h \times (km/h) = km$. Thus, the product is a displacement of magnitude 40 km, towards east. 1



Commonly Made Error

► Students find it difficult to understand that the first case shows the multiplication of a vector quantity with a constant, whereas the second case shows the multiplication of a scalar quantity with a vector quantity



Answering Tip

► Students should carefully find out the difference between the multiplication in both the cases and then answer the same.

Q. 2. Does the nature of a vector change when it is multiplied by a scalar ? [U]

Ans. The nature of a vector may or may not be changed when it is multiplied. ½

For example, when a vector is multiplied by a pure number like 1, 2, 3, etc., then the nature of the vector does not change. ½

On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes. For example, when acceleration (vector) is multiplied by mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration. 1



Commonly Made Error

► Students get confused in the nature and direction of a vector quantity and answer the same affirmatively only.



Answering Tip

► Students should remember the basic rules of multiplication of a vector and their corresponding examples before answering this question.

Q. 3. Why the magnitude of the rectangular component of a vector can't be greater than the magnitude of the vector itself ? [R]

Ans. The magnitude of the rectangular component of a vector itself cannot be greater than the magnitude of vector itself because the rectangular component of a vector A are $A_x = A \cos \theta$ and $A_y = A \sin \theta$. As $\sin \theta$ and $\cos \theta$ both are ≤ 1 , so both A_x and A_y cannot be greater than A . 2

Q. 4. If $\vec{A} + \vec{B} = \vec{C}$ prove that $C = (A^2 + B^2 + 2AB \cos \theta)^{1/2}$, where θ is the angle between A and B . [A]

Ans. $\vec{C} = \vec{A} + \vec{B}$

$$\therefore C^2 = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B})$$

$$C^2 = \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A}$$

$$= A^2 + B^2 + 2\vec{A} \cdot \vec{B}$$

$$C = (A^2 + B^2 + 2AB \cos \theta)^{1/2} \quad 2$$

Q. 5. Under what conditions does the equality $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ hold good ?

[A] [A] [NCT 2010]

Ans. $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

So, $|\vec{A} + \vec{B}|$ will be equal to $|\vec{A} - \vec{B}|$ only when

$$2AB \cos \theta = -2AB \cos \theta$$

Or, $4AB \cos \theta = 0$

So, either $AB = 0$ or $\cos \theta = 0$

$AB = 0$ is not possible since both A and B are non-zero.

So, $\cos \theta = 0$, i.e., $\theta = 90^\circ$

So, \vec{A} and \vec{B} are perpendicular to each other, then

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$



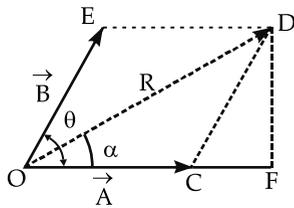
Short Answer Type Questions-II

(3 marks each)

Q. 1. Find resultant vector of the summation of two vectors A and B having angle θ between them.

[A] [MSE Chandigarh 2008]

Ans. Let there be two vectors \vec{A} and \vec{B} with angle θ between them.



Using parallelogram law of vector addition resultant. ½

$$\vec{R} = \vec{A} + \vec{B} \quad \frac{1}{2}$$

OC is extended upto F and perpendicular DF is drawn on it.

Considering right angled ΔOFD

$$\begin{aligned} OD^2 &= OF^2 + DF^2 \\ &= (OC + CF)^2 + DF^2 \\ &= (A + B\cos \theta)^2 + (B\sin \theta)^2 \end{aligned}$$

$$R^2 = A^2 + B^2 + 2AB\cos \theta \quad \frac{1}{2}$$

($\because \cos^2 \theta + \sin^2 \theta = 1$)

$$R = \sqrt{A^2 + B^2 + 2AB\cos \theta} \quad \dots(i) \quad \frac{1}{2}$$

$$\tan \alpha = \frac{DF}{OF} = \frac{DF}{OC+CF} \quad \frac{1}{2}$$

$$= \frac{B \sin \theta}{A+B \cos \theta}$$

$$\alpha = \tan^{-1} \frac{B \sin \theta}{A+B \cos \theta} \quad \dots(ii) \quad \frac{1}{2}$$

Q. 2. Find the magnitude and directions of the vectors

$$\hat{i} + \hat{j} \text{ and } \hat{i} - \hat{j}.$$

[A]

Ans. (i) Magnitude of vectors $\vec{i} + \vec{j}$

$$= |\hat{i} + \hat{j}|$$

$$= \sqrt{1^2 + 1^2} = \sqrt{2} \quad \frac{1}{2}$$

$$\tan \theta = \frac{|\hat{j}|}{|\hat{i}|} = \frac{1}{1} = 1 \quad \frac{1}{2}$$

or $\theta = \tan^{-1} 1 = 45^\circ$ with x -axis

(ii) Magnitude of vectors $\hat{i} - \hat{j}$

$$= |\hat{i} - \hat{j}|$$

$$= \sqrt{1^2 + (-1)^2} = \sqrt{2} \quad 1$$

$$\tan \theta = \frac{|-\hat{j}|}{|\hat{i}|} = \frac{-1}{1} = -1$$

$$\begin{aligned} \theta &= \tan^{-1}(-1) \\ &= -45^\circ \text{ with } x\text{-axis} \quad 1 \end{aligned}$$

Q. 3. Give an analytical method to find the vector sum

of three vectors \vec{P} , \vec{Q} and \vec{R} .

[A]

Ans. Let \vec{P} , \vec{Q} and \vec{R} be represented in component from, i.e.,

$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

$$\vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k} \quad 1$$

Let \vec{S} be their summation vector, i.e.,

$$\vec{S} = \vec{P} + \vec{Q} + \vec{R}$$

$$\begin{aligned} &= (P_x \hat{i} + P_y \hat{j} + R_z \hat{k}) + (Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}) \\ &\quad + (R_x \hat{i} + R_y \hat{j} + R_z \hat{k}) \quad 1 \end{aligned}$$

Addition of vectors obey the commutative as well as associative laws

$$\therefore \vec{S} = (P_x + Q_x + R_x) \hat{i} + (P_y + Q_y + R_y) \hat{j} + (P_z + Q_z + R_z) \hat{k}$$

$$S_x = P_x + Q_x + R_x,$$

$$S_y = P_y + Q_y + R_y \text{ and}$$

$$S_z = P_z + Q_z + R_z. \quad 1$$



Commonly Made Errors

- ▶ Students get confused between the use of triangle law and parallelogram law of vector addition.
- ▶ Different values from the extended diagram to be used for calculations generally are taken incorrectly.



Answering Tips

- ▶ Students must remember to learn the conditions where triangle law of parallelogram law of vector additions are to be used.
- ▶ The use of different values to get the result must be used carefully considering the basic trigonometric relationship.



Long Answer Type Questions

(5 marks each)

Q.1. Define null vector. What are its properties? What is its physical significance? **[R]**

Ans. It is defined as a vector having zero magnitude and acting in the arbitrary direction. It is denoted by $\vec{0}$. 1

Properties of null vector :

(i) The addition or subtraction of zero vector from a given vector is again the same vector.

$$\begin{aligned} \text{i.e., } \vec{A} + \vec{0} &= \vec{A} \\ \vec{0} + \vec{A} &= \vec{A} \\ \vec{A} - \vec{0} &= \vec{A} \end{aligned} \quad 1$$

(ii) The multiplication of zero vector by a non-zero real number is again the zero vector.

i.e., $n \cdot \vec{0} = \vec{0}$ 1

(iii) If $n_1 \vec{A} = n_2 \vec{B}$, where n_1 and n_2 are non-zero real numbers. Then the relation will hold good.

if $\vec{A} = \vec{B} = \vec{0}$
i.e., both \vec{A} and \vec{B} are null vectors. 1

Physical significance of null vector : It is useful in describing the physical situation involving vector quantities.

e.g., $\vec{A} - \vec{A} = \vec{0}$
 $\vec{0} \times \vec{A} = \vec{0}$ 1

Q.2. What do you understand by rectangular resolution of a vector ? Resolve it into its two rectangular components. [R]

Ans. It is defined as the factors of splitting a given vector in two or three component vectors at right angles to each other. The component vectors are called rectangular component of the given vector.

Let \vec{R} be the given vector acting in X-Y plane at an angle θ with X-axis. Perpendiculars CA and CB are drawn on X and Y axes respectively. O and C are joined. $\vec{OC} = \vec{R}$ If \vec{P} and \vec{Q} be the rectangular components of \vec{R} along X and Y axes respectively, then 1

$\vec{OA} = \vec{P}$ or $OA = P$
and $\vec{OB} = \vec{Q}$ or $OB = Q$

Now in right angled ΔOAC ,

$\sin \theta = \frac{AC}{OC} = \frac{Q}{P}$... (i) 1/2

or $Q = R \sin \theta$

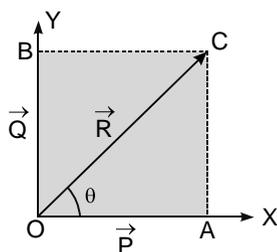
and $\cos \theta = \frac{OA}{OC} = \frac{P}{R}$

or $P = R \cos \theta$... (ii) 1/2

Also $OC^2 = OA^2 + AC^2$

or $R^2 = P^2 + Q^2$

or $R = \sqrt{P^2 + Q^2}$



and $\tan \theta = \frac{AC}{OA} = \frac{Q}{P}$ 1

Also, according to Δ law of vector addition,

$\vec{R} = \vec{P} + \vec{Q}$
 $= P\hat{i} + Q\hat{j}$

or $\vec{R} = (R \cos \theta) \hat{i} + (R \sin \theta) \hat{j}$ 1

Thus if \vec{A}_x and \vec{A}_y be the two rectangular components of \vec{A} along X and Y axes respectively, then $\vec{A} = A_x \hat{i} + A_y \hat{j}$ 1



Commonly Made Error

► There is a high possibility of committing error while using trigonometric functions diagrammatically.



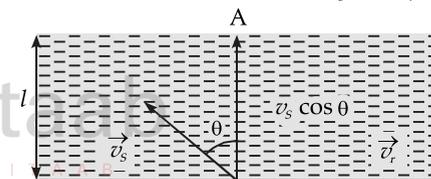
Answering Tip

► Students should do the resolution of the vectors cautiously. They should use basic trigonometric functions via a rectangular diagram and go ahead step by step.

Q.3. Find the way for a swimmer who wants to cross the river in the shortest time. [AI] [R]

Ans. Let \vec{v}_s and \vec{v}_r be the velocities of swimmer and river respectively.

Let \vec{v} = resultant velocity of v_s and v_r



Let the swimmer begins to swim at an angle θ with the line OA where OA is \perp to the flow of river.

If t = time taken to cross the river, then

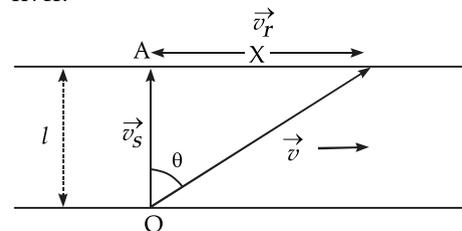
$t = \frac{l}{v_s \cos \theta}$ 1

where l = breadth of river

For t to be minimum, $\cos \theta$ should be maximum, i.e., $\cos \theta = 1$

This is possible if $\theta = 0^\circ$ 1

Thus, we conclude that the swimmer should swim in a direction \perp to the direction of flow of river.



$v = \sqrt{v_s^2 + v_r^2}$ 1

$\tan \theta = \frac{v_r}{v_s} = \frac{X}{l}$

or $X = l \frac{v_r}{v_s}$ 1

If $\theta = 0^\circ \Rightarrow \cos \theta = 1$
then shortest time,

$$t = \frac{l}{v_s} \quad 1$$



Commonly Made Error

- ▶ Students commit error while proving that the shortest distance will come up when the angles made with flow is 0° .



Answering Tip

- ▶ Students should be able to understand that there is an angle made due to the flow of the river only and then they should calculate the value of the cosine of the angle according to the basic trigonometric calculations.

Topic-2 Projectile Motion



Revision Notes

▶ Projectile :

- (a) Projectile is defined as a body thrown with some initial velocity except vertically upward or downward and then allowed to move under the action of gravity alone, without being propelled by an engine or fuel or any source. The path followed by a projectile is known as its **trajectory**.
 - (b) The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.
 - (c) Due to the vertical component of velocity, the body rises vertically upward and due to the horizontal component of velocity the body shifts horizontally simultaneously.
- ▶ The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.
 - ▶ Due to the vertical component of velocity, the body rises vertically upward and due to the horizontal component of velocity the body shifts horizontally simultaneously.
- #### ▶ Centripetal force :
- (a) It is the force required to move the body in circular path with a constant angular velocity.
 - (b) The centripetal force acts on the particle along the radius which is directed towards the centre of circular path.
 - (c) The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path, therefore the work done by the centripetal force is zero.
 - (d) The centripetal force is provided in different ways, in different types of circular motions.

Scan to know more about this topic



Projectile motion



Key Words

- ▶ **Angular displacement** of the object moving around a circular path is defined as the angle traced out by radius vector at the centre of circular path in given time. It is vector quantity.
- ▶ **Angular velocity** of an object in circular motion is defined as the time rate of change of its angular displacement.
- ▶ **Angular acceleration** of an object in circular motion is defined as the time rate of change of its angular velocity.
- ▶ **Uniform circular motion** is the motion when a point object is moving on a circular path with a constant speed.



Key Formulae

- ▶ For motion along X-axis, $v_x = u_x + a_x t$ and $x = x_0 + u_x t + \frac{1}{2} a_x t^2$
- ▶ For motion along Y-axis, $v_y = u_y + a_y t$ and $y = y_0 + u_y t + \frac{1}{2} a_y t^2$

➤ Velocity of projective at an instant of its flight is

$$v = \sqrt{v_x^2 + v_y^2}$$

and

$$\tan \beta = \frac{v_y}{v_x}$$

➤ Angular projection of projectile :

(i) Time of flight, $T = \frac{2u \sin \theta}{g}$

(ii) Maximum height, $h = \frac{u^2 \sin^2 \theta}{2g}$

(iii) Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

(iv) Maximum horizontal range $R_{max} = \frac{u^2}{g}$ for $\theta = 45^\circ$

(v) Range is same for for angles θ and $(90^\circ - \theta)$ if u & g remains unchanged

➤ Circular Motion

- $\omega = \theta/t$
- $\omega = 2\pi v = \frac{2\pi}{t}$
- $a_c = \omega^2 r = \omega v = v^2/r$
- $a_T = r\alpha$

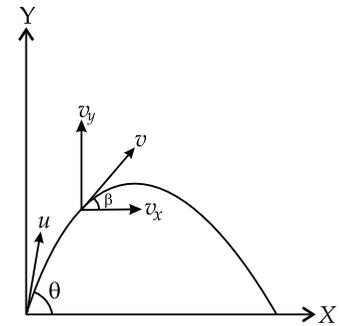
where, a_c = centripetal acceleration

a_T = tangential acceleration

ω = angular velocity

v = frequency

T = Time period



u = initial speed
 θ = angle of projection



Very Short Answer Type Questions

(1 mark each)

Q. 1. What furnishes the centripetal acceleration for the Earth to go round the Sun ? [U]

Ans. The gravitational pull of the Sun at the Earth. 1

Q. 2. Name the quantity which remains unchanged during the flight of an oblique projectile. [A]

Ans. Horizontal component of velocity. 1

Q. 3. A body in uniform horizontal circular motion possess variable velocity. Does it mean that the kinetic energy of the body is also variable ? [A]

Ans. No, the magnitude of velocity in uniform motion is constant. Hence, the K.E. will not change. 1

Q. 4. Is the rocket in flight is an example of projectile ? [A]

Ans. No, because it is propelled by combustion of fuel and does not move under the effect of gravity alone. 1

Q. 5. What is the angle between velocity vector and acceleration vector in uniform circular motion. [A]

Ans. 90° . 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. Is circular motion possible at constant speed or at constant velocity ? Explain. [R]

Ans. Circular motion is possible at constant speed because in circular motion, the magnitude of the velocity, i.e., speed remains constant while the direction of motion changes continuously. 2

Q. 2. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

[R] [MSE Chandigarh 2008]

Ans. The horizontal range is maximum for $\theta = 45^\circ$ and it is given by

$$R_{max} = \frac{u^2}{g} \quad \frac{1}{2}$$

The maximum height attained,

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad \frac{1}{2}$$

Therefore, for $\theta = 45^\circ$

$$H_{max} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} \quad \dots(ii)$$

From the equation (i) and (ii),

$$R_{\max} = 4H_{\max} \quad 1$$

Q. 3. If both the speed of a body and radius of its circular path are doubled, what will happen to the centripetal acceleration? [A]

Ans. Centripetal acceleration

$$\begin{aligned} a &= v^2/r \\ \text{When } v' &= 2v \\ \text{and } r' &= 2r, \\ \text{then } a' &= v'^2/r' \\ &= (2v)^2 / (2r) = 2a \quad 2 \end{aligned}$$

Q. 4. Two bombs of 5 kg and 10 kg are thrown from a tower with the same velocity in the same direction. [R]

- (i) Which bomb will reach the ground first?
 (ii) If the bombs are thrown in the same direction with different velocity. What would be the effect?

Ans. (i) Both the bombs will reach the ground simultaneously because the time of flight doesn't depend upon the mass of projectile. 1

(ii) On throwing with different velocities, the bomb thrown with lesser velocity will reach the ground earlier.

$$\begin{aligned} T &= \frac{2u \sin \theta}{g} \\ \text{i.e., } T &\propto u \quad 1 \end{aligned}$$

Q. 5. In long jumping, does it matter how high you jump? What factors determine the span of the jump? [R]

Ans. If an athlete takes jump with velocity u and at an angle θ with horizontal, then

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad \frac{1}{2}$$

$$\text{and } R = \frac{u^2 \sin 2\theta}{g} \quad \frac{1}{2}$$

$$\begin{aligned} \therefore \frac{R}{H} &= \frac{u^2 \sin 2\theta}{g} \times \frac{2g}{u^2 \sin^2 \theta} \\ &= \frac{2 \sin 2\theta}{\sin^2 \theta} = \frac{2 \times 2 \sin \theta \cos \theta}{\sin^2 \theta} \end{aligned}$$

$$= 4 \cot \theta$$

$$\text{or } R = 4H \cot \theta \quad 1$$

Thus, the span of the jump depends on the height of the jump and the angle at which the athlete jumps.



Commonly Made Error

► Students consider this problem directly with the horizontal motion and calculate the horizontal distance.



Answering Tip

Students should think of this problems as the projectile motion and then deduce the desired relationship.

Short Answer Type Questions-II

(3 marks each)

Q. 1. A particle has displacement equation

- (i) $x_A = 2t + 7$
 (ii) $x_B = 3t^2 + 2t + 6$
 (iii) $x_C = 5t^3 + 4t$

Which of them has uniform acceleration? [A]

Ans. (i) $v = \frac{dx_A}{dt} = 2$

and $a = \frac{d^2x_A}{dt^2} = 0$

As per this eqn. the particle has no acceleration at all. 1

(ii) $v = \frac{dx_B}{dt} = 3 \times 2t + 2 + 0 = 6t + 2$
 $a = \frac{d^2x_B}{dt^2} = 6$

Here, acceleration is uniform. 1

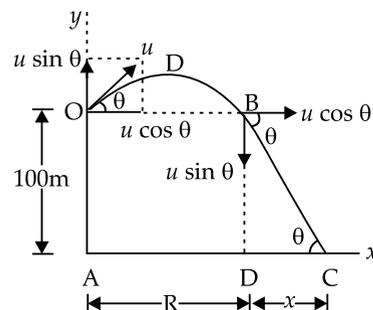
(iii) $v = \frac{dx_C}{dt} = 5 \times 3t^2 + 4 = 15t^2 + 4$

$$\begin{aligned} a &= \frac{d^2x_C}{dt^2} \\ &= 15 \times 2t = 30t \end{aligned}$$

Here, acceleration depend upon time so it is not uniform. 1

Q. 2. A machine gun is mounted on the top of a tower 100 m high. At what angles should the gun be inclined to cover a maximum range of firing on the ground below? The muzzle speed of bullet is 150 m/s. Take $g = 10 \text{ m/s}^2$. [A]

Ans.



OA is the tower.

If u is the velocity of the muzzle speed of the bullet, then it has two components. $u \cos \theta$ horizontally and $u \sin \theta$ vertically.

ODB is a projectile path for which range = $R =$

$$OB = AD = \frac{u^2 \sin 2\theta}{g} = \frac{150^2 \sin 2\theta}{10}$$

In $\triangle BDC$, $\angle BCD = \theta$

$$\therefore \cot \theta = \frac{DC}{BD} = \frac{x}{100}$$

$$\therefore x = 100 \cot \theta$$

\therefore Final range of the bullet = $AC = AD + DC$

$$\text{Or, } AC = R + x$$

$$\text{Or, } R_{\text{final}} = \frac{150^2 \sin 2\theta}{10} + 100 \cot \theta$$

R_{final} is maximum when

$$\frac{dR_{\text{final}}}{d\theta} = 0$$

$$\text{Or, } \frac{d}{d\theta} \left[\frac{150^2 \sin 2\theta}{10} + 100 \cot \theta \right] = 0$$

$$\text{Or, } \frac{d}{d\theta} [2250 \sin^2 \theta + 100 \cot \theta] = 0$$

$$\text{Or, } 2250 \times 2 \cos^2 \theta - 100 \operatorname{cosec}^2 \theta = 0$$

$$\text{Or, } 4500(\cos^2 \theta - \sin^2 \theta) - \frac{100}{\sin^2 \theta} = 0$$

$$\text{Or, } 45 \sin^2 \theta - 90 \sin^4 \theta - 1 = 0$$

$$\text{Or, } 90 \sin^4 \theta - 45 \sin^2 \theta + 1 = 0$$

$$\therefore \sin^2 \theta = \frac{45 \pm \sqrt{45^2 - 4 \times 90 \times 1}}{2 \times 90}$$

$$\therefore \sin^2 \theta = 0.48 \text{ or } 0.006$$

Discarding the second value, for maximum value of $\sin \theta$,

$$\sin^2 \theta = 0.48$$

$$\text{Or } \sin \theta = 0.69$$

$$\therefore \theta = 43.6^\circ$$



Commonly Made Error

► Students commit error in solving quadratic equation.



Answering Tip

► As the answer demands mathematical solution, students should make them well-versed with different quadratic formulas.

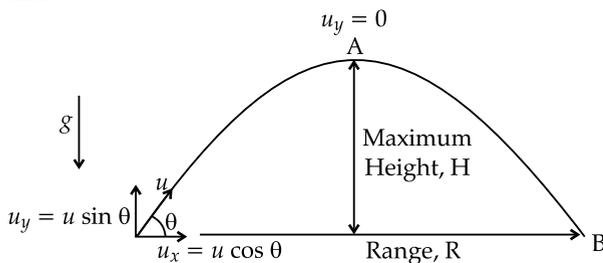


Long Answer Type Questions

(5 marks each)

Q. 1. For an angular projection given to a projectile, find : (i) Maximum height, (ii) Time of flight, (iii) Horizontal range. [A] [KVS 2013]

Ans.



(i) **Maximum height:** Maximum height of the point of the projectile from where it starts returning back.

For maximum height only u_y component of velocity is considered.

Maximum height is attained when the u_y becomes 0.

$$\therefore 0 = (u \sin \theta)^2 - gH$$

$$\therefore H = \frac{u^2 \sin^2 \theta}{2g}$$

(ii) **Time of flight:** Time of flight is the total time required to traverse the path from point to projection to maximum height and from maximum height to the plane of projection.

After traversing the path OAB , vertical displacement of the particle is 0.

$$\text{So, } 0 = u \sin \theta \times T - \frac{1}{2} gT^2$$

$$\therefore T = \frac{2u \sin \theta}{g}$$

(iii) **Horizontal range:** Horizontal range is the total horizontal distance covered by the projectile during the flight time.

For horizontal range only u_x component of velocity is considered.

$$R = u \cos \theta \times T$$

$$\text{Or } R = u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$\therefore R = \frac{u^2 \sin^2 \theta}{g}$$



Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of \vec{A} , \vec{B} and \vec{C} are respectively 5, 4 and 3 respectively, then

the angle between \vec{A} and \vec{C} is

- (A) $\sin^{-1}\left(\frac{3}{5}\right)$ (B) $\cos^{-1}\left(\frac{3}{5}\right)$
 (C) $\cos^{-1}\left(\frac{4}{5}\right)$ (D) $\sin^{-1}\left(\frac{4}{5}\right)$

Ans. Option (B) is correct.

Explanation:

$$\vec{A} = \vec{B} + \vec{C}$$

Or, $\vec{A} - \vec{C} = \vec{B}$

Or, $(\vec{A} - \vec{C})^2 = (\vec{B})^2$

Or, $A^2 + C^2 - 2\vec{A} \cdot \vec{C} = B^2$

Or, $25 + 9 - 2AC \cos \theta = 16$

Or, $34 - 2 \times 5 \times 3 \cos \theta = 16$

Or, $30 \cos \theta = 18$

Or, $\cos \theta = \frac{18}{30} = \frac{3}{5}$

$\therefore \theta = \cos^{-1}\left(\frac{3}{5}\right)$

Q. 2. If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$, find the

component of \vec{A} along \vec{B} is

- (A) $\frac{5}{\sqrt{2}}$ (B) $\frac{3}{\sqrt{2}}$
 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Ans. Option (A) is correct.

Explanation: Component along \vec{B} is $\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$

$$= \frac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{1^2 + 1^2}}$$

$$= \frac{5}{\sqrt{2}}$$

Q. 3. A particle is projected making an angle 60° with the horizon, with K amount of kinetic energy.

The kinetic energy of the particle at the highest position is

- (A) 0 (B) $\frac{K}{2}$
 (C) $\frac{K}{4}$ (D) K

Ans. Option (C) is correct.

Explanation: Initial velocity = v

$$\text{Initial kinetic energy} = \frac{1}{2}mv^2 = K$$

$$\text{At the highest position velocity} = v \cos 60^\circ = \frac{v}{2}$$

$$\therefore \text{Kinetic energy} = \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}mv^2\right) = \frac{K}{4}$$

Q. 4. Which quantity of a projectile remains unchanged?

- (A) Momentum
 (B) Kinetic energy
 (C) Vertical component of velocity
 (D) Horizontal component of velocity

Ans. Option (D) is correct.

Explanation: There is no acceleration in horizontal direction. Hence, the horizontal component of velocity remains unchanged.

Q. 5. Which one of the following is the correct expression of centripetal acceleration?

- (A) $4\pi^2v^2R$ (B) $\frac{4\pi^2v^2}{R}$
 (C) $2\pi vR$ (D) $\frac{2\pi v}{R}$

Ans. Option (A) is correct.

Explanation: $a_c = \omega^2R$

Putting $\omega = 2\pi v$

$$a_c = 4\pi^2v^2R$$

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
 (B) Both A and R are true but R is NOT the correct explanation of A
 (C) A is true but R is false
 (D) A is false and R is true

Q. 1. Assertion (A): When a 30 dyne force is inclined to y-axis at an angle 60° , the vertical and horizontal components of the force are 15 dyne and $15\sqrt{3}$ dyne respectively.

Reason (R): When a vector \vec{A} is inclined to y -axis at an angle θ , the vertical and horizontal components of the vector are $A \cos\theta$ and $A \sin\theta$ respectively.

Ans. Option (A) is correct.

Explanation: When a vector \vec{A} is inclined to y -axis at an angle θ , the vertical and horizontal components of the vector are $A \cos\theta$ and $A \sin\theta$ respectively. 30 dyne force is inclined to y -axis at an angle 60° . So, the vertical component = $30 \cos 60^\circ = 15$ dyne. The horizontal component = $30 \sin 60^\circ = 15\sqrt{3}$ dyne.

So, the assertion and reason both are true and the reason explains the assertion.

Q. 2. Assertion (A): A projectile is projected at an angle θ and then at an angle $90^\circ - \theta$, keeping the velocity same. In both the cases, the range will be same.

Reason (R): Range of a projectile = $R = \frac{u^2 \sin 2\theta}{g}$

Ans. Option (C) is correct.

Explanation: $R = \frac{u^2 \sin 2\theta}{g}$



NCERT Corner

FreeKItaab

Q. 1. State, for each of the following physical quantities. If it is a scalar or a vector :

Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Ans.

Scalars	Vectors
volume	acceleration
mass	velocity
speed	displacement
density	angular velocity
number of moles	angular frequency

Q. 2. Pick out the two scalar quantities in the following list :

Force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.

Ans. Work, Current.

Q. 3. Pick out the only vector quantity in the following list :

Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Ans. Impulse, as Impulse = Force \times time

Q. 4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :

When angle of projection = $90^\circ - \theta$, then

$$R' = \frac{u^2 \sin 2(90^\circ - \theta)}{g}$$

$$= \frac{u^2 \sin(180^\circ - 2\theta)}{g}$$

$$= \frac{u^2 \sin 2\theta}{g} = R$$

So, the assertion is true but the reason is false.

Q. 3. Assertion (A): For uniform circular motion, the displacement and acceleration are directed towards the centre along the radius.

Reason (R): Centripetal acceleration is represented as $\frac{\omega^2}{R}$.

Ans. Option (D) is correct.

Explanation: For uniform circular motion, the displacement is directed along the tangent and centripetal acceleration is directed towards the centre along the radius. So, the assertion is false. Centripetal acceleration is represented as $\omega^2 R$. So, the reason is also false.

(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Ans. (a) No, adding any two scalars is not meaningful, but the scalars of same dimensions, (i.e., of same nature) can be added.

(b) No, adding a scalar to a vector of the same dimension is not meaningful as a scalar cannot be added to a vector.

(c) Yes, multiplying any vector by any scalar is meaningful in algebraic operation. It is because when any vector is multiplied by any scalar, then we get a vector having magnitude equal to scalar number of times the magnitude of the given vector. For example, when acceleration a is multiplied by mass m , we get force $F = ma$ which is a meaningful operation.

(d) Yes, it is product of two scalar gives a meaningful result. For example, when power P is multiplied by time t , then we get work done (W), i.e., $W = Pt$, which is a useful algebraic operation.

(e) No, as the two vectors of same dimensions (i.e., of the same nature) can only be added. So addition of any two vectors of same dimension is a meaningful algebraic operation.

(f) Addition of a component of a vector to the same vector can be done by the law of vector addition. So algebraic operation is a meaningful operation.

Q. 5. Read each statement below carefully and state with reasons, if it is true or false :

- (a) The magnitude of a vector is always a scalar.
- (b) Each component of a vector is always a scalar.
- (c) The total path length is always equal to be magnitude of the displacement vector of a particle.
- (d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.
- (e) Three vectors not lying in a plane can never add up to give a null vector.

Ans. (a) True, magnitude is pure number.

(b) False, each component is vector.

(c) True, only if particle moves along a straight line and in same direction otherwise false.

(d) True, because-

Total path length \geq Magnitude of displacement vector

(e) True, because these three vectors cannot represent three sides of triangle which taken in same order.

Q. 6. Establish the following vector inequalities geometrically or otherwise:

(a) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

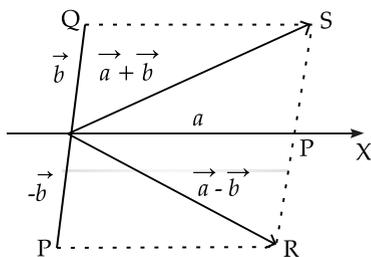
(b) $|\vec{a} + \vec{b}| \geq ||\vec{a}| + |\vec{b}||$

(c) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(d) $|\vec{a} - \vec{b}| \geq ||\vec{a}| - |\vec{b}||$

When does the equality sign above apply?

Ans. (a) As-



Length of one side of triangle is always less than sum of lengths of other two sides.

So, $|\vec{a} + \vec{b}| < |\vec{a}| + |\vec{b}|$ (I)

If the two vectors are acting along same straight line and in same direction, then

$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ (II)

(b) From ΔOPS ,
 $OS + PS > OP$

$\Rightarrow OS > |OP - PS|, OS > |OP - OQ|$ (III)

Modulus of $(OP - PS)$ has been taken, as LHS is positive but RHS may be negative if $OP < PS$.

From eq.(III)

$|\vec{a} + \vec{b}| > ||\vec{a}| - |\vec{b}||$ (IV)

If \vec{a} and \vec{b} are acting along straight line but in opposite direction,

$|\vec{a} + \vec{b}| = ||\vec{a}| - |\vec{b}||$ (V)

from eq (IV) & (V)

$|\vec{a} + \vec{b}| \geq ||\vec{a}| - |\vec{b}||$

(c) From ΔORP , $OR < OP + PR$

So, $|\vec{a} - \vec{b}| < ||\vec{a}| + |\vec{b}||$ (VI)

Now, two vectors are acting along straight line in opposite direction-

$|\vec{a} - \vec{b}| = ||\vec{a}| + |\vec{b}||$ (VII)

from eq. (VI) & (VII)

$|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(d) From ΔOPR , $OR > |OP - PR|$

$|\vec{OP} - \vec{PR}|$ has been taken, because LHS = positive,

RHS may be negative

if $OP < PR$,

$\therefore |\vec{a} - \vec{b}| > ||\vec{a}| - |\vec{b}||$

Q. 7. Given $\vec{a} + \vec{b} + \vec{c} - \vec{d} = 0$, which of the following statements are correct:

(a) $(\vec{a}, \vec{b}, \vec{c})$ and \vec{d} must each be a null vector,

(b) The magnitude of $(\vec{a} + \vec{c})$ equals the magnitude of $(\vec{b} + \vec{d})$,

(c) The magnitude of \vec{a} can never be greater than the sum of the magnitudes of \vec{b}, \vec{c} and \vec{d} ,

(d) $\vec{b} + \vec{c}$ must lie in the plane of \vec{a} and \vec{d} if \vec{a} and \vec{d} are not collinear, and in the line of \vec{a} and \vec{d} , if they are collinear?

Ans. (a) Incorrect

In order to make, $\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$ it is not necessary to have all the four given vectors to be null vectors. There are many other combinations which can give the sum zero.

(b) Correct

$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$

$\vec{a} + \vec{c} = -(\vec{b} + \vec{d})$

Taking modulus on both the sides, we get:

$$|\vec{a} + \vec{c}| = |-(\vec{b} + \vec{d})| = |\vec{b} + \vec{d}|$$

Hence, the magnitude of $(\vec{a} + \vec{c})$ is the same as the magnitude of $(\vec{b} + \vec{d})$

(c) Correct

$$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$$

$$\vec{a} = -(\vec{b} + \vec{c} + \vec{d})$$

Taking modulus both sides, we get the magnitude of \vec{a} to be equal to the magnitude of $(\vec{b} + \vec{c} + \vec{d})$:

$$|\vec{a}| = |(\vec{b} + \vec{c} + \vec{d})|$$

$$|\vec{a}| \leq |\vec{b}| + |\vec{c}| + |\vec{d}| \quad \dots(i)$$

Now, $(\vec{b} + \vec{c} + \vec{d})$ is the sum of vectors \vec{b} , \vec{c} and \vec{d} . Therefore, the magnitude of $(\vec{b} + \vec{c} + \vec{d})$ is less than, or equal to the sum of the magnitudes of \vec{b} , \vec{c} and \vec{d} . Hence, the magnitude of \vec{a} can never be greater than the sum of the magnitudes of \vec{b} , \vec{c} and \vec{d} . Equation (i) shows that the magnitude of \vec{a} is equal to or less than the sum of the magnitudes of \vec{b} , \vec{c} and \vec{d} .

(d) Correct

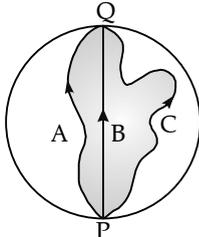
$$\text{For, } \vec{a} + \vec{b} + \vec{c} + \vec{d} = \vec{0}$$

$$\vec{a} + (\vec{b} + \vec{c}) + \vec{d} = \vec{0}$$

The resultant sum of the three vectors \vec{a} , $(\vec{b} + \vec{c})$, and \vec{d} can be zero only if $(\vec{b} + \vec{c})$ lie in the same plane as \vec{a} and \vec{d} .

If \vec{a} and \vec{d} are collinear, then it implies that the vector $(\vec{b} + \vec{c})$ is in the line of \vec{a} and \vec{d} . This implication holds true in this scenario and the vector sum of all the vectors will be zero.

- Q. 8.** Three girls A, B and C were skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Figure. (i) What is the magnitude of the displacement vector for each? (ii) For which girl is this equal to the actual length of path skate?

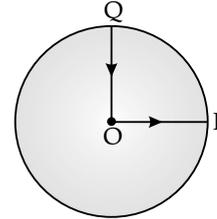


- Ans. (i)** PQ = diameter
= displacement for each girl
= $2r = 2 \times 200 = 400$ m

Since, displacement vector does not depend upon the actual path length and it is the shortest distance between initial and final position, so in the case of each girl the displacement is 400 m.

- (ii) For girl B, the displacement is equal to the actual length of path skate.

- Q. 9.** A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity and (c) average speed of the cyclist?



Ans. Given, radius of circular park = 1 km

- (a) As cyclist returns to its initial state, therefore the net displacement of the cyclist is zero.

(b) Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}}$

$$= \frac{0}{\text{Total time taken}}$$

$$= 0$$

- (c) Total distance travelled by the cyclist

$$= OP + \text{arc } PQ + QO$$

$$= r + \left(\frac{1}{4} \times 2\pi r\right) + r$$

$$= 1 + \left(\frac{1}{2} \times \frac{22}{7} \times 1\right) + 1$$

$$= 2 + \frac{11}{7}$$

$$= \frac{25}{7} \text{ km}$$

Total time taken = 10 min

$$= \frac{10}{60} \text{ h} = \frac{1}{6} \text{ h}$$

∴ Average speed of the cyclist

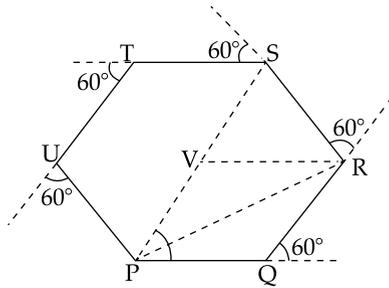
$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{25/7}{1/6} = \frac{150}{7}$$

$$= 21.43 \text{ km/h}$$

- Q. 10.** On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Ans. The path followed by the motorist is a regular hexagon with side 500 m, as shown in the given figure



Let the motorist start from point P.
The motorist takes the third turn at S.
 \therefore Magnitude of displacement = $PS = PV + VS$
 $= 500 + 500 = 1000$ m
Total path length = $PQ + QR + RS = 500 + 500 + 500 = 1500$ m
The motorist takes the sixth turn at point P, which is the starting point.

\therefore Magnitude of displacement = 0
Total path length = $PQ + QR + RS + ST + TU + UP$

$$= 500 + 500 + 500 + 500 + 500 + 500 = 3000 \text{ m}$$

The motorist takes the eighth turn at point R

\therefore Magnitude of displacement = PR

$$= \sqrt{PQ^2 + QR^2 + 2(PQ)(QR)\cos 60^\circ}$$

$$= \sqrt{500^2 + 500^2 + (2 \times 500 \times 500 \times \cos 60^\circ)}$$

$$= \sqrt{250000 + 250000 + \left(500000 \times \frac{1}{2}\right)}$$

$$= 866.03 \text{ m}$$

Total path length

$$= \text{Circumference of the hexagon} + PQ + QR$$

$$= 6 \times 500 + 500 + 500 = 4000 \text{ m}$$

The magnitude of displacement and the total path length corresponding to the required turns is shown in the given table

Turn	Magnitude of displacement (m)	Total path length (m)
Third	1000	1500
Sixth	0	3000
Eighth	866.03	4000

Q. 11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is

- (a) the average speed of the taxi,
(b) the magnitude of average velocity? Are the two equal?

Ans. (a) Total distance travelled = 23 km

$$\text{Total time taken} = 28 \text{ min} = \frac{28}{60} \text{ h}$$

$$\therefore \text{Average speed of the taxi} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$\Rightarrow \frac{23}{\left(\frac{28}{60}\right)} = 49.29 \text{ km/h}$$

(b) Distance between the hotel and the station = 10 km = Displacement of the car

$$\therefore \text{Average velocity} = \frac{10}{\left(\frac{28}{60}\right)} = 21.43 \text{ km/h}$$

Therefore, the two physical quantities (average speed and average velocity) are not equal.

Q. 12. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall?

[A1] [NCERT TB Q. No. 15]

Ans. Step 1 : Using

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

when

$$H = 25 \text{ m}, u = 40 \text{ m/s}$$

and

$$g = 9.8 \text{ m/s}^2$$

$$25 = \frac{40^2 \sin^2 \theta}{2 \times 9.8}$$

$$\text{i.e., } \sin^2 \theta = \frac{490}{40^2}$$

$$\sin \theta = \frac{\sqrt{490}}{40} = 0.5534$$

$$\text{i.e., } \theta = 33.6^\circ$$

Step 2 :

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{40^2 \sin 2(33.6)}{9.8} \text{ m}$$

$$= \frac{40^2 \sin 67.2}{9.8} \text{ m}$$

$$= \frac{40^2 \times 0.9219}{9.8} \text{ m}$$

$$= 150.514 \text{ m}$$

Q. 13. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball? **[NCERT TB Q. No. 16]**

Ans. From the formula the horizontal range is given by

$$R = \frac{u^2 \sin 2\theta}{g} \quad \dots(i)$$

$$\text{For } R = R_{\max}, \theta = 45^\circ, \text{ i.e., } \sin 2\theta = \sin 90^\circ = 1$$

Putting the given value is eq. (i), we

$$\therefore R_{\max} = \frac{u^2}{g}$$

$$\Rightarrow 100 = \frac{u^2}{g} \quad (\because R_{\max} = 100 \text{ given}) \dots(\text{ii})$$

Suppose H = height upto which the ball goes when the cricketer throws it with velocity u . Since, the final velocity of the ball, $v = 0$.

\therefore Applying the relation, $v^2 - u^2 = 2as$,
(\because here, $v = 0, a = -g, s = H$)

$$\text{or} \quad H = \frac{u^2}{2g}$$

$$H = \frac{1}{2} \left(\frac{u^2}{g} \right) \\ = \frac{1}{2} \times (100) \quad [\text{by using (ii)}]$$

$$H = 50 \text{ m.}$$

Q. 14. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ? [A1] [NCERT TB Q. No. 17]

Ans. Given, Radius of the horizontal circle, $r = 80 \text{ cm} = 0.80 \text{ m}$, frequency = $f = \frac{14}{25} \text{ rps}$

Angular speed of revolution of the stone is given by

$$\omega = 2\pi f \\ \Rightarrow \omega = 2 \times \frac{22}{7} \times \left(\frac{14}{25} \right) \\ \Rightarrow \omega = \frac{88}{25} \text{ rad/sec.}$$

\therefore Magnitude of acceleration produced in the stone will be equal to the magnitude of centripetal acceleration.

$$= r\omega^2 \\ = 0.80 \times \left(\frac{88}{25} \right)^2 \\ = 0.80 \times \frac{88}{25} \times \frac{88}{25} \\ = 9.90 \text{ ms}^{-2}$$

We know that, the direction of the acceleration is towards the centre of the circle along its radius.

Q. 15. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity. [NCERT TB Q. No. 18]

Ans. Given : $r = 1 \text{ km} = 1000 \text{ m}$;
 $v = 900 \text{ kmh}^{-1}$
 $= 900 \times \frac{1000}{3600} \text{ ms}^{-1}$
 $= 250 \text{ ms}^{-1}$

The centripetal acceleration of the aircraft is

$$a = \frac{v^2}{r} = \frac{(250)^2}{1000}$$

$$= \frac{62500}{1000} = 62.5 \text{ ms}^{-2}$$

Acceleration due to gravity,
 $g = 9.8 \text{ ms}^{-2}$

$$\therefore \frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{a}{g} \\ = \frac{62.5}{9.8}$$

or $a = 6.38 g$

Q. 16. Read each statement below carefully and state, with reasons, if it is true or false :

- The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.
- The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
- The acceleration vector of a particle in uniform circular motion over one cycle is a null vector.

[NCERT TB Q. No. 19]

Ans. (a) **False**—The net acceleration of a particle in circular motion is towards the centre only if its speed is constant.

(b) **True**—A particle released at any point of its path will always move along the tangent to the path at the point.

(c) **True**—For any two diametrically opposite points on the circumference, the acceleration vectors are equal and opposite. Hence, the acceleration vector average over one completely cycle is null vector.

Q. 17. The position vector of a particle is given by

$$\vec{r} = 3.0t \hat{i} - 2.0t^2 \hat{j} + 4.0\hat{k} \text{ m}$$

where t is in seconds and the coefficients have the proper units for \vec{r} to be in metres.

- Find the \vec{v} and \vec{a} of the particle ?
- What is the magnitude and direction of velocity of the particle at $t = 2.0 \text{ s}$? [NCT 2009, 10]

[NCERT TB Q. No. 20]

Ans. The position vector (\vec{r}) of the particle is

$$\vec{r} = 3.0t \hat{i} - 2.0t^2 \hat{j} + 4.0\hat{k} \text{ m} \quad \dots(\text{i})$$

(a) velocity $\vec{v}(t)$ of the particle is given by

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt} (3.0t \hat{i} - 2.0t^2 \hat{j} + 4.0\hat{k}) \\ \vec{v}(t) = 3\hat{i} - 4t \hat{j} + 0 \quad \dots(\text{ii})$$

Also, acceleration $\vec{a}(t)$ of the particle is given by

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d}{dt} (3\hat{i} - 4t \hat{j})$$

$$= \frac{d}{dt}(3\hat{i} - 4t\hat{j}) \text{ [by using (ii)]}$$

$$= 0 - 4\hat{j}$$

$$\vec{a}(t) = -4\hat{j} \text{ ms}^{-2} \quad \dots\text{(iii)}$$

(b) At time t , the velocity of the particle is given by using to equation (ii).

$$\vec{v}(t) = 3.0\hat{i} - 4t\hat{j} \quad \text{[by using (ii)]}$$

$$\therefore \text{At } t = 2\text{s,}$$

$$v = 3.0\hat{i} - 4 \times 2\hat{j}$$

$$= 3.0\hat{i} - 8.0\hat{j}$$

\therefore Its magnitude is

$$v = \sqrt{3^2 + (-8)^2}$$

$$= \sqrt{9 + 64}$$

$$= \sqrt{73} = 8.544 \text{ ms}^{-1}$$

and, direction of v is given by

$$\therefore \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$= \tan^{-1}\left(\frac{-8}{3}\right)$$

$$= -70^\circ \text{ with } x\text{-axis}$$

Q. 18. A particle starts from the origin at $t = 0$ s with a velocity of $10.0 \hat{j}$ and moves in the x - y plane with a constant acceleration of $(8.0 \hat{i} + 2.0 \hat{j}) \text{ ms}^{-2}$

(a) At what time is the x -coordinate of the particle 16 m? What is the y -coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

[NCERT TB Q. No. 21]

Ans. (a) Velocity of the particle, $\vec{u} = 10.0 \hat{j} \text{ m/s}$

Acceleration of the particle $\vec{a} = (8.0 \hat{i} + 2.0 \hat{j})$

Also,

$$\text{But, } \vec{a} = \frac{d\vec{v}}{dt} = 8.0\hat{i} + 2.0\hat{j}$$

$$d\vec{v} = (8.0\hat{i} + 2.0\hat{j})dt$$

Integrating both sides:

$$\vec{v} = 8.0\hat{i}t + 2.0\hat{j}t + \vec{u}$$

Where,

\vec{u} = Velocity vector of the particle at $t = 0$

\vec{v} = Velocity vector of the particle at time t

$$\text{Again, } \vec{v} = \frac{d\vec{r}}{dt}$$

$$d\vec{r} = \vec{v} dt = (8.0\hat{i}t + 2.0\hat{j}t + \vec{u})dt$$

Integrating

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\hat{i} + \frac{1}{2}2.0t^2\hat{j}$$

$$= \vec{u}t + 4.0t^2\hat{i} + t^2\hat{j}$$

$$= (10.0\hat{j})t + 4.0t^2\hat{i} + t^2\hat{j}$$

$$x\hat{i} + y\hat{j} = 4.0t^2\hat{i} + (10t + t^2)\hat{j}$$

Since, the motion of the particle is confined to the

x - y plane, on equating the coefficients of \hat{i} and \hat{j} , we get:

$$x = 4t^2$$

$$\text{or, } t = \left(\frac{x}{4}\right)^{1/2}$$

$$\text{And } y = 10t + t^2$$

When $x = 16\text{m}$:

$$t = \left(\frac{16}{4}\right)^{1/2} = 2\text{s}$$

$$\therefore y = 10 \times 2 + (2)^2 = 24\text{m}$$

(b) Velocity of the particle is given by:

$$\vec{v} = 8.0t\hat{i} + 2.0t\hat{j} + \vec{u}$$

at $t = 2\text{s}$

$$\vec{v} = 8.0 \times 2\hat{i} + 2.0 \times 2\hat{j} + 10\hat{j}$$

$$= 16\hat{i} + 14\hat{j}$$

\therefore Speed of the particle:

$$|\vec{v}| = \sqrt{(16)^2 + (14)^2}$$

$$= \sqrt{256 + 196} = \sqrt{452}$$

$$= 21.26 \text{ m/s}$$

Q. 19. (i) \hat{i} and \hat{j} are unit vector along X -and Y -axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

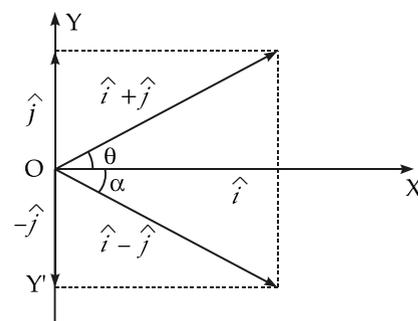
(ii) What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? You may use graphical method.

[NCERT TB Q. No. 22]

Ans. (i) Magnitudes of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$ vectors: \hat{i}

and \hat{j} are the unit vectors along X -and Y -axis respectively.

$$|\hat{i}| = |\hat{j}| = 1$$



The magnitude of vector $\hat{i} + \hat{j}$ is given by

$$\begin{aligned}
 |\hat{i} + \hat{j}| &= \sqrt{1^2 + 1^2 + 2 \times 1 \times 1 \times \cos 90^\circ} \\
 &= \sqrt{2+0} = \sqrt{2} \\
 &= 1.414 \text{ units.}
 \end{aligned}$$

The magnitude of vector $\hat{i} - \hat{j}$ is given by

$$\begin{aligned}
 |\hat{i} - \hat{j}| &= \sqrt{1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 90^\circ} \\
 &= \sqrt{2-0} = \sqrt{2} \\
 &= 1.414 \text{ units.}
 \end{aligned}$$

Directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$:

Let θ be the angle made by $\hat{i} + \hat{j}$ with \hat{i} , i.e., with X-axis.

By definition of scalar product of two vectors

$$(\hat{i} + \hat{j}) \cdot \hat{i} = |\hat{i} + \hat{j}| |\hat{i}| \cos \theta$$

or $\hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{i} = \sqrt{2} \times 1 \times \cos \theta$

or $1 + 0 = \sqrt{2} \cos \theta$

($\because \hat{i} \cdot \hat{i} = 1$ and $\hat{i} \cdot \hat{j} = 0$)

or $\cos \theta = \frac{1}{\sqrt{2}} = \cos 45^\circ$
 $\theta = 45^\circ$

Also let α be the angle made by $\hat{i} - \hat{j}$ with \hat{i} , i.e., with X-axis

Then $(\hat{i} - \hat{j}) \cdot \hat{i} = |\hat{i} - \hat{j}| |\hat{i}| \cos \alpha$

or $\hat{i} \cdot \hat{i} - \hat{j} \cdot \hat{i} = \sqrt{2} \times 1 \times \cos \alpha$

or $1 - 0 = \sqrt{2} \cos \alpha$

or $\cos \alpha = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$\alpha = 45^\circ$

As $-\hat{j}$, is -ve, so $\hat{i} - \hat{j}$ makes -45° with $-\hat{j}$.

Thus, $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$ act at 45° and -45° with X-axis respectively.

(ii) (a) Let us now determine the component of

$\vec{A} = 2\hat{i} + 3\hat{j}$ in the direction of $(\hat{i} + \hat{j})$

Let $\vec{B} = \hat{i} + \hat{j}$

$\therefore \vec{A} \cdot \vec{B} = AB \cos \beta$
 $= (A \cos \beta) B$

or $A \cos \beta = \frac{\vec{A} \cdot \vec{B}}{B}$

Magnitude of the component of \vec{A} in the direction of \vec{B} , i.e., $(\hat{i} + \hat{j})$ is $A \cos \beta$

$$= \frac{\vec{A} \cdot \vec{B}}{B} = \frac{(2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|}$$

$$= \frac{(2\hat{i} \cdot \hat{i} + 3\hat{j} \cdot \hat{j})}{\sqrt{2}}$$

($\because \hat{j} \cdot \hat{i} = \hat{i} \cdot \hat{j} = 0$)

$$= \frac{2+3}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

If \hat{n}_1 = unit vector along $(\hat{i} + \hat{j})$, then

$$\hat{n}_1 = \frac{(\hat{i} + \hat{j})}{|\hat{i} + \hat{j}|} = \frac{(\hat{i} + \hat{j})}{\sqrt{2}}$$

\therefore Component of A along $(\hat{i} + \hat{j})$ = Magnitude of the component of A along $\hat{i} + \hat{j}$ i.e., \hat{n}_1

$$= \frac{5}{\sqrt{2}} \cdot \frac{(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{5}{2}(\hat{i} + \hat{j})$$

(b) Let us now determine the component of \vec{A} along $\hat{i} - \hat{j}$.

Let \hat{n}_2 = unit vector acting along $\hat{i} - \hat{j}$

$$\hat{i} - \hat{j} = |\hat{i} - \hat{j}| \hat{n}_2 = \sqrt{2} \hat{n}_2$$

$\therefore \hat{n}_2 = \frac{(\hat{i} - \hat{j})}{\sqrt{2}}$

\therefore Magnitude of the component of \vec{A} along $(\hat{i} - \hat{j})$

$$= \vec{A} \cdot \hat{n}_2 = (2\hat{i} + 3\hat{j}) \cdot \frac{\hat{i} - \hat{j}}{\sqrt{2}}$$

$$= \frac{2-3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

\therefore Component of \vec{A} along $(\hat{i} - \hat{j})$

$$= \left(\frac{\vec{A} \cdot \hat{n}_2}{\hat{n}_2} \right) \hat{n}_2$$

$$= \left(-\frac{1}{\sqrt{2}} \right) \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right)$$

$$= -\frac{1}{2}(\hat{i} - \hat{j})$$

Q. 20. For any arbitrary motion in space, which of the following relations are true:

(a) $\vec{v}_{average} = \left(\frac{1}{2} \right) (\vec{v}(t_1) + \vec{v}(t_2))$

(b) $\vec{v}_{average} = \frac{[\vec{r}(t_2) - \vec{r}(t_1)]}{(t_2 - t_1)}$

- (c) $\vec{v}(t) = \vec{v}(0) + \vec{a}t$
 (d) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + \left(\frac{1}{2}\right)\vec{a}t^2$
 (e) $\vec{a}_{average} = \frac{[\vec{v}(t_2) - \vec{v}(t_1)]}{(t_2 - t_1)}$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

[NCERT TB Q. No. 23]

Ans. (a) False.

It is given that the motion of the particle is arbitrary. Therefore, the average velocity of the particle cannot be given by this equation.

- (b) True.
The arbitrary motion of the particle can be represented by this equation.
 (c) False.
The motion of the particle is arbitrary. The acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of the particle in space.
 (d) False.
The motion of the particle is arbitrary; acceleration of the particle may also be non-uniform. Hence, this equation cannot represent the motion of particle in space.
 (e) True.
The arbitrary motion of the particle can be represented by this equation.

Q. 21. Read each statement below carefully and state, with reasons and examples, if it is true or false :

- (a) A scalar quantity is the one that is conserved in a process.
 (b) A scalar quantity is the one that can never take negative values.
 (c) A scalar quantity must be dimensionless
 (d) A scalar quantity is the one that does not vary from one point to another in space.
 (e) A scalar quantity has the same value for observers with different orientations of the axes.

[NCERT TB Q. No. 24]

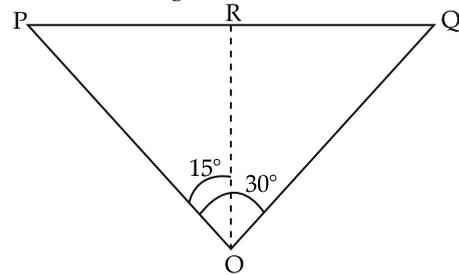
Ans. (d) *Explanation:*

- (a) **False**, because energy is not conserved during inelastic collision.
 (b) **False**, because temperature and electrical potential energy can be negative.
 (c) **False**, scalar quantity like mass density are not dimensionless.
 (d) **False**, because gravitational potential energy vary from point to point.
 (e) **True**, as the R values of scalar does not change with orientation of axes.

Q. 22. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30° , what is the speed of the aircraft?

[AI] [NCERT TB Q. No. 25]

Ans. The positions of the observer and the aircraft are shown in the figure.



Height of the aircraft from ground, $OR = 3400$ m
 Angle subtended between the positions, $\angle POQ = 30^\circ$
 Time = 10 s

In ΔPRO :

$$\tan 15^\circ = \frac{PR}{OR}$$

$$PR = OR \tan 15^\circ = 3400 \times \tan 15^\circ$$

ΔPRO is similar to ΔRQO .

$$\therefore PR = RQ$$

$$PQ = PR + RQ$$

$$= 2PR = 2 \times 3400 \tan 15^\circ$$

$$= 6800 \times 0.268 = 1822.4 \text{ m}$$

$$\therefore \text{Speed of the aircraft} = \frac{1822.4}{10} = 182.24 \text{ m/s}$$



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

Q. 1. The angle between $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$ is

- (A) 45° (B) -45°
 (C) 90° (D) 180°

Ans. Option (C) is correct.

Explanation: Given :

$$\vec{A} = \hat{i} + \hat{j}, \vec{B} = \hat{i} - \hat{j}$$

$$\therefore |\vec{A}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2},$$

$$|\vec{B}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$= \frac{(\hat{i} + \hat{j})(\hat{i} - \hat{j})}{\sqrt{2} \times \sqrt{2}} = 0$$

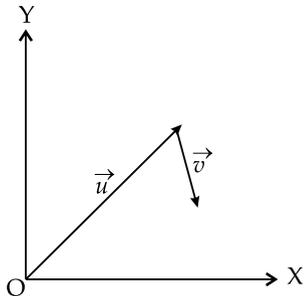
$$\cos \theta = \cos 90^\circ$$

$$\therefore \theta = 90^\circ$$

Q. 2. Figure shows the orientation of two vectors u and v in the XY plane.

If $\vec{u} = a\hat{i} + b\hat{j}$ and

$\vec{v} = p\hat{i} + q\hat{j}$



Which of the following is correct?

- (A) a and p are positive while b and q are negative.
- (B) a, p and b are positive while q is negative.
- (C) a, q and b are positive while p is negative.
- (D) a, b, p and q are all positive.

Ans. Option (B) is correct.

Explanation: According to figure, in $\vec{u} = a\hat{i} + b\hat{j}$, both a & b are positive while in $\vec{v} = p\hat{i} + q\hat{j}$, p is positive and q is negative. Thus a, b and p are positive and q is negative.

Q. 3. The component of a vector \vec{r} along x -axis will have maximum value if

- (A) \vec{r} is along positive Y-axis
- (B) \vec{r} is along positive X-axis
- (C) \vec{r} makes an angle of 45° with the X-axis
- (D) \vec{r} is along negative Y-axis

Ans. Option (B) is correct.

Explanation: If \vec{r} is at angle θ with x -axis, then component r of \vec{r} along x -axis $= r \cos \theta$. It will be maximum if $\cos \theta = \text{maximum} = 1$ or $\theta = 0^\circ$ i.e., \vec{r} is along positive x -axis.

Q. 4. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45° , its range will be

- (A) 60 m
- (B) 71 m
- (C) 100 m
- (D) 141 m

Ans. Option (C) is correct.

Explanation: $R = \frac{u^2 \sin 2\theta}{g}$; $R \propto \sin 2\theta$

$$\therefore \frac{R_1}{R_2} = \frac{\sin 2 \times 15^\circ}{\sin 2 \times 45^\circ} = \frac{\sin 30^\circ}{\sin 90^\circ} = \frac{1}{2}$$

$$R_2 \Rightarrow 2R_1 \Rightarrow 2 \times 50\text{m} = 100\text{m}$$

Q. 5. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are :

- (A) Impulse, pressure and area
- (B) Impulse and area
- (C) Area and gravitational potential
- (D) Impulse and pressure

Ans. Option (B) is correct.

Explanation: Impulse is a vector quantity. Area of a surface is a vector which is along normal to the surface in the outward direction.

Q. 6. Three vectors \vec{A} , \vec{B} and \vec{C} add up to zero. Find which is false.

- (A) $(\vec{A} \times \vec{B}) \times \vec{C}$ is not zero unless \vec{B}, \vec{C} are parallel.
- (B) $(\vec{A} \times \vec{B}) \cdot \vec{C}$ is not zero unless \vec{B}, \vec{C} are parallel.
- (C) If $\vec{A}, \vec{B}, \vec{C}$ define a plane, $(\vec{A} \times \vec{B}) \times \vec{C}$ is in that plane.
- (D) $(\vec{A} \times \vec{B}) \cdot \vec{C} = |\vec{A}| |\vec{B}| |\vec{C}| \rightarrow C^2 = A^2 + B^2$.

Ans. Option (C) is correct.

Q. 7. It is found that $|\vec{A} + \vec{B}| = |\vec{A}|$. This necessarily implies,

- (A) $\vec{B} = \vec{1}$.
- (B) \vec{A}, \vec{B} are antiparallel.
- (C) \vec{A}, \vec{B} are perpendicular.
- (D) $\vec{A} \cdot \vec{B} \leq 0$.

Ans. Option (B) is correct.

Explanation: If $|\vec{A} + \vec{B}| = |\vec{A}|$, then either $\vec{B} = 0$ or $\vec{B} = -2\vec{A}$. Both are satisfied when \vec{A} and \vec{B} are anti-parallel.

Very Short Answer Type Questions

Q. 8. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown Fig. If he maintains constant speed of

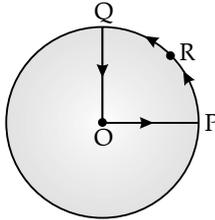
10 ms^{-1} , what is his acceleration at point R in magnitude and direction? [AI]

[NCERT Exemplar Q. No. 16]

Ans. Centripetal acceleration at R is given by the relation,

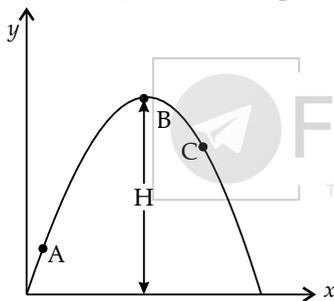
$$a_c = \frac{v^2}{r}$$

$$\Rightarrow a_c = \frac{v^2}{R} = \frac{(10)^2}{1000} = \frac{100}{10^3} = 0.1 \text{ m/s}^2 \text{ along RO.}$$

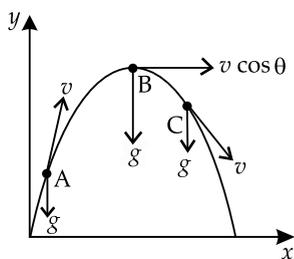


Q. 9. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig. where x and y indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.

[NCERT Exemplar Q. No. 17]



Ans. Direction of the velocities at point A, B and C are along the tangent of the path at that point. Acceleration at each point is the acceleration due to gravity vertically downwards.



Q. 10. A ball thrown from a roof top at an angle of 45° above the horizontal. It hits the ground a few seconds later. At what point during motion, does the ball have

- (a) greatest speed.
- (b) smallest speed.
- (c) greatest acceleration.

Explain. [NCERT Exemplar Q. No. 18]

Ans. (a) It has greatest speed just before it hits the ground.
(b) It has smallest speed at the highest point reached.
(c) Acceleration is always constant throughout the journey and is vertically downward equal to g .

Q. 11. A football is kicked into the air vertically upwards. What is its

- (a) acceleration, and (b) velocity at the highest point?

[NCERT Exemplar Q. No. 19]

Ans. (a) Acceleration will always be vertical downward and is called acceleration due to gravity (g).
(b) When football reaches the highest point it is momentarily at rest velocity, $v=0$.

As, it is continuously retarded by acceleration due to gravity (g).

Q. 12. A, B and C are three non-collinear, non coplanar vectors. What can you say about direction of $\vec{A} \times (\vec{B} \times \vec{C})$? [NCERT Exemplar Q. No. 20]

Ans.

$$\vec{B} \times \vec{C} = BC \sin \theta \hat{n}$$

\hat{n} = unit vector perpendicular to the plane containing \vec{B} and \vec{C}

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \hat{n} (BC \sin \theta)$$

$$= (BC \sin \theta) A \sin \alpha \hat{P}$$

\hat{P} = unit vector perpendicular to the plane containing \vec{A} and $(\vec{B} \times \vec{C})$

Hence, $\vec{A} \times (\vec{B} \times \vec{C})$ will lie in the plane of \vec{B} and \vec{C} , and is perpendicular to \vec{A} .

Short Answer Type Questions

Q. 13. A boy travelling in an open car moving on a levelled road with constant speed tosses a ball vertically up in the air and catches it back. Sketch the motion of the ball as observed by a boy standing on the footpath. Give explanation to support your diagram.

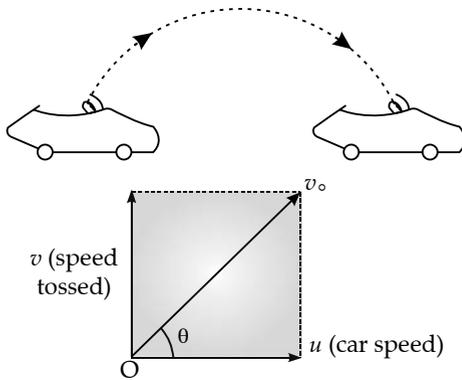
[NCERT Exemplar Q. No. 21]

Ans The horizontal speed of the ball is same as that of the car, therefore ball as well as car travels

equal horizontal distance. u = car speed of car, v = vertical speed of ball

$$v_0 = \sqrt{u^2 + v^2}$$

$$\theta = \tan^{-1} \left(\frac{v}{u} \right)$$



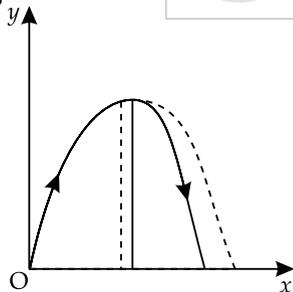
For a ground-based observer, the ball is a projectile with speed v_0 and the angle of projection θ with horizontal in as shown above.

Q. 14. In dealing with motion of projectile in air, we ignore effect of air resistance on motion. This gives trajectory as a parabola as you have studied. What would the trajectory look like if air resistance is included? Sketch such a trajectory and explain why you have drawn it that way.

[NCERT Exemplar Q. No. 23]

Ans. If air resistance is included the horizontal component of velocity will not be constant and obviously, trajectory will change.

Due to air resistance, energy as well as horizontal component of velocity of the particle keep on decreasing making the fall steeper as shown in the figure.



Q. 15. (a) Earth can be thought of as a sphere of radius 6400 km. Any object (or a person) is performing circular motion around the axis of Earth due to Earth's rotation (period 1 day). What is acceleration of object on the surface of the Earth (at equator) towards its centre? What is it at latitude θ ? How does these accelerations compare with $g = 9.8 \text{ m/s}^2$?

(b) Earth also moves in circular orbit around Sun once every year with on orbital radius of $1.5 \times 10^{11} \text{ m}$. What is the acceleration of Earth (or any object on the surface of the Earth) towards the centre of the Sun? How does this acceleration compare with $g = 9.8 \text{ m/s}^2$?

$$\left(\text{Hint: acceleration} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \right)$$

[NCERT Exemplar Q. No. 25]

Ans (a) Radius of Earth, $R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$
Time period, $T = 1 \text{ day} = 24 \times 60 \times 60 = 26400 \text{ s}$

$$\begin{aligned} a_c &= \frac{4\pi^2 R}{T^2} \\ &= \frac{4 \times (22/7)^2 \times 6.4 \times 10^6}{(24 \times 60 \times 60)^2} \\ &= \frac{4 \times 484 \times 6.4 \times 10^6}{49 \times (24 \times 3600)^2} = 0.034 \text{ m/s}^2 \end{aligned}$$

At latitude θ , radius of circular path is $R \cos \theta$.
So, at latitude θ ,

$$\begin{aligned} a_c &= 0.034 \cos \theta \text{ m/s}^2 \\ \therefore \text{ at equator, } \frac{a_c}{g} &= \frac{0.034}{9.8} = \frac{1}{288} \end{aligned}$$

at latitude θ ,

$$\frac{a_c}{g} = \frac{\cos \theta}{288}$$

(b) Orbital radius = $1.5 \times 10^{11} \text{ m}$

$$\begin{aligned} T = 1 \text{ year} &= 365 \text{ days} = 365 \times 26400 \\ &= 3.15 \times 10^7 \text{ s} \end{aligned}$$

$$\begin{aligned} a_c &= \frac{4\pi^2 R}{T^2} \\ &= \frac{4 \times \left(\frac{22}{7}\right)^2 \times 1.5 \times 10^{11}}{(3.15 \times 10^7)^2} \\ &= 5.97 \times 10^{-3} \text{ m/s}^2 \\ \therefore \frac{a_c}{g} &= \frac{5.9 \times 10^{-3}}{9.8} = \frac{1}{1642} \end{aligned}$$

Q. 16. If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, then match the relations

in column I with the angle θ between \vec{A} and \vec{B} in column II.

Column I		Column II	
(a)	$\vec{A} \cdot \vec{B} = 0$	(i)	$\theta = 0^\circ$
(b)	$\vec{A} \cdot \vec{B} = +8$	(ii)	$\theta = 90^\circ$
(c)	$\vec{A} \cdot \vec{B} = 4$	(iii)	$\theta = 180^\circ$
(d)	$\vec{A} \cdot \vec{B} = -8$	(iv)	$\theta = 60^\circ$

[NCERT Exemplar Q. No. 27]

Ans. (a) (ii) because $\cos 90^\circ = 0$, hence $|\vec{A}| |\vec{B}| \cos 90^\circ = 2 \times 4 \times 0 = 0$

(b) (i) because $\cos 0^\circ = 1$, hence $|\vec{A}| |\vec{B}| \cos 0^\circ = 2 \times 4 \times 1 = +8$

(c) (iv) because $\cos 60^\circ = \frac{1}{2}$, hence $|\vec{A}| |\vec{B}| \cos 60^\circ = 2 \times 4 \times \frac{1}{2} = +4$

(d) (iii) because $\cos 180^\circ = -1$, hence $|\vec{A}| |\vec{B}| \cos$

$$180^\circ = 2 \times 4 \times (-1) = -8$$

Q. 17. If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, then match the relations

in column I with the angle θ between A and B in column II. AI

	Column I		Column II
(a)	$ \vec{A} \times \vec{B} = 0$	(i)	$\theta = 30^\circ$
(b)	$ \vec{A} \times \vec{B} = 8$	(ii)	$\theta = 45^\circ$
(c)	$ \vec{A} \times \vec{B} = 4$	(iii)	$\theta = 90^\circ$

(d) $ \vec{A} \times \vec{B} = 4\sqrt{2}$	(iv) $\theta = 0^\circ$
--	-------------------------

[NCERT Exemplar Q. No. 28]

Ans. (a) (iv) because $\sin \theta = 0^\circ$, hence $|\vec{A}| |\vec{B}| \sin 0^\circ = 2 \times 4 \times 0 = 0$

(b) (iii) because $\sin 90^\circ = 1$, hence $|\vec{A}| |\vec{B}| \sin 90^\circ = 2 \times 4 \times 1 = 8$

(c) (i) because $\sin 30^\circ = \frac{1}{2}$,

$$\text{hence } |\vec{A}| |\vec{B}| \sin 30^\circ = 2 \times 4 \times \frac{1}{2} = 4$$

(d) (ii) because $\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$,

$$\text{hence, } |\vec{A}| |\vec{B}| \sin 45^\circ = 2 \times 4 \times \frac{\sqrt{2}}{2} = 4\sqrt{2}$$



Long Answer Type Questions

Q. 18. A hill is 500 m high. Supplies are to be sent across the hill using a cannon that can hurl packets at a speed of 125 m/s over the hill. The cannon is located at a distance of 800m from the foot of hill and can be moved on the ground at a speed of 2 m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10 \text{ m/s}^2$.

AI [NCERT Exemplar Q. No. 29]

Ans. Given: Speed of packets = 125 m/s

$$\text{Height of hill} = 500 \text{ m}$$

$$d = 800 \text{ m}$$

To cross the hill in shortest time the vertical velocity should be such that it just crosses the height of hill.

The minimum vertical velocity required for crossing the hill is given by

$$v_{\perp}^2 \geq 2gh = 10,000$$

$$v_{\perp} > 100 \text{ m/s}$$

As cannon can haul packets with a speed of 125m/s, so the maximum value of horizontal velocity, v_{\perp} will be

$$v_{\perp} = \sqrt{125^2 - 100^2} = 75 \text{ m/s}$$

The time taken to reach the top of the hill with velocity v_{\perp} is given by

$$\frac{1}{2} g T^2 = h$$

putting values, $T = 10 \text{ s}$

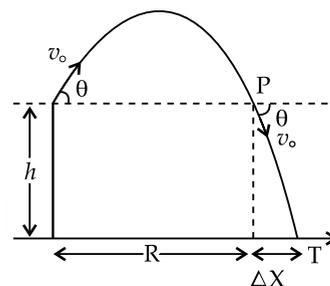
In 10 s the horizontal distance covered 10×75

$$= 750 \text{ m.}$$

So, cannon has to be moved through a distance of 50 m on the ground.

So, total time taken (shortest) by the packet to reach ground across the hill = $\frac{50}{2} \text{ s} + 10 \text{ s} + 10 \text{ s} = 45 \text{ s}$.

Q. 19. A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$.



If a target farther away by distance Δx (beyond R) has to be hit with the same gun (Fig), show that it could be achieved by raising the gun to a height at least

$$h = \Delta x \left[1 + \frac{\Delta x}{R} \right]$$

(Hint: This problem can be approached in two different ways:

- (i) Refer to the diagram: target T is at horizontal distance and below point of projection $y = -h$.
- (ii) From point P in the diagram: Projection at speed v_0 at an angle θ below horizontal with height h and horizontal range Δx .)

[NCERT Exemplar Q. No. 30]

Ans. As, $R = \frac{v_0^2}{g}$... (1)

Displacement along y -axis,

$$-h = (v_0 \sin \theta)t + \frac{1}{2}(-g)t^2 \quad \dots(2)$$

Displacement along x -axis is $R + \Delta x$

$$\text{Time required} = \frac{(R + \Delta x)}{v_0 \cos \theta} \quad \dots(3)$$

Substituting t in eq.(2),

$$h = (-v_0 \sin \theta) \times \left(\frac{R + \Delta x}{v_0 \cos \theta} \right) + \frac{1}{2}g \left(\frac{R + \Delta x}{v_0 \cos \theta} \right)^2$$

$$h = -(R + \Delta x) \tan \theta + \frac{1}{2}g \frac{(R + \Delta x)^2}{v_0^2 \cos^2 \theta}$$

For maximum range, $\theta = 45^\circ$,

$$h = -(R + \Delta x) \times 1 + \frac{1}{2}g \frac{(R + \Delta x)^2}{v_0^2 \left(\frac{1}{2}\right)}$$

$$h = -(R + \Delta x) + \frac{(R + \Delta x)^2}{R} \quad \therefore R = \left(\frac{v_0^2}{g} \right)$$

$$= -R - \Delta x + \left(R + \frac{\Delta x^2}{R} + 2\Delta x \right) = \Delta x + \frac{\Delta x^2}{R}$$

$$h = \Delta x \left(1 + \frac{\Delta x}{R} \right).$$

Q. 20. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal (See Fig.).

- (a) Find an expression of range on the plane surface (distance on the plane from the point of projection at which particle will hit the surface).

(b) Time of flight.

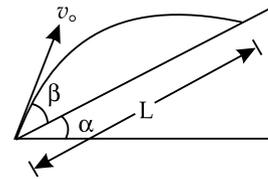
(c) β at which range will be maximum.

[Hint : This problem can be solved in two different ways :

(i) Point P at which particle hits the plane can be seen as intersection of its trajectory (parabola) and straight line. Remember particle is projected at an angle $(\alpha + \beta)$ w.r.t. horizontal.

(ii) We can take x -direction along the plane and y -direction perpendicular to the plane. In that case resolve g (acceleration due to gravity) in two different components, g_x along the plane and g_y perpendicular to the plane. Now the problem can be solved as two independent motions in

x and y directions respectively with time as a common parameter.)



[NCERT Exemplar Q. No. 31]

Ans.

(a) Time of flight = T

Considering motion along vertical direction,

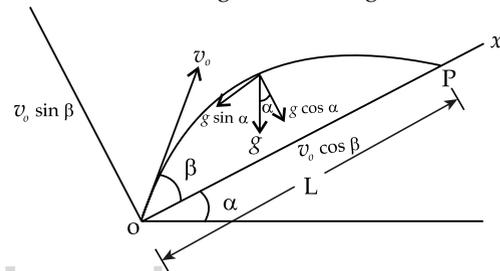
$$y = u_y T + \frac{1}{2} a_y T^2$$

$$\text{or, } 0 = (v_0 \sin \beta) T - \frac{1}{2} (-g \cos \alpha) T^2$$

$$\text{or, } (v_0 \sin \beta) T = \frac{1}{2} (-g \cos \alpha) T^2$$

$$\therefore T = (2v_0 \sin \beta) / (g \cos \alpha)$$

Now, considering motion along OX,



$$x = u_x t + \frac{1}{2} a_x t^2$$

$$L = v_0 \cos \beta T + \frac{1}{2} (-g \sin \alpha) T^2$$

$$L = T \left[v_0 \cos \beta - \frac{1}{2} g \sin \alpha \times \frac{2v_0 \sin \beta}{g \cos \alpha} \right]$$

$$= \frac{2v_0^2 \sin \beta}{g \cos^2 \alpha} [\cos \beta \cos \alpha - \sin \alpha \sin \beta]$$

$$\Rightarrow L = \frac{2v_0^2 \sin \beta}{g \cos^2 \beta} \cos(\alpha + \beta)$$

(b) Time of flight evaluated in (a).

$$T = \frac{2v_0 \sin \beta}{g \cos \alpha}$$

(c) To find maximum range, $\sin \beta \cos(\alpha + \beta)$ should be maximum. $\sin \beta \cos(\alpha + \beta)$

$$= \sin \beta (\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$= \frac{1}{2} (2 \cos \alpha \cos \beta \sin \beta - 2 \sin \alpha \sin^2 \beta)$$

$$= \frac{1}{2} [\cos \alpha \sin 2\beta - \sin \alpha (1 - 2 \cos \beta)]$$

$$= \frac{1}{2} [\cos \alpha \sin 2\beta + 2 \sin \alpha \cos 2\beta - \sin \alpha]$$

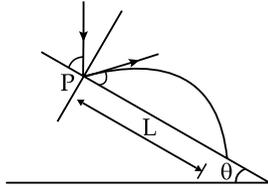
$$= \frac{1}{2} [\sin(2\beta + \alpha) - \sin \alpha]$$

For maximum value, $\sin(2\beta + \alpha) = 1$ [since, $\sin \alpha$ is constant]

$$\sin(2\beta + \alpha) = 1 \Rightarrow 2\beta + \alpha = \frac{\pi}{2}$$

$$\beta = \frac{\pi}{4} = -\frac{\alpha}{2}$$

Q. 21. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_0 and rebounds elastically (Fig). Find the distance along the plane where it will hit second time.



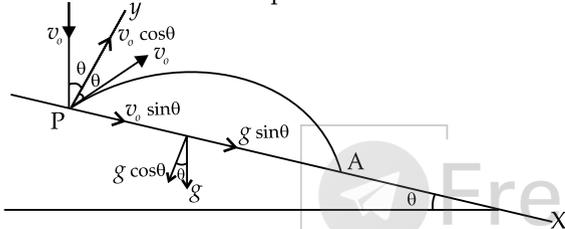
(Hint: (i) After rebound, particle still has speed v_0 to start.

(ii) Work out angle particle speed has with horizontal after it rebounds.

(iii) Rest is similar to if particle is projected up the incline.) [NCERT Exemplar Q. No. 32]

Ans. Time of flight, $T = \frac{2v_0 \cos \theta}{g \cos \theta} = \frac{2v_0}{g}$

Considering motion along x -axis,
 $X = PA = L$, $u_x = v_0 \sin \theta$, $a_x = g \sin \theta$, $t = T = 2v_0/g$
 From kinematics equation-



$$x = u_x t + \frac{1}{2} a_x t^2$$

$$L = v_0 \sin \theta t + \frac{1}{2} g \sin \theta t^2$$

$$= (v_0 \sin \theta)(T) + \frac{1}{2} g \sin \theta T^2$$

$$= (v_0 \sin \theta) \left(\frac{2v_0}{g} \right) + \frac{1}{2} g \sin \theta \times \left(\frac{2v_0}{g} \right)^2$$

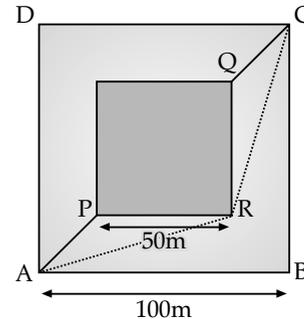
$$= \frac{2v_0^2}{g} \sin \theta + \frac{1}{2} g \sin \theta + \frac{4v_0^2}{g^2}$$

$$\text{Distance, } L = \frac{4v_0^2}{g} \sin \theta.$$

Q. 22. A man wants to reach from A to the opposite corner of the square C (Fig.). The sides of the square are 100 m. A central square of 50 m is filled with sand. Outside this square, he can walk only at a speed of 1 m/s. In the central square, he can walk only at a speed of v m/s ($v < 1$). What is smallest value of v for which he

can reach faster via a straight path through the sand than any path in the square outside the sand? [NCERT Exemplar Q. No. 37]

Ans. Consider the straight line path APQC through the sand. Time taken to go from A to C via this path



$$T_{\text{sand}} = \frac{AP + QC}{1} + \frac{PQ}{v}$$

$$= \frac{25\sqrt{2} + 25\sqrt{2}}{1} + \frac{50\sqrt{2}}{v}$$

$$= 50\sqrt{2} \left[\frac{1}{v} + 1 \right]$$

The shortest path outside the sand will be ARC.

$$T_{\text{outside}} = \frac{AR + RC}{1}$$

$$AR = \sqrt{75^2 + 25^2} = \sqrt{75 \times 75 + 25 \times 25}$$

$$= 5 \times 5\sqrt{9+1} = 25\sqrt{10} \text{ m}$$

$$RC = AR = \sqrt{75^2 + 25^2} = 25\sqrt{10} \text{ m}$$

$$T_{\text{outside}} = 2AR/1 = 2 \times 25\sqrt{10} = 50\sqrt{10} \text{ s}$$

For $T_{\text{sand}} < T_{\text{outside}}$

$$50\sqrt{2} \left(\frac{1}{v} + 1 \right) < 2 \times 25\sqrt{10}$$

$$\frac{2\sqrt{2}}{2} \left(\frac{1}{v} + 1 \right) = \sqrt{10}$$

$$\Rightarrow \left(\frac{1}{v} + 1 \right) < \frac{2\sqrt{10}}{2\sqrt{2}}$$

$$\Rightarrow \frac{1}{v} < \frac{\sqrt{5}}{2} \times 2 - 1 \Rightarrow \frac{1}{v} < \sqrt{5} - 1$$

$$\Rightarrow v > \frac{1}{\sqrt{5} - 1} \approx 0.81 \text{ m/s} \Rightarrow v > 0.81 \text{ m/s}$$



COMPETENCY BASED QUESTIONS

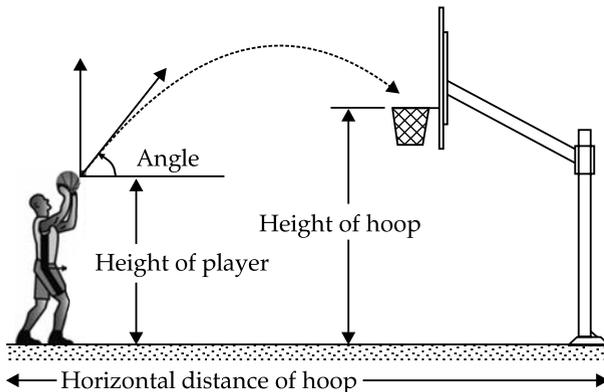
These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

Using the laws of physics, it is possible to make a successful jump shot in basketball every time. The trajectory of a basketball is always a parabola. It is projectile when it is launched up into the air, and

this is due to the affects of gravity as well as the force put on the ball by the player that properly follows the properties of a projectile.



The velocity of the ball changes as it moves through the air, but the velocity of the first half of the path matches with the last half of the path. It may deviate from its path if the ball collides with either the backboard or another player interfering with its path.

From the information of the height of the player, the horizontal distance and height of the hoop, the player can easily calculate the speed to be imparted to the ball and the angle for a sure success.

If player's height is 1.27 meters standing at a distance of 2 meters from the hoop which is at a height of 3.05 meters, he needs an angle of 55° and velocity of the ball 7 m/s to be imparted to make his shot a success. (If he uses a smaller angle he has to shoot with a greater velocity.)

If he stands 6 meters distance from the hoop, he needs an angle 60° and a velocity 9.5 m/s to be successful.

To improve chances of an accurate shot, often a player includes a backspin on the ball as he launches it for a shot. The backspin ensures that the ball enters the hoop, especially if the shot is a "soft shot." Soft shot is when the ball is shot at a low angle and low velocity, the player adds a backspin because if the ball winds up hitting the rim, the spin will help it to enter the hoop. The backspin changes the velocity direction (once it hits the rim) to the opposite direction of the rim rather than bouncing it out.

Q. 1. The trajectory of a basketball is always a

- (A) Straight line
- (B) Circle
- (C) Parabola
- (D) May be a circle, may be a parabola

Ans. Option (C) is correct.

Explanation: The trajectory of a basketball is always a parabola when it is launched up into the air, and this is due to the affects of gravity as well as the force put on the ball by the player and its property follows the properties of a projectile.

The velocity of ball changes as it moves through the air, but the velocity of the first half of the path matches with the last half of the path.

Q. 2. When the basketball deviates from its parabolic path?

- (A) If the ball collides with the backboard
- (B) If the ball collide with another player interfering with its path
- (C) If a backspin is applied to the ball
- (D) Both (A) and (B)

Ans. Option (D) is correct.

Explanation: The basketball may deviate from its path if the ball collides with either the backboard or another player interfering with its path. Backspin does not deviate the ball from its trajectory. It improves the chances of an accurate shot.

Q. 3. To calculate the speed to be imparted to the ball and the angle for an accurate shot the required information are

- (A) Height of the player
- (B) Horizontal distance of the hoop
- (C) Height of the hoop
- (D) All of the above

Ans. Option (D) is correct.

Q. 4. If the horizontal distance of the hoop from a player increases

- (A) Velocity and angle both should increase
- (B) Velocity and angle both should decrease
- (C) Velocity should increase but angle should decrease
- (D) Velocity should decrease but angle should increase

Ans. Option (A) is correct.

Explanation: In the given example in the passage, if the distance increases from 2 m to 6 m, then the angle should change from 55° to 60° and the velocity of the ball changes from 7 m/s to 9.5 m/s.

Q. 5. Why backspin is applied on basketball while launching it for a shot?

- (A) The backspin ensures that the ball follows parabolic path
- (B) The backspin ensures that the ball enters the hoop
- (C) The backspin ensures that the ball does not collide with an interfering player
- (D) The backspin ensures that the ball moves faster

Ans. Option (B) is correct.

Explanation: To improve chances of an accurate shot, often a player includes a backspin on the ball as he launches it for a shot. The backspin ensures that the ball enters the hoop, especially if the shot

is a "soft shot." Soft shot is when the ball is shot at a low angle and low velocity, the player adds a backspin because if the ball winds up hitting the rim, the spin will help it to enter the hoop.

The backspin changes the velocity direction (once it hits the rim) to the opposite direction of the rim rather than bouncing it out.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

When the football travels through the air, it always follows a parabolic path because the movement of the ball in the vertical direction is influenced by the force of gravity. As the ball travels up, gravity slows it down until it stops for a moment at its peak height; the ball then comes down and gravity accelerates it until it hits the ground.

When a footballer kicks a ball he has to consider 3 factors:

- The velocity at which the ball leaves his feet
- The angle of kick
- The rotation of the ball

The rotation of the ball determines how the ball will slow down in flight. The velocity of the ball and the angle of the kick are the major factors those determine:

- How long the ball will remain in air (hang-time)
- How high the ball will go
- How far will the ball go

When the ball leaves the footballer's foot, it moves in two directions - horizontally and vertically.

If the ball is kicked at a steep angle, then it will have more velocity in the vertical direction than in the horizontal direction - the ball will go high, have a long hang-time, but travel a short distance.

If the ball is kicked at a shallow angle, it will have more velocity in the horizontal direction than in the vertical direction - the ball will not go very high, will have a short hang-time, but will travel a long distance.

The footballer decides on the best angle in view of his field position.

Q. 1. A footballer kicks a ball with velocity 60 m/s at an angle α . For which of the following values of α , the hang-time will be highest?

- (A) 15° (B) 30°
(C) 45° (D) 60°

Ans. Option (D) is correct.

Explanation: Hang time = Time of flight = $T = (2u \sin\alpha)/g$
 u remaining constant, T increases as $\sin\alpha$ increases. Hence, for $\alpha = 60^\circ$, T will be maximum.

Q. 2. Ball A is kicked at an angle 30° and ball B is kicked at an angle 60° with same velocity. Which one will have larger vertical component of velocity?

Ans. Ball B will have larger vertical component of velocity. Vertical component is $u \sin\alpha$.

u remaining constant, vertical component increases as $\sin\alpha$ increases. Hence, the ball B will have larger vertical component of velocity.

Q. 3. As the angle of the kick is increased (keeping velocity constant), the distance traveled by the ball increases to a maximum at 45° and then decreases. Why?

Ans. Horizontal range = $\frac{(u^2 \sin^2 \alpha)}{g}$.

Value of $\sin 2\alpha$ is maximum when $2\alpha = 90^\circ$, i.e., at $\alpha = 45^\circ$. Beyond that angle, the value of $\sin 2\alpha$ decreases. For this reason the distance traveled by the ball increases to a maximum at 45° and then decreases.

Q. 4. A player can impart a maximum speed 40 m/s to a football. At which angle he should kick the ball to pass it on to another player of his team standing a distance 80m?

Ans. Range = $\frac{(u^2 \sin 2\alpha)}{g}$

Or, $80 = \frac{(1600 \sin 2\alpha)}{10}$

Or, $800 = 1600 \sin 2\alpha$

Or, $\sin 2\alpha = \frac{1}{2}$

Or, $2\alpha = 30^\circ$

$\therefore \alpha = 15^\circ$

Q. 5. State a difference between projectile and a free fall.

Ans. Free fall starts with no initial velocity whereas projectile starts with an initial upward velocity.



UNIT-III LAWS OF MOTION

Study Time
Maximum Time: 3 : 30 hours
Maximum Questions: 100

CHAPTER

4

LAWS OF MOTION



Syllabus

Intuitive concept of force, Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion; Law of conservation of linear momentum and its applications; Equilibrium of concurrent forces, Static and kinetic friction, law of friction, rolling friction, lubrication; Dynamics of uniform circular motion : Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on a banked road).

Topic-1 Newton's Laws of Motion

TOPIC - 1

Newton's Laws of Motion

.... P. 71

TOPIC - 2

Friction & Dynamics of Circular Motion

.... P. 77



Revision Notes

- **Newton's I law** : A body continues to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some non-zero external force to change the state. This law defines forces and is also called law of inertia.
- **Inertia** of a body is of three types as follows :
 - (i) **Inertia of rest** of a body is inability to change by itself, its state on its own.
 - (ii) **Inertia of motion** of a body is the inability to change by itself its state of uniform motion i.e., body in uniform motion can neither accelerate nor retard on its own and comes to rest.
 - (iii) **Inertia of direction** of a body is inability to change by itself its direction of motion, i.e., body continues to move along the same straight line unless compelled by some external force to change it.
- **Linear momentum** : Linear momentum (\vec{p}) of a body is measured by the product of the mass (m) of the body and its velocity (\vec{v}) i.e.,

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector quantity. Its direction is same as the direction of velocity of the body. The S.I. unit of linear momentum is kg ms^{-1} .

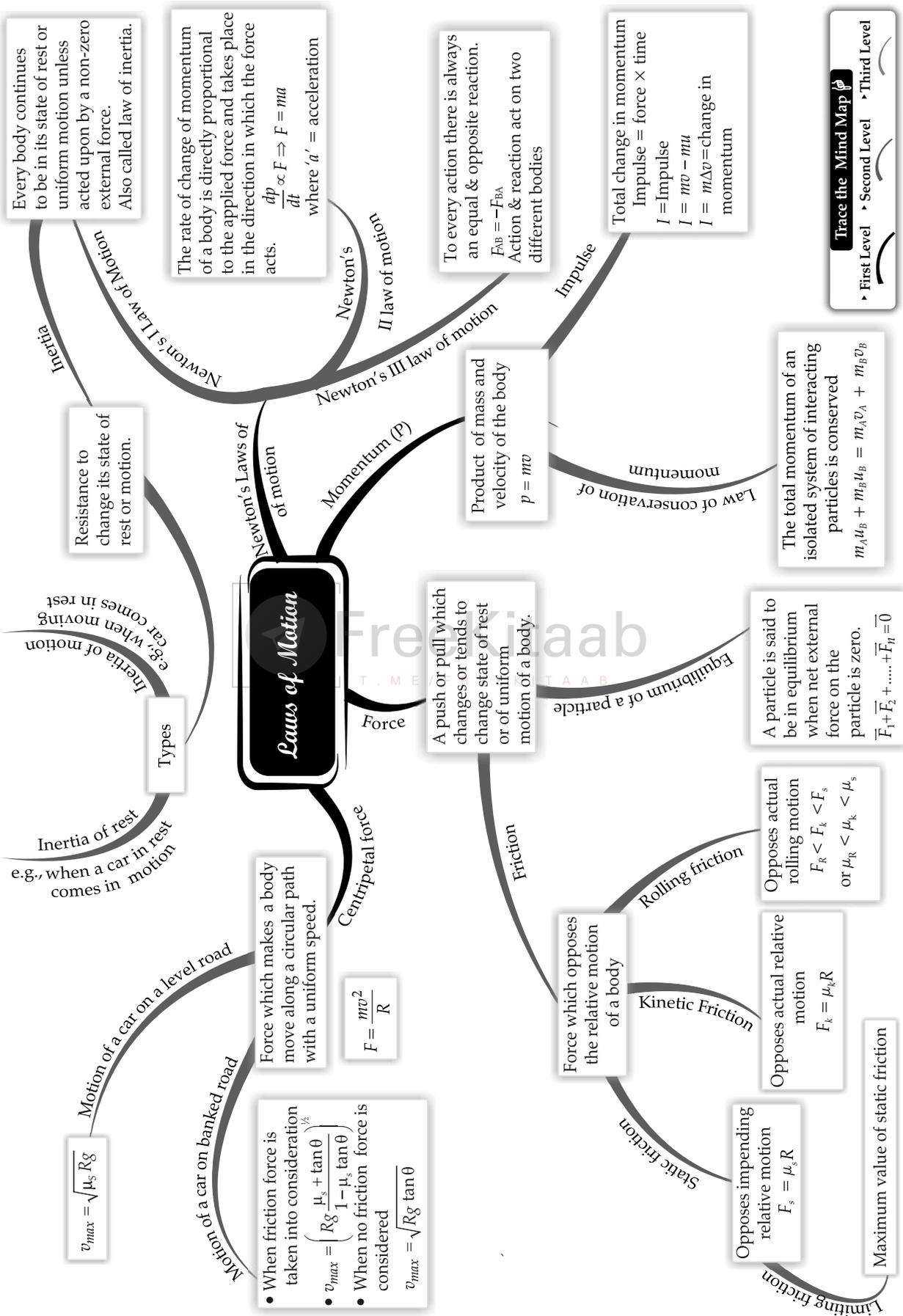
- **Newton's II law** : The rate of change of linear momentum ($\vec{p} = m\vec{v}$) of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force, i.e.,

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

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more about
this topic



Inertia and law
of conservation
of momentum



or

$$\vec{F} = k \frac{d\vec{p}}{dt} = k \frac{d}{dt}(m\vec{v})$$

$$= km \left(\frac{d\vec{v}}{dt} \right) = kma$$

where $\frac{d\vec{v}}{dt} = \vec{a}$, which is called acceleration of the body. Force can be defined in such a way that $k = 1$, then Newton's second law is written as

$$\vec{F} = m \frac{d\vec{v}}{dt} = m\vec{a}$$

➤ **Newton's III law :** To every action, there is always an equal and opposite reaction. The action and reaction act on different bodies, so they never cancel each other.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

\vec{F}_{AB} = Force exerted on A by B
 \vec{F}_{BA} = Force exerted on B by A
Hence, $\vec{F}_{AB} + \vec{F}_{BA} = \vec{0}$

(i) **Principle of conservation of linear momentum :** From this principle, in an isolated system, the vector sum of the linear momentum of all the bodies of the system is conserved and is unaffected due to their mutual action and reaction. The total linear momentum of all the bodies in the system is given by

$$\vec{p} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n = M \vec{v}_{c.m} = \text{constant}$$

where, M is the total mass of the system and $\vec{v}_{c.m}$ is the velocity of the centre of mass of the system.

(ii) **Rocket propulsion :** The propulsion of a rocket is based on the principle of conservation of linear momentum of Newton's third law of motion.

Suppose,

M_0 = Initial mass of rocket,

$\frac{\Delta M}{\Delta t}$ = Rate of ejection of fuel,

M = Mass of rocket at any instant,

\vec{v} = Relative velocity of ejected gases w.r.t. rocket.

Then, thrust on the rocket in the absence of gravity = $\frac{\Delta M}{\Delta t} \times \vec{v}$

Acceleration of the rocket in the absence of gravity = $\frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M}$

Thrust on the rocket in the presence of gravity,

$$\vec{F} = \frac{\Delta M}{\Delta t} \times \vec{v} - M \vec{g}$$

Acceleration of the rocket in the presence of gravity,

$$\vec{a} = \frac{\Delta M}{\Delta t} \times \frac{\vec{v}}{M} - \vec{g}$$

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The Rocket Equation



Mnemonics

Newton's Laws of Motion: Newton, Newton don't kick cow She may move ahead little bit now Newton hears her MAAA sound Cow gives Newton a kick rebound

Interpretation: 1st two lines of the rhyme depicts the 1st law of motion

Newton's 1st law: A body continues its state of

rest or state of motion unless it is acted upon by an unbalanced force.

3rd line depicts the 2nd law of motion

Newton's 2nd law: $F = ma$

Last line depicts the 3rd law of motion

Newton's 3rd law: Every action has its equal and opposite reaction.



Key Words

- **Force** is an external effort in the form of push or pull which can try to produce motion in a body at rest, or stops or try to stop a moving body or can change or try to change the direction of motion of the body.
- **Inertia** is the inherent property of a body, by virtue of which, the body doesn't change its state of rest or of uniform motion along a straight line, on its own. It depends upon the mass of the body.
- **Impulse** : When a large force acts on a body for a short time, then the measure of the total effect of force is called impulse of force. It can be found out

$$\text{Impulse} = \text{Force} \times \text{Time} = \vec{F}_{av} \times \Delta t$$



Key Formulae

- **Linear Momentum :**

$$\vec{p} = m \vec{v}$$

- **Force**

$$= \frac{d\vec{p}}{dt} = m \vec{a}$$

- **Impulse** = Force \times time

$$\vec{I} = \vec{F} \times t = m(\vec{v} - \vec{u})$$

- **Principle of Conservation of Linear Momentum**

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n = \text{Constant}$$

- **Recoil velocity of gun**

$$(\vec{v}_2) = \frac{-m_1 \vec{v}_1}{m_2}$$

m_2 = Mass of gun

m_1 = Mass of bullet

v_1 = Velocity of bullet



Very Short Answer Type Questions

(1 mark each)

Q. 1. Rocket works on which principle of conservation ? [R] [NCT 2010]

Ans. Law of conservation of linear momentum. 1

Q. 2. Why does a cricketer move his hands backwards when holding a catch ?

[A] [I] [U] [NCT 2009, KVS 2008]

Ans. In order to save himself from getting hurt, a cricketer moves his hands backwards while holding a catch. The cricketer increases the time interval by doing so. Increased time duration decreases the impulse, so the cricketer does not get hurt more. 1

Q. 3. If the net external force acting on a body is zero, then the body at rest continues to remain at rest and a body in motion continues to move with

uniform motion. What is the name given to this property of the body ? [R] [NCT 2008]

Ans. Inertia. 1

Q. 4. In which direction the standing passengers of a moving bus fall if the driver suddenly applies brakes to stop the bus ? [U]

Ans. The passengers will get jerk in forward direction because of inertia of motion. 1

Q. 5. An astronaut comes out of a spaceship accelerating in space with an acceleration $a \text{ ms}^{-2}$. Neglecting the gravitational force of nearby stars, what is the acceleration of the astronaut at the moment he is out of the spaceship ? [A] [A] [I]

Ans. Zero, because no external force is acting on the astronaut. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. Newton's first law of motion is the law of inertia. Explain. [U]

Ans. According to Newton's first law of motion, a body can't change its state of rest or of uniform motion along a straight line unless an external force acts on it. It means that the natural tendency of the material body to remain continue in the state of rest or that of uniform motion, which is termed as inertia. Thus, Newton's first law is the law of inertia. 2

Q. 2. When a man jumps out of a boat, then it is pushed away. Why? [U] [NCT 2008]

Ans. This is due to the Newton's third law of motion. When the man jumps out of the boat, he applies a force on it in the backward direction and in turn, the reaction of the boat on the man pushes him out of the boat. 2

Q. 3. Why Newton's second law of motion is not applicable to the motion of a rocket?

[A] [MSE Chandigarh 2002]

Ans. Newton's second law, i.e., $\vec{F} = m\vec{a}$ is applicable

only if the mass (m) of the body remains constant. In case of the rocket, the mass continuously decreases and hence $\vec{F} = m\vec{a}$ is not applicable. 2



Commonly Made Error

- ▶ Students connect it with the acceleration due to gravity and may forget to include the variable mass of the rocket.



Answering Tip

- ▶ Students should be aware of the phenomenon of the motion of a rocket and its related conditions.

Q. 4. How is second law consistent with the first law? [U]

Ans. From second law, force $F = m \times a$. If $F = 0$, then $a = 0$, i.e., no external force means no acceleration. It implies no change in state of rest or of uniform motion in a straight line which is nothing but first law of motion. 2

Q. 5. A bullet of mass 0.04 kg moving with a speed of 90 ms⁻¹ enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet? [A] [KVS 2014]

Ans. The retardation ' a ' of the bullet (assumed constant) is given by

$$\begin{aligned} a &= -\frac{u^2}{2s} \\ &= -\frac{90 \times 90}{2 \times 0.6} \text{ ms}^{-2} \\ &= -6750 \text{ ms}^{-2} \end{aligned} \quad 1$$

The retarding force, by the second law of motion is

$$\begin{aligned} &= 0.04 \text{ kg} \times 6750 \text{ ms}^{-2} \\ &= 270 \text{ N} \end{aligned}$$

Q. 6. Vehicles stop on applying brakes. Does this phenomenon violate the principle of conservation of momentum? [U]

Ans. The law of conservation of momentum is not violated by stopping vehicles on applying brakes. Some retarding force is being applied due to brakes and the vehicles come to rest such that the total loss of its momentum is equal to the impulse of the applied force. Thus the law of conservation of momentum is not violated. 2

Q. 7. Second law of motion is strictly applicable to a single point particle then how is it applied to a rigid body or a system of particles? [U]

Ans. In this case force \vec{F} refers to the total external force, on the system (internal forces in the system are not included in \vec{F}) and \vec{a} refers to the acceleration of the whole system, i.e., acceleration of the centre of mass of the system. 2



Short Answer Type Questions-II

(3 marks each)

Q. 1. State and prove impulse-momentum theorem. [R]

Ans. Impulse-momentum theorem states that the impulse of force on a body is equal to the change in momentum of the body.

$$\text{i.e.,} \quad \vec{J} = \vec{F} t = \vec{p}_2 - \vec{p}_1 \quad 1$$

Proof. According to Newton's Second law of motion, we know that

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\text{or} \quad \vec{F} dt = d\vec{p} \quad \dots(i) \frac{1}{2}$$

When $\vec{F} = \text{constant force}$ acting on the body.

Suppose \vec{p}_1 and \vec{p}_2 be the linear momenta of the body at time $t = 0$ and t respectively.

∴ Integrating equation (i) within these limits, we get

$$\int_0^t \vec{F} dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} \quad \frac{1}{2}$$

$$\Rightarrow \vec{F} \int_0^t dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$$

$$\vec{F} [t]_0^t = [p]_{\vec{p}_1}^{\vec{p}_2}$$

$$\vec{F} t = \vec{p}_2 - \vec{p}_1$$

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \quad 1$$



Commonly Made Error

- Students commit error while doing the various steps of integration here and thus, may obtain a different/incorrect result.



Answering Tip

- Students should try to solve the basic integration problems first by remembering the rules of finite integrals and then, apply the Newton's second law of motion to get the desired result.

Q. 2. In the case of rocket propulsion derive expression for U A I

- Velocity of rocket at any instant,
- Thrust on the rocket,
- Speed of the rocket when whole fuel is burnt.

Ans. At time $t = 0$

Let the mass of rocket with fuel = m_0

Velocity of the rocket with respect to earth = v_0

At time $t = t$

Mass of rocket with fuel = m

and velocity of rocket w.r.t. earth = v

such that $m < m_0$ and $v > v_0$ 1/2

At time $(t + dt)$

If at time $(t + dt)$, the mass of fuel burnt is dm which gives gases of velocity v_g and velocity of rocket becomes $(v + dv)$.

Using the law of conservation of momentum

$$mv = (m - dm)(v + dv) - dm v_g$$

(v_g is with $-ve$ sign because it acts downward)

$$\text{i.e., } mdv = -(v_g + v) dm$$

(neglecting $dmdv$ being very small)

$$\text{i.e., } mdv = -udm \quad \frac{1}{2}$$

(u is the relative velocity of gases with respect to the rocket)

(a) Velocity of rocket at any instant

Using $mdv = -udm$

(∵ velocity increase with decrease in mass)

Integrating

$$\int_{v_0}^v dv = - \int_{m_0}^m u \frac{dm}{m} = u \int_{m_0}^m \frac{dm}{m}$$

$$\text{i.e., } v - v_0 = -u[\log_e m - \log_e m_0]$$

$$= u[\log_e m_0 - \log_e m]$$

$$\text{i.e., } v = v_0 + u \log_e \frac{m_0}{m}$$

If $v_0 = 0$,

$$v = u \log_e \frac{m_0}{m} \quad 1$$

(b) Thrust on the rocket

Using, $mdv = -udm$,

(neglecting $-ve$ sign because it only represents direction)

we get, $\frac{mdv}{dt} = u \frac{dm}{dt}$

(dividing both sides by dt)

or $m \times a = u \frac{dm}{dt}$

(where acceleration, $a = \frac{dv}{dt}$)

$$\text{or } F = \frac{udm}{dt} \quad \frac{1}{2}$$

(c) Speed of the rocket when whole fuel is burnt

Using, $v = v_0 + u \log_e \frac{m_0}{m}$

where, $v = v_e$, speed of empty rocket

and $m = m_e$, mass of empty rocket

we get, $v_e = v_0 + u \log_e \frac{m_0}{m_e} \quad \frac{1}{2}$



Commonly Made Error

- Students commit error while applying different formulas and integration methods.



Answering Tip

- Students should ensure about the formulas to be used for each expressions and then remember the basic integral rules to derive each expression.

Topic-2 Friction & Dynamics of Circular Motion



Revision Notes

➤ **Equilibrium of Concurrent forces :** Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero.

(i) **Conditions of equilibrium of concurrent forces :** If $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots$ are the concurrent forces acting at the same point, then the point will be in equilibrium if

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = \vec{0}$$

➤ **Friction :** Friction is an opposing force which comes into play when one body actually moves (slides or rolls) or even tries to move over the surface of another body. Frictional forces arise due to interlocking of irregularities present on the two surfaces which are in contact. From modern view, the frictional force arises due to strong atomic or molecular force of attraction between the two surfaces at the points of actual contact.

It is of two types :

(a) **Internal friction :** It arises on account of relative motion between every two layers of liquid. It is also known as viscosity of liquid.

(b) **External friction :** It arises when two bodies in contact with each other try to move or there is an actual relative motion between the two. It is also known as contact friction. Further it is of four types:

(i) **Static friction** is an opposing force which comes into play when one body tends to move over the surface of another body, but the actual motion has yet not started. It is a self-adjusting force.

(ii) **Limiting friction** is the maximum value of static friction. Limiting friction is the maximum opposing force that comes into play of one body is just at the average of moving over the surface of another body.

(iii) **Dynamic or kinetic friction** is the opposing force that acts between two surfaces in contact when one body is actually moving over the surface of another body.

(iv) **Rolling friction** is an opposing force that comes into play when one body is actually rolling over the surface of another body.

➤ **Laws of limiting friction :**

(a) The magnitude of the force of limiting friction (F) between two bodies in contact is directly proportional to the normal reaction (R) between them, i.e., $F \propto R$.

(b) The direction of the force of limiting friction is always opposite to the direction in which the body moves or tends to move.

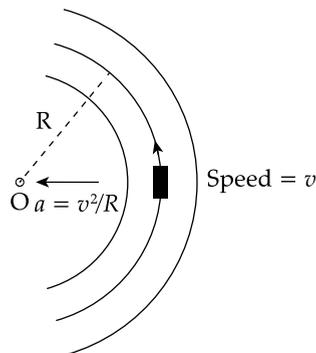
(c) The force of limiting friction is independent of the area of contact.

(d) The force of limiting friction between any two bodies in contact depends on the nature of the surfaces in contact.

➤ **Coefficient of limiting friction** between two surfaces in contact (μ) is defined as the ratio of the force of limiting friction (F) and normal reaction (R) between them, i.e.,

$$\mu = F/R$$

➤ **Motion of car on banked road:**



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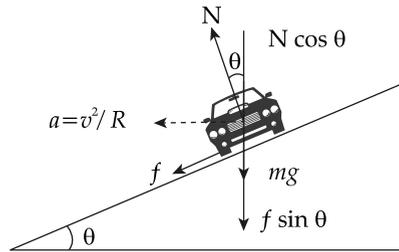
Equilibrium of Forces

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Friction

$$V_{\max} = \sqrt{\mu_s Rg}$$



$$V_{\max} = \left[\frac{rg(\mu_s + \tan \theta)}{1 - \mu_s \tan \theta} \right]$$

For $\mu_s = 0$ i.e., for a frictionless banked road

$$V_{\max} = \sqrt{rg \tan \theta}$$

where, μ_s is the coefficient of friction and R is the radius of the circle.

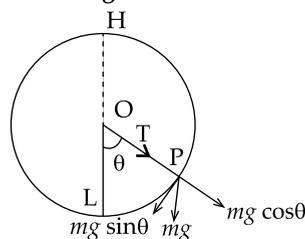
Key Terms

- **Centripetal force** is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle.
- **Angle of friction** is the angle in which the direction of resultant of the force of friction and normal reaction makes with the direction of normal reaction. It is represented by θ .
$$\tan \theta = \mu$$
- **Angle of repose** is the maximum angle of inclination of a plane with the horizontal, at which the body placed on the plane is just in limiting equilibrium.
- **Centrifugal force** is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

Key Formulae

- **Angle of friction,** $\tan \theta = \mu$.
- **Angle of repose,** $\mu = \frac{F}{R} = \tan \theta$.
- **Centripetal force** $= \frac{mv^2}{r} = m\omega^2$.
- $= mr(2\pi v)^2$.
- $\tan \theta = v^2/rg$.
- $\tan \theta = \frac{h}{\sqrt{b^2 - h^2}} = v^2/rg$.
- h = height between outer edge and inner edge
 b = breadth of road
- Maximum speed of car on circular banked road $= v_{\max} = \left[\frac{rg(\mu_s + \tan \theta)}{(1 - \mu_s \tan \theta)} \right]^{1/2}$

- **At any position of angular displacement θ along a vertical circle**



$$T = \frac{mv^2}{r} + mg \cos \theta.$$

➤ At lowest point of vertical circle, $\theta = 0^\circ$; $T_L = \frac{mv_L^2}{r} + mg.$

➤ At highest point of vertical circle, $\theta = 180^\circ$

$$T_H = \frac{mv_H^2}{r} - mg.$$

➤ Minimum velocity at highest point, at H

$$v_H = \sqrt{gr}.$$

➤ Minimum velocity at lowest point, at L

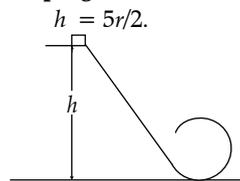
$$v_L = \sqrt{5gr}.$$

➤ When,

$$\theta = 90^\circ$$

$$v = \sqrt{3gr}.$$

➤ Height through which a body should fall for looping the vertical loop



Very Short Answer Type Questions

(1 mark each)

Q. 1. Is force of friction independent of path ?

Ans. No, the frictional force is a non-conservative force, i.e., the work done by it depends upon the actual path followed by a body. 1

Q. 2. How can the friction be reduced ?

Ans. We can reduce friction by using lubricants, ball bearings, polished surfaces and air cushions etc. 1

Q. 3. Mention a factor on which coefficient of friction depends.

Ans. The coefficient of friction depends upon the nature of the surfaces in contact. 1

Q. 4. Why banking of road at turnings are required ?

Ans. Banking of road at turnings are require to provide the necessary centripetal force to avoid the chance of skid. 1

Q. 5. Why rockets are given conical shapes ?

Ans. The rockets are given conical shapes so as to reduce atmospheric friction. 1

Short Answer Type Questions-I

(2 marks each)

Q. 1. What is rolling friction ? [NCT 2011]

Ans. The opposition offered to the circular motion of objects like ring, disc, sphere, cylinder etc. on another surface is called rolling friction. The coefficient of rolling friction (μ_r) is smaller than the coefficient of kinetic friction (μ_k). The sliding friction can be decreased a lot by converting it into rolling friction. 2

Q. 2. What is pseudo force ?

Ans. The fictitious or imaginary force used to balance an actual or material force is called pseudo force,

e.g., centrifugal force, $F = \frac{-mv^2}{r}$ is a pseudo

force. Pseudo forces do not have material sources, they are simply produced because of acceleration or rotation of the frame itself. 2

Q. 3. Why a horse has to apply more force to start a cart than to keep it moving ? Explain.

[NCT 2009]

Ans. Static friction comes into play when the horse applies force to start the motion in the cart, on the other hand, kinetic friction comes into play when the cart is moving. 2

Q. 4. Sometimes we need to increase friction. Why ?

[NCT 2009]

Ans. Sometimes friction between two surfaces decreases to such an extent that it is difficult to move on that surface. So, friction needs to be increased. For example, vehicles can't move on a road covered with snow. In such cases we have to throw sand on the road to increase the friction. 2

Q. 5. Why automobiles tyres have generally irregular projections over their surface ? [NCT 2009]

Ans. The automobile tyres have generally irregular projections over their surface so as to :

(i) Increase friction,

(ii) Increase the grip with the ground and thus avoiding their skidding. 2



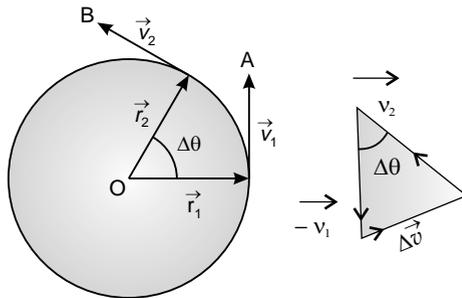
Short Answer Type Questions-II

(3 marks each)

Q. 1. Define centripetal acceleration. Find the expression for it. Give one example of centripetal force. [R+A] [KVS 2013]

Ans. Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

Expression : Consider a particle of mass m , moving with a constant speed v and uniform angular velocity ω , on a circular path of radius r with centre at O .



angular speed $\omega = \frac{\Delta\theta}{\Delta t}$... (1)

Let \vec{v}_1 and \vec{v}_2 be the velocity vectors. Then

$$\Delta\theta = \omega\Delta t = \frac{|\Delta\vec{v}|}{|\vec{v}|}$$

Here $|\vec{v}_1| = |\vec{v}_2| = |\vec{v}|$

or $\frac{|\Delta\vec{v}|}{\Delta t} = |\vec{v}| \omega = (\omega r)\omega = \omega^2 r$ 1

when $\Delta t \rightarrow 0$ then $\frac{|\Delta\vec{v}|}{\Delta t}$ represents the magnitude of centripetal acceleration, which is given by

$$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \omega^2 r = \left(\frac{v}{r}\right)^2 r = \frac{v^2}{r}$$

then, $|\vec{a}| = \omega^2 r = \frac{v^2}{r}$

Example of centripetal acceleration is a stone moved around tied to the string. 1

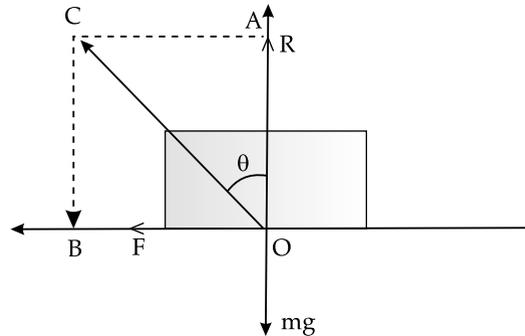
Q. 2. Define coefficient of friction and angle of friction and hence derive a relation between them.

[R+U] [KVS 2013; MSE Chandigarh 2008]

Ans. The coefficient of friction between any two surfaces in contact is defined as the ratio of the force of limiting friction and normal reaction between them.

$$\mu = \frac{F}{R}$$
 1

Angle with the resultant of force of limiting friction F and normal reaction R makes with the direction of normal reaction R is angle of friction. 1



Relation :

In ΔAOC $\tan \theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{F}{R} = \mu$
Hence, $\mu = \tan \theta$ 1



Commonly Made Error

While deriving the coefficient of friction and the angle of friction, students commit error in using the correct angle and its component arms.

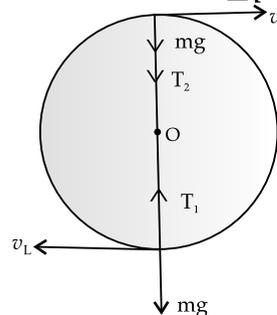


Answering Tip

Students must ensure themselves about the angle they are going to find and then, find the relationship using the suitable trigonometric function.

Q. 3. Derive a relation for the velocity at the lowest point and the highest point for looping the loop of a vertical circle. [A] [MSE Chandigarh 2009]

Ans.



Consider an object of mass m completing vertical circle of radius r with velocities v_L at the lowest point and v_T at the highest point.

Minimum velocity at the highest point

Here, centripetal force = weight of the object + tension in the string

i.e., $\frac{mv_L^2}{r} = mg + T_2$
 $= mg + 0$
 (if T_2 is taken as zero; since $T_2 \geq 0$)
 i.e., $v_T = \sqrt{gr}$ 1
 Minimum velocity at the lowest point.
 Here, centripetal force = tension in the string
 - weight of the object
 i.e., $\frac{mv_L^2}{r} = T_1 - mg$

Also by using the law of conservation of energy, total energy of the object at T = kinetic energy of the object at L . 1

i.e., $(P.E. + K.E.)_T = K.E._L$
 i.e., $mg2r + \frac{1}{2}mv_T^2 = \frac{1}{2}mv_L^2$
 i.e., $v_L^2 = v_T^2 + 4gr$
 but $v_T = \sqrt{gr}$
 $\therefore v_L^2 = 4gr + gr = 5gr$
 i.e., $v_L = \sqrt{5gr}$ 1



Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. Application of lubricants cannot reduce

- (A) Static friction (B) Sliding friction
 (C) Rolling friction (D) Inertia

Ans. Option (D) is correct.

Q. 2. A body moving with constant speed in a horizontal circle. Which of the following parameters remain constant?

- (A) Velocity (B) Acceleration
 (C) Kinetic energy (D) Centripetal force

Ans. Option (C) is correct.

Explanation: When a body moves with constant speed in a horizontal circle, the magnitudes of its velocity, acceleration and centripetal force acting on it does not change; but their direction changes. Kinetic energy is a scalar quantity. It has no direction. Its magnitude remains constant since mass of the body and the speed remain constant.

Q. 3. A lift having mass 100 kg is rising up with an acceleration 4 m/s². The tension in the string is

- (A) 1380 N (B) 1160 N
 (C) 2380 N (D) 580 N

Ans. Option (A) is correct.

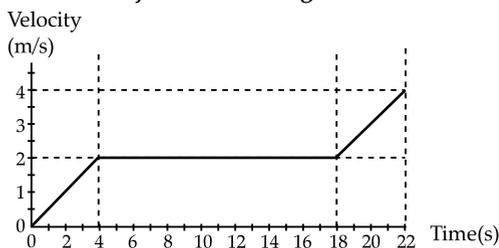
Explanation: When the lift is rising up,

$$T = m(g+a)$$

Or, $T = 100 \times (9.8 + 4)$

$\therefore T = 1380 \text{ N}$

Q. 4. In the following graph, the total impulse acted on the object of mass 2 kg in first 20 seconds is



- (A) 4 Ns (B) 2 Ns
 (C) 6 Ns (D) 8 Ns

Ans. Option (C) is correct.

Explanation: In first 4s, the velocity of the object has increased from 0 to $\frac{2m}{s}$.

So, acceleration during this period = $a = \frac{\text{Change of velocity}}{\text{Time difference}} = \frac{2}{4} = 0.5 \frac{m}{s^2}$.

So, force acting = $F = m \times a = 2 \times 0.5 = 1 \text{ N}$

So, impulse (in first 4 s) = $F \times \Delta t = 1 \times 4 = 4 \text{ Ns}$

From 4s to 18s, the velocity is constant. So, no impulse acts in this region.

In next 2 s, the velocity increases by $\frac{2m}{s}$.

So acceleration during this period = $a' = \frac{\text{Change of velocity}}{\text{Time difference}} = \frac{2}{4} = 0.5 \frac{m}{s^2}$.

So, force acting = $F' = m \times a' = 2 \times 0.5 = 1 \text{ N}$

So, impulse (in 18s to 20s) = $F' \times \Delta t' = 1 \times 2 = 2 \text{ Ns}$

So, total impulse = $4 \text{ Ns} + 2 \text{ Ns} = 6 \text{ Ns}$

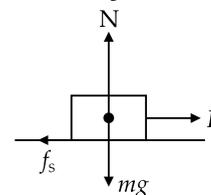
Q. 5. A force of 49 N is just able to move a block of mass 10 kg on a rough horizontal surface. The coefficient of friction is

- (A) 0.5 (B) 1.0
 (C) 0 (D) 0.8

Ans. Option (A) is correct.

Explanation: The box is just able to move when $F = f_s$

(F = force applied, f_s = force of static friction)



Since $f_s = \mu N$

$\therefore F = \mu N$

Or, $\mu = \frac{F}{N}$

$$\text{Or, } \mu = \frac{49}{mg}$$

$$\text{Or, } \mu = \frac{49}{(10 \times 9.8)}$$

$$\therefore \mu = 0.5$$

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
 (B) Both A and R are true but R is NOT the correct explanation of A
 (C) A is true but R is false
 (D) A is false and R is true

Q. 1. Assertion (A): Impulse has a dimension of linear momentum.

Reason (R): Impulse = Force \times time.

Ans. Option (B) is correct.

Explanation: Dimension of impulse = dimension of linear momentum = $[MLT^{-1}]$

So, the assertion is true.

Impulse = Force \times time. The reason is also true, but it does not explain the assertion.

Q. 2. Assertion (A): A gun recoils when it fires a bullet.

Reason (R): Newton's 2nd law of motion explains the phenomenon.

Ans. Option (C) is correct.

Explanation: A gun recoils when it fires a bullet. This phenomenon is explained by Newton's 3rd law of motion. So, the assertion is true but the reason is false.

Q. 3. Assertion (A): The force of friction increases as the roughness of a surface increases.

Reason (R): As the roughness of surface increases, the irregularities of the surface increases. Hence, large force is required to overcome the irregularities.

Ans. Option (A) is correct.

Explanation: Friction is caused by the irregularities on the two surfaces in contact. Irregularities on the two surfaces lock into one another. When one attempts to move any surface, he has to apply a force to overcome interlocking. On rough surfaces, there are a larger number of irregularities and larger force is to be applied. So the force of friction increases with the roughness of the surface.

So, assertion and reason both are true and the reason explains the assertion.



NCERT Corner

FreeKItaAb

Q. 1. Give the magnitude and direction of the net force acting on

- (a) a drop of rain falling down with a constant speed.
 (b) a cork of mass 10 g floating on water.
 (c) a kite skill fully held stationary in the sky.
 (d) a car moving with a constant velocity of 30 km/h on a rough road.
 (e) a high-speed electron in space far from all material objects, and free of electric and magnetic fields.

[For Simplicity in numerical calculations, take $g = 10 \text{ ms}^{-2}$]

- Ans.** (a) Acceleration, $a = 0$, net force $F = ma = 0$
 (b) Net force, $F = 0$, as its weight is being balanced by the upthrust.
 (c) From Newton's I law, as kite is stationary, So, net force, $F = 0$.
 (d) As, velocity v constant
 \therefore acceleration, $a = 0$.
 $F = ma = 0$.
 (e) Net force, $F = 0$ because there is no fields.

Q. 2. A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the new force on the pebble:

- (a) during its upward motion.
 (b) during its downward motion.
 (c) at the highest point where it is momentarily at rest. Do your answer alter if the pebble

was thrown at an angle of 45° with horizontal direction?

Ans. (a) Force $F = mg = 0.05 \times 9.8$
 $= 0.49 \text{ N} \downarrow$

(b) In this situation also

$$F = mg = 0.05 \times 9.8$$

$$= 0.49 \text{ N} \downarrow$$

(c) Again, $F = mg = 0.05 \times 9.8$
 $= 0.49 \text{ N} \downarrow$

No, the answer does not alter.

At the highest point there is only horizontal component of velocity and this force has no effect on the horizontal component of velocity.

Q. 3. Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg. A1

- (a) just after it is dropped from the window of a stationary train.
 (b) just after it is dropped from the window of a train running at a constant velocity of 36 km/h.
 (c) just after it is dropped from the window of a train accelerating with 1 m/s^2 .
 (d) lying on the floor of a train which is accelerating with 1 ms^{-2} the stone being at rest relative to the train. Neglect air resistance throughout.

Ans. (a) Net force, $F = mg$.
 $= 0.1 \times 10 = 1.0 \text{ N}$

Direction—vertically downwards.

- (b) Speed of train is constant, so acceleration = 0.
No force acts on stone because of this motion.
∴ Force on stone is same i.e., 1.0 N.
- (c) As, acceleration = 1 m/s²
∴ $F_1 = ma = 0.1 \times 1 = 0.1 \text{ N}$, horizontal
Net force on stone –
 $F = mg = 0.1 \times 10$ (F_1 is not considered as it stops acting as stone is released)
= 1 N
It is vertically downwards.
- (d) Stone is lying on the floor of train.
So, acceleration is same for stone and train.
∴ $F = ma$
= 0.1×1
= 0.1 N
along the direction of motion of the train.

Q. 4. One end of a string of length l is connected to particle of mass m and the other to a small peg on smooth horizontal table. If the particle moves in a circle with speed v , the net force on the particle (directed towards the centre) is : (T is the tension in the string)

- (i) $T = \frac{mv^2}{l}$ (ii) $T - \frac{mv^2}{l}$
(iii) $T + \frac{mv^2}{l}$ (iv) 0

Ans. (i) The net force on the particle is T because tension T provides the necessary centripetal force, i.e.,

$$T = \frac{mv^2}{l}$$

Q. 5. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms⁻¹. How long does the body take to stop ?

Ans. Force

$$F = ma$$

$$a = \frac{F}{m} = -\frac{50}{20}$$

$$= -2.5 \text{ ms}^{-2}$$

Using,

$$v = u + at$$

When,

$$u = 15 \text{ ms}^{-1}, v = 0$$

We get,

$$0 = 15 - 2.5 \times t$$

$$t = \frac{15}{2.5}$$

or

$$t = 6 \text{ s.}$$

Q. 6. A constant force acting on a body of mass 3 kg changes its speed from 2 ms⁻¹ to 3.5 ms⁻¹ in 25 s. The direction of motion of the body remains same. Find magnitude and direction of force.

Ans. Step 1. Using $v = u + at$

$$\text{When } v = 3.5 \text{ ms}^{-1}, u = 2 \text{ ms}^{-1}, t = 25 \text{ s}$$

$$\text{We get, } 3.5 = 2 + a \times 25$$

$$a = \frac{3.5 - 2}{25}$$

$$= 0.06 \text{ ms}^{-2}$$

Step 2. Force = $m \times a = 3 \times 0.06$

$$= 0.18 \text{ N acting in the direction of motion of the body.}$$

Q. 7. A body of mass 5 kg is acted upon by two perpendicular forces, 6 N and 8 N. The resultant acceleration of the body is

(a) 2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w. r. t. 6 N force.

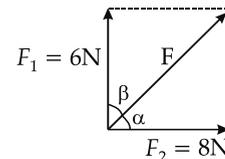
(b) 0.2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{4}{3}\right)$ w. r. t. 6 N force.

(c) 2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w. r. t. 8 N force.

(d) 0.2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ w. r. t. 8 N force.

Ans. (a, c)

Explanation:



Given situation is shown in figure

$m = 5 \text{ kg}$

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{6^2 + 8^2} = 10 \text{ N}$$

∴ Acceleration of body,

$$a = \frac{F}{m} = \frac{10}{5} = 2 \text{ m/s}^2$$

direction of acceleration

$$\tan \alpha = \frac{F_1}{F_2} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{3}{4}\right) \text{ with respect to } 8 \text{ N of force.}$$

$$\tan \beta = \frac{F_2}{F_1} = \frac{8}{6} = \frac{4}{3}$$

$$\therefore \beta = \tan^{-1}\left(\frac{4}{3}\right) \text{ with respect to } 6 \text{ N of force.}$$

Q. 8. The driver of a three wheeler moving with a speed of 36 kmh⁻¹ sees a child standing in the middle of the road and brings his vehicle to rest in 4 s, just in time to save the child. What is the average retarding force on the vehicle ? The mass of the three wheeler is 400 kg and the mass of the driver is 65 kg. A1

Ans. Using,

$$v = u + at$$

When,

$$u = 36 \text{ kmh}^{-1}$$

$$= \frac{36 \times 1000}{3600} \text{ m/s}$$

$$= 10 \text{ ms}^{-1}$$

$$v = 0, t = 4 \text{ s}$$

We get,

$$0 = 10 + a \times 4$$

or

$$a = -2.5 \text{ ms}^{-2}$$

Total mass of the driver and the three wheeler,

$$M = 65 + 400$$

$$= 465 \text{ kg}$$

Retarding force

$$F = 465 \times 2.5 \text{ N}$$

$$= 1162.5 \text{ N}$$

Q. 9. A rocket with a lift-off mass 20,000 kg is blasted upwards with net initial acceleration of 5 ms^{-2} , calculate initial thrust of the blast.

Ans. Here, total acceleration

$$a' = (g + a)$$

$$\text{or } a' = (9.8 + 5)$$

$$= 14.8 \text{ ms}^{-2}$$

\therefore Thrust of blast of rocket

$$= m \times a'$$

$$= 20000 \times 14.8$$

$$= 2.96 \times 10^5 \text{ N}$$

Q. 10. A body of mass 0.40 kg moving initially with a constant speed of 10 m/s to the north subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied at $t = 0$, the position of the body at that time to be $x = 0$ and predict its position at $t = -5 \text{ s}$, 25 s , 100 s .

Ans. Mass of the body, $m = 0.40 \text{ kg}$

Initial speed of the body, $u = 10 \text{ m/s}$ due north

Force acting on the body, $F = -8.0 \text{ N}$

Acceleration produced in the body,

$$a = \frac{F}{m} = \frac{-8.0}{0.40} = -20 \text{ m/s}^2$$

At $t = -5 \text{ s}$

Acceleration, $a' = 0$ and $u = 10 \text{ m/s}$

$$s = ut + \frac{1}{2} a' t^2$$

$$= 10 \times (-5) = -50 \text{ m}$$

At $t = 25 \text{ s}$

Acceleration, $a'' = -20 \text{ m/s}^2$ and $u = 10 \text{ m/s}$

$$s' = ut' + \frac{1}{2} a'' t'^2$$

$$= 10 \times 25 + \frac{1}{2} \times (-20) \times (25)^2$$

$$= 250 + 6250 = -6000 \text{ m}$$

At $t = 100 \text{ s}$

For $0 \leq t \leq 30 \text{ s}$

$$a = -20 \text{ m/s}^2$$

$$u = 10 \text{ m/s}$$

$$s_1 = ut + \frac{1}{2} a t^2$$

$$= 10 \times 30 + \frac{1}{2} \times (-20) \times (30)^2$$

$$= 300 - 9000$$

$$= -8700 \text{ m}$$

For $30 \text{ s} < t \leq 100 \text{ s}$

As per the first equation of motion, for $t = 30 \text{ s}$,

final velocity is given as: $v = u + at$

$$= 10 + (-20) \times 30 = -590 \text{ m/s}$$

Velocity of the body after 30 s = -590 m/s

For motion between 30 s to 100 s, i.e., in 70 s:

$$s_2 = vt + \frac{1}{2} a t^2$$

$$= -590 \times 70 = -41300 \text{ m}$$

$$\therefore \text{Total distance, } s'' = s_1 + s_2$$

$$= -8700 - 41300$$

$$= -50000 \text{ m}$$

Q. 11. A truck starts from rest and accelerates uniformly at 2.0 ms^{-2} . At $t = 10 \text{ s}$, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at $t = 11 \text{ s}$? (Neglect air resistance.)

Ans. (a) Initial velocity of the truck, $u = 0$

Acceleration, $a = 2 \text{ m/s}^2$

Time $t = 10 \text{ s}$

As per the first equation of motion, final velocity is given as:

$$v = u + at$$

$$= 0 + 2 \times 10 = 20 \text{ m/s}$$

The final velocity of the truck and hence, of the stone is 20 m/s .

At $t = 11 \text{ s}$, the horizontal component (v_x) of velocity, in the absence of air resistance, remains unchanged, i.e.,

$$v_x = 20 \text{ m/s}$$

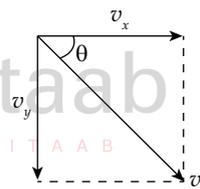
The vertical component (v_y) of velocity of the stone is given by the first equation of motion as:

$$v_y = u + a_y \Delta t$$

Where, $\Delta t = 11 - 10 = 1 \text{ s}$ and $a = g = 10 \text{ m/s}^2$

$$\therefore v_y = 0 + 10 \times 1 = 10 \text{ m/s}$$

The resultant velocity (v) of the stone is given as:



$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{20^2 + 10^2} = \sqrt{400 + 100}$$

$$= \sqrt{500} = 22.36 \text{ m/s}$$

(b) Let θ be the angle made by the resultant velocity with the horizontal component of velocity, v_x

$$\therefore \tan \theta = \left(\frac{v_y}{v_x} \right)$$

$$\theta = \tan^{-1} \left(\frac{10}{20} \right)$$

$$= \tan^{-1} (0.5)$$

$$= 26.57^\circ$$

When the stone is dropped from the truck, the horizontal force acting on it becomes zero. However, the stone continues to move under the influence of gravity. Hence, the acceleration of the stone is 10 m/s^2 and it acts vertically downward.

Q. 12. A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 ms^{-1} . What is the trajectory of the bob if the string is cut when the bob is:

- (a) at one of its extreme position.
(b) at its mean position.

Ans. (a) We know that at each extreme position, the instantaneous velocity of the bob is zero. If the string is cut at the extreme position, it is under the action of 'g' only, hence, the bob will fall vertically downwards.

(b) When the bob is at the mean position, it is affected by gravity. At mean position of the bob is having velocity of 1 ms^{-1} along the tangent to the arc which is in the horizontal direction. If the string is cut at the mean position, the bob will behave as a horizontal projectile. Hence, it will follow a parabolic path.

Q. 13. A man of mass 70 kg stands on a weighing scale in a lift which is moving :

- (a) upward with uniform speed of 10 m/s ?**
- (b) downward with a uniform acceleration of 5 m/s^2 ?**
- (c) upward with uniform acceleration of 5 m/s^2 . What would be the readings on the scale in each case ?**
- (d) What would be the reading if the lift mechanism failed and it came down freely under gravity ?**

Ans. Given, mass of man (m) = 70 kg
In each case the weighing scale will read the reaction R, i.e. the apparent weight.

(a) As lift is moving upward with a uniform speed, therefore its acceleration $a = 0$

$$\begin{aligned} \therefore \text{Normal reaction } w = R &= mg \\ &= 70 \times 10 \text{ N} \\ &= 700 \text{ N} \end{aligned}$$

w acts vertically downward and R acts vertically upwards.

$$\begin{aligned} \therefore \text{Reading on weighing scale} &= \frac{700}{10} = 70 \text{ kg} \end{aligned}$$

(b) Acceleration of the lift $a = 5 \text{ m/s}^2$ (\downarrow)

$$\begin{aligned} \therefore \text{Normal reaction, } R &= m(g - a) \\ &= 70(10 - 5) \text{ N} \\ &= 70 \times 5 \text{ N} = 350 \text{ N} \end{aligned}$$

$$\begin{aligned} \therefore \text{Reading on weighing scale} &= \frac{350 \text{ N}}{10 \text{ m/s}^2} \\ &= 35 \text{ kg.} \end{aligned}$$

(c) Acceleration of the lift $a = 5 \text{ m/s}^2$ (\uparrow)

$$\begin{aligned} \therefore \text{Normal reaction } R &= m(g + a) \\ &= 70(10 + 5) \\ &= 1050 \text{ N} \end{aligned}$$

$$\begin{aligned} \therefore \text{Reading on weighing scale} &= \frac{1050}{10 \text{ m/s}^2} = 105 \text{ kg.} \end{aligned}$$

(d) Acceleration of the lift when it is falling freely under gravity

$$a = g (\downarrow)$$

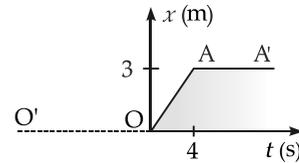
$$\begin{aligned} \therefore \text{Normal reaction } R &= m(g - a) \\ &= m(g - g) = 0 \end{aligned}$$

\therefore Reading on weighing scale = 0
This is the state of weightlessness.

Q. 14. Figure below shows the position-time graph of a particle of mass 4 kg. What is the

- (a) force on the particle for $t < 0$, $t > 4 \text{ s}$, $0 < t < 4 \text{ s}$?**
- (b) impulse at $t = 0$ and $t = 4 \text{ s}$? (Consider one dimensional motion only.)**

[MSE Chandigarh 2009]



Ans. Mass of particle = $m = 4 \text{ kg}$

(a) (1) For $t < 0$, the position-time graph is $O'O$ which means displacement of the particles is zero, i.e., the particle is at rest. Therefore, force on the particle must be zero,

$$\text{i.e., } F = 0$$

(2) For $t > 4 \text{ s}$, the position-time graph AA' is parallel to time axis. Therefore the particle remains at a distance of 3 m from the origin, and, it is at rest. Hence, no force is acting on the particle for this interval, i.e., $F = 0$.

(3) For $0 < t < 4 \text{ s}$, the particle is changing its position continuously. The position-time graph (OA) during this interval represents uniform motion of the particle, i.e., it moves with a constant speed and thus its acceleration is zero. Therefore

$$\begin{aligned} F &= ma \\ &= 0 \text{ during this interval} \end{aligned}$$

(b) (1) Impulse at $t = 0$

We know that,

$$\begin{aligned} \text{Impulse} &= \text{change in momentum} \\ &= mv - mu \\ &= m(v - u) \end{aligned} \quad \dots(\text{i})$$

Hence, $u = 0$ as the particle is at rest before $t = 0$. After $t = 0$, the particle has a constant velocity v where $v = \text{slope of line } OA = \text{velocity from point } O \text{ to point } A$

$$= \frac{3 \text{ m}}{4 \text{ s}} = 0.75 \text{ ms}^{-1} \quad \dots(\text{ii})$$

From equation (i) and (ii), we get

$$\begin{aligned} \text{impulse} &= 4(0.75 - 0) \\ &= 4 \times \frac{3}{4} = 3 \text{ kg ms}^{-1} \end{aligned}$$

Impulse at $t = 4 \text{ s}$

Before $t = 4 \text{ s}$, the particle has a constant velocity

$$u = \frac{3}{4} \text{ ms}^{-1}$$

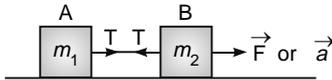
After $t = 4 \text{ s}$, the particle is at rest, i.e., $v = 0$

\therefore By definition of impulse, we get

$$\begin{aligned} \text{Impulse} &= mv - mu \\ &= m(v - u) \\ &= 4\left(0 - \frac{3}{4}\right) \\ &= -3 \text{ kg ms}^{-1}. \end{aligned}$$

Q. 15. Two bodies A and B of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force $F = 600$ N is applied to (a) B, (b) A along the direction of string. What is the tension in the string in each case?

Ans. Given : $F = 600$ N
Suppose $m_1 = 10$ kg
and $m_2 = 20$ kg
be the masses lying on a frictionless horizontal table.



Suppose T be the tension in the string and ' a ' be the acceleration of the system, in the direction of force applied.

(a) If force is applied on the heavier mass i.e., B

Then, equations of motion of A and B are

$$m_1 a = T \quad \dots(i)$$

$$m_2 a = F - T \quad \dots(ii)$$

Dividing equation (ii) by equation (i), we get

$$\frac{m_2}{m_1} = \frac{F - T}{T} = \frac{F}{T} - 1$$

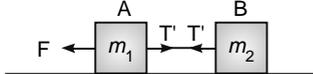
or $\frac{20}{10} = \frac{F}{T} - 1$

or $\frac{F}{T} = 2 + 1 = 3$

or $T = \frac{F}{3} = \frac{600}{3}$

$$= 200 \text{ N}$$

(b) If the force is applied on lighter mass i.e. A,



Suppose T be the tension in the string in this case
Then, equations of motion of A and B are

$$F - T = m_1 a \quad \dots(iii)$$

and $T = m_2 a \quad \dots(iv)$

Equation (iii) and (iv) gives

$$\frac{F - T}{T} = \frac{m_1 a}{m_2 a}$$

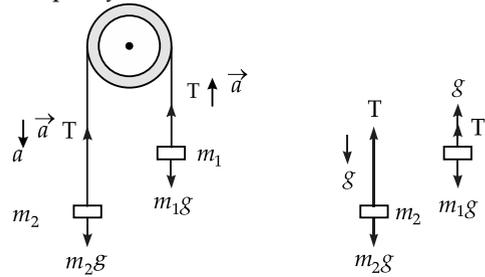
or $\frac{F}{T} - 1 = \frac{m_1}{m_2} = \frac{10}{20} = \frac{1}{2}$

or $\frac{F}{T} = 1 + \frac{1}{2} = \frac{3}{2}$

or $T = \frac{2}{3} F = \frac{2}{3} \times 600$
 $= 400 \text{ N}$

Q. 16. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string when the masses are released. **AI**

Ans. Suppose m_1 and m_2 be the masses suspended at the ends of a light inextensible string passing over the pulley.



$$\therefore m_1 = 8 \text{ kg}, m_2 = 12 \text{ kg}$$

Suppose, $T =$ tension in the string

$a =$ common acceleration with which

m_1 moves upward and m_2 moves downward

The equations of motion of m_1 and m_2 are given by

$$T - m_1 g = m_1 a \quad \dots(i)$$

and $m_2 g - T = m_2 a \quad \dots(ii)$

Adding equations (i) and (ii),

$$(m_2 - m_1) g = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_1) g}{m_1 + m_2} \quad \dots(iii)$$

\therefore From equations (i) and (iii),

$$T = m_1 g + m_1 \frac{(m_2 - m_1) g}{m_1 + m_2}$$

$$T = \frac{m_1 g}{m_1 + m_2} (m_1 + m_2 + m_2 - m_1)$$

or $T = \frac{2m_1 m_2}{m_1 + m_2} g \quad \dots(iv)$

Putting $m_1 = 8$ kg and $m_2 = 12$ kg and $g = 10 \text{ ms}^{-2}$, in equation (iii) and (iv), we get

$$a = \frac{(12 - 8)}{(8 + 12)} \times 10$$

or $= \frac{4}{20} \times 10 = 2 \text{ ms}^{-2}$

From eq. (i) $T = m_1 a + m_1 g$
 $= 8 \times 2 + 8 \times 10 = 96 \text{ N}$

Q. 17. A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei, the products must be emitted in opposite directions.

Ans. Suppose $m =$ mass of the nucleus at rest

$u =$ its initial velocity $= 0$ as it is at rest

Also suppose m_1, m_2 be the masses of the two smaller nuclei also called product nuclei and \vec{v}_1, \vec{v}_2 be their respective velocities.

When \vec{p}_i and \vec{p}_f be the initial and final momentum of the nucleus and the two nuclei respectively, then

$$\vec{p}_i = m \vec{u} = 0 \quad \dots(i)$$

$$\text{and} \quad \vec{p}_f = m_1 \vec{v}_1 + m_2 \vec{v}_2 \quad \dots(ii)$$

Now according to the law of conservation of linear momentum, we know that

$$\vec{p}_i = \vec{p}_f$$

$$\Rightarrow 0 = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$\Rightarrow m_2 \vec{v}_2 = -m_1 \vec{v}_1$$

$$\Rightarrow \vec{v}_2 = -\frac{m_1 \vec{v}_1}{m_2} \quad \dots(iii)$$

The negative sign in equation (iii) show that \vec{v}_1 and \vec{v}_2 are in opposite directions, i.e., the two smaller nuclei are emitted in opposite directions.

Q. 18. Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 ms⁻¹ collide and rebound with the same speed. What is the impulse imparted to each ball due to the other ?

Ans. Given : Mass of each ball, $m = 0.05$ kg
Speed of each ball, $v = 6$ ms⁻¹

∴ Initial momentum of each ball

$$\begin{aligned} &= m \vec{v} \\ &= (0.05)(6) \text{ kg ms}^{-1} \\ &= 0.30 \text{ kg ms}^{-1} \end{aligned}$$

As after collision, the direction of velocity of each ball is reversed on rebounding.

∴ Final momentum of each ball will be

$$\begin{aligned} \vec{p}_f &= m(-\vec{v}) \\ &= 0.05(-6) \\ &= -0.30 \text{ kg ms}^{-1} \end{aligned}$$

∴ Impulse imparted to each ball

$$= \vec{p}_f - \vec{p}_i$$

change in momentum of each ball

$$\begin{aligned} &= -0.30 - (0.30) \\ &= -0.60 \text{ kg ms}^{-1} \end{aligned}$$

or magnitude of impulse imparted by one ball due to collision with the other - 0.6 kg ms⁻¹. The two impulses are opposite in directions.

Q. 19. A shell of mass 0.02 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 ms⁻¹, what is the recoil speed of the gun ?

Ans. Given : mass of gun, $m_g = 100$ kg, mass of shell $m_b = 0.02$ kg, velocity of shell, $v_b = 80$ ms⁻¹

Total initial momentum = 0

(∵ the gun and shell have no initial velocity as they are initially at rest)

Total final momentum = momentum of bullet + momentum of gun

$$= m_b v_b + m_g v_g$$

Applying law of conservation of momentum,

$$0 = m_b v_b + m_g v_g$$

$$\text{i.e.,} \quad m_b v_b = -m_g v_g$$

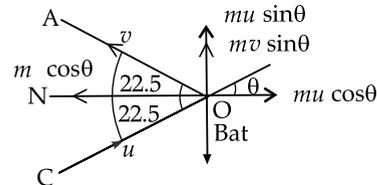
$$\text{i.e.,} \quad 0.02 \times 80 = -100 \times v_g$$

$$\begin{aligned} \text{or} \quad v_g &= -\frac{0.02 \times 80}{100} \\ &= -0.016 \text{ ms}^{-1} \end{aligned}$$

Hence, recoil speed of the gun is = 0.016 ms⁻¹.

Q. 20. A batsman deflects a ball by an angle of 45° without changing its initial speed, which is equal to 54 kmh⁻¹. What is the impulse imparted to the ball ? Mass of the ball is 0.15 kg.

Ans. The ball struck by the bat is deflected back such that total angle is 45°.



Change in momentum in vertical direction,

$$= mv \sin \theta - m u \sin \theta$$

$$= 0 \quad (\text{since, } v = u \text{ \& } \theta \text{ is same})$$

So, net change in momentum is only due to that in horizontal direction.

Now, initial momentum of ball = $mu \cos \theta$

$$= \frac{0.15 \times 54 \times 1000 \times \cos 22.5}{3600}$$

$$= 0.15 \times 15 \times 0.9239 \text{ along NO}$$

Final momentum of ball = $mu \cos \theta$ along ON

Impulse = change in momentum

$$= mu \cos \theta - (-mu \cos \theta)$$

$$= 2mu \cos \theta$$

$$= 2 \times 0.15 \times 15 \times 0.9239$$

$$\text{i.e.,} \quad \text{Impulse} = 4.16 \text{ kg ms}^{-1}$$

Q. 21. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with speed 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

Ans. Frequency of revolution of stone is given by

$$f = 40 \text{ rev/min} = \frac{40}{60} \text{ rev/s}$$

Mass of stone, $m = 0.25$ kg

Radius of circle, $r = 1.5$ m

By formula angular speed of the stone,

$$\omega = 2\pi f$$

$$\text{or} \quad \omega = 2\pi \times \frac{40}{60}$$

$$= \frac{4\pi}{3} \text{ rad s}^{-1}$$

T = tension in the string = ?

T_{max} = maximum tension in the string = 200 N

i.e.,
$$T = \frac{mv^2}{r} = mr\omega^2$$

or
$$T = 0.25 \times 1.5 \times \left(\frac{4\pi}{3}\right)^2 \text{ N}$$

or
$$T = 0.25 \times 1.5 \times \frac{16}{9} \times 9.87$$

($\because \pi^2 = 9.87$)

or
$$T = 6.58 \text{ N} \approx 6.6 \text{ N}$$

As the string can withstand a maximum tension of 200 N,

$$\therefore T_{\max} = \frac{mv_{\max}^2}{r}$$

or
$$v_{\max} = \sqrt{\frac{rT_{\max}}{m}}$$

$$= \sqrt{\frac{1.5 \times 200}{0.25}}$$

or
$$v_{\max} = 34.64 \text{ ms}^{-1}$$

Q. 22. If in Q. 21 the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks ?

- (a) The stone jerks radially outwards.
- (b) The stone flies off tangentially from the instant the string breaks.
- (c) The stone flies off at the angle with the tangent whose magnitude depends on the speed of particle.

Ans. (b) Correctly describes the trajectory of the stone after the string breaks, i.e., the stone flies off tangentially from the instant the string breaks.

The velocity always acts tangentially to the circle at each point in the circular motion. At the time, the string breaks, the particle continues to move in the tangential direction according to Newton's first law of motion.

Q. 23. Explain why :

[AI]

- (a) a horse cannot pull a cart and run in empty space, [NCT 2009]
- (b) passengers are thrown forward from their seats when a speeding bus stops suddenly, [NCT 2008]
- (c) it is easier to pull a lawn mover than to push it,
- (d) a cricketer moves his hands backwards when holding a catch. [NCT 2008]

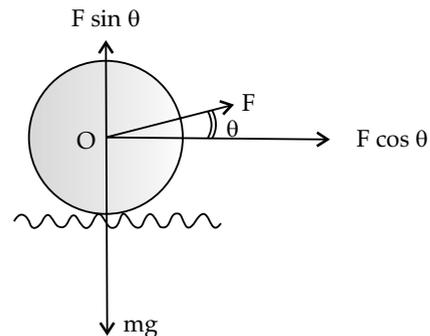
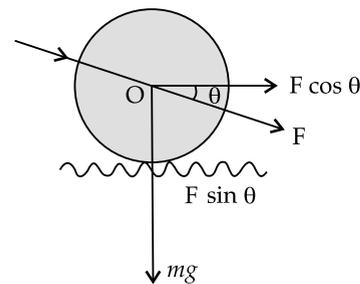
Ans. (a) While trying to pull a cart, a horse pushes the ground backwards with a certain force at an angle. The ground offers an equal reaction in opposite direction; on the feet of the horse. The forward component of this reaction is responsible for the motion of cart.

In empty space, there is no reaction and hence horse cannot pull the cart and run.

- (b) This is due to inertia of motion possessed by the passenger in a speeding bus. The lower portion of the body of passenger (which is in contact with the bus) comes to rest. But due to inertia of motion, the upper portion of the body tends

to keep on moving. As a result, the passenger is thrown forward.

- (c) Consider the free body diagram in the two cases, the lawn mover is represented by the point O.



The forces acting on the lawn mover are (i) applied force F, (ii) weight of the mover W downward, (iii) force of friction acting opposite to the direction of motion, (iv) reaction of ground on the mover (R).

In case of pushing the mover, the horizontal component of push $F \cos \theta$ in forward direction and vertical component $F \sin \theta$ in downward direction. The total downward force = $W + F \sin \theta$.

Therefore, the reaction

$$R = W + F \sin \theta$$

(Newton's third law of motion) and force of friction,

$$f = \mu R$$

$$= \mu(W + F \sin \theta) \quad (\because W = mg)$$

where, μ = coefficient of friction.

When P be the net forward force, then

$$P = F \cos \theta - f$$

$$= F \cos \theta - \mu(mg + F \sin \theta) \quad \dots(i)$$

In case of pull, the forward component of pull

$$= F \cos \theta$$

and vertical component = $F \sin \theta$

where R' be the normal reaction in the case, then

$$F' = W - F \sin \theta$$

$$= mg - F \sin \theta$$

\therefore if f' be the force of friction in this case, then net force

$$P' = F \cos \theta - f' = F \cos \theta - \mu R'$$

$$= F \cos \theta - \mu(mg - F \sin \theta) \quad \dots(ii)$$

From equation (i) and (ii), we see that $P' > P$. Therefore, it is easier to pull than to push the lawn mover.

- (d) While holding a catch, the impulse received by the hands, $F \times t =$ change in momentum of the

ball is constant. By moving his hands backwards, the cricketer increases the time of impact (t) to complete the catch. Since, t increases, F decreases and as a reaction, the ball hurts hands lesser.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

Q. 1. A ball is travelling with uniform translatory motion. This means that

- (A) It is at rest.
 (B) The path can be a straight line or circular and the ball travels with uniform speed.
 (C) All parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.
 (D) The centre of the ball moves with constant velocity and the ball spins about its centre uniformly.

Ans. Option (C) is correct.

Q. 2. A metre scale is moving with uniform velocity. This implies

- (A) The force acting on the scale is zero, but a torque about the centre of mass can act on the scale.
 (B) The force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero.
 (C) The total force acting on it need not be zero but the torque on it is zero.
 (D) Neither the force nor the torque need to be zero.

Ans. Option (B) is correct.

Q. 3. A cricket ball of mass 150 g has an initial velocity

$\vec{u} = (3\hat{i} + 4\hat{j}) \text{ ms}^{-1}$ and a final velocity $\vec{v} = -(3\hat{i} + 4\hat{j}) \text{ ms}^{-1}$ after being hit. The change in momentum (final momentum-initial momentum) is (in kg m s^{-1}) A1

- (A) Zero
 (B) $-(0.45\hat{i} + 0.6\hat{j})$
 (C) $-(0.9\hat{i} + 1.2\hat{j})$
 (D) $-5(\hat{i} + \hat{j})$.

Ans. Option (C) is correct.

Explanation:

Here, $m = 150 \text{ g} = 0.15 \text{ kg}$

$$\vec{u} = (3\hat{i} + 4\hat{j}) \text{ m/s}$$

$$\vec{v} = -(3\hat{i} + 4\hat{j}) \text{ m/s}$$

Initial momentum, $P_i = m\vec{u}$

$$\vec{p}_i = (0.15 \text{ kg})(3\hat{i} + 4\hat{j}) \text{ m/s}$$

$$= (0.45\hat{i} + 0.6\hat{j}) \text{ kg m/s}$$

$$\begin{aligned} \text{Final momentum, } \vec{p}_f &= (0.15 \text{ kg})(-3\hat{i} - 4\hat{j}) \text{ m/s} \\ &= (-0.45\hat{i} - 0.6\hat{j}) \text{ kg m/s} \end{aligned}$$

Change in momentum,

$$\begin{aligned} \Delta \vec{p} &= \vec{p}_f - \vec{p}_i = (-0.45\hat{i} - 0.6\hat{j}) \text{ kg m/s} \\ &\quad - (0.45\hat{i} + 0.6\hat{j}) \text{ kg m/s} \\ &= -(0.9\hat{i} + 1.2\hat{j}) \text{ kg m/s} \end{aligned}$$

Q. 4. In the previous Que. 3, the magnitude of the momentum transferred during the hit is

- (A) Zero
 (B) 0.75 kg ms^{-1}
 (C) 1.5 kg ms^{-1}
 (D) 14 kgms^{-1}

Ans. Option (C) is correct.]

Explanation: By previous solution,

$$\Delta \vec{p} = -(0.9\hat{i} + 1.2\hat{j})$$

$$\begin{aligned} \text{Magnitude} &= |\Delta \vec{p}| = \sqrt{(0.9)^2 + (1.2)^2} \\ &= \sqrt{0.81 + 1.44} \\ &= 1.5 \text{ kg m/s} \end{aligned}$$

Q. 5. Conservation of momentum in a collision between particles can be understood from

- (A) Conservation of energy.
 (B) Newton's first law only.
 (C) Newton's second law only
 (D) Both newton's second and third law.

Ans. Option (D) is correct.

Q. 6. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is

- (A) Frictional force along westward.
 (B) Muscle force along southward.
 (C) Frictional force along south-west.
 (D) Muscle force along south-west.

Ans. Option (C) is correct.

Explanation: Direction of change in momentum will represent direction of force.

Q. 7. A body of mass 2 kg travels according to the law $x(t) = pt + qt^2 + rt^3$ where $p = 3 \text{ ms}^{-1}$, $q = 4 \text{ ms}^{-2}$, and $r = 5 \text{ ms}^{-3}$.

The force acting on the body at $t = 2$ seconds is

- (A) 136 N
- (B) 134 N
- (C) 158 N
- (D) 68 N

Ans. Option (A) is correct.

Explanation: Here, $x(t) = pt + qt^2 + rt^3$
where $p = 3 \text{ m/s}$, $q = 4 \text{ m/s}^2$ and $r = 5 \text{ m/s}^3$
 $m = 2 \text{ kg}$

$$\text{velocity, } v = \frac{dx}{dt} = \frac{d}{dt}(pt + qt^2 + rt^3)$$

$$= p + 2qt + 3rt^2$$

$$\text{Acceleration, } a = \frac{dv}{dt} = 2q + 6rt$$

$$\left(\frac{dv}{dt}\right)_{t=2} = a = 2(4 \text{ m/s}^2) + 6(5 \text{ m/s}^3) \times (2 \text{ s})$$

$$\left(\frac{dv}{dt}\right)_{t=2} = 8 \text{ m/s}^2 + 60 \text{ m/s}^2 = 68 \text{ m/s}^2$$

\therefore Force acting on the body of mass 2 kg is
 $F = ma = (2 \text{ kg})(68 \text{ m/s}^2)$
 $= 136 \text{ N}$

Q. 8. A body with mass 5 kg is acted upon by a force, $\vec{F} = (-3\hat{i} + 4\hat{j}) \text{ N}$. If its initial velocity at $t = 0$ is $(u) = (6\hat{i} - 12\hat{j}) \text{ ms}^{-1}$, the time at which it will just have velocity along the y -axis is

- (A) Never
- (B) 10 s
- (C) 2 s
- (D) 15 s

Ans. Option (B) is correct.

Explanation: Given

$$m = 5 \text{ kg}, \vec{F} = -(3\hat{i} + 4\hat{j}) \text{ N}, u = 6\hat{i} - 12\hat{j} \text{ m/s}$$

The acceleration of the body is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{-(3\hat{i} + 4\hat{j}) \text{ N}}{5 \text{ kg}}, = -\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right) \text{ m/s}^2$$

Velocity of the body along x -axis at any time t is

$$v_x = u_x + a_x t = 6 - \frac{3}{5}t$$

As the body will have a velocity along y -axis, When its velocity along x -axis will be zero.

$$\text{i.e., } v_x = 0$$

$$\text{or } 6 - \frac{3}{5}t = 0 \text{ or } t = \frac{30}{3} = 10 \text{ s}$$

Q. 9. A car of mass m starts from rest and acquires a velocity along east $\vec{v} = v\hat{i}$ ($v > 0$) in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is

- (A) $\frac{mv}{2}$ eastward and is exerted by the car engine.
- (B) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road.
- (C) More than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road.
- (D) $\frac{mv}{2}$ exerted by the engine.

Ans. Option (B) is correct.

Explanation: Given, mass of the car = m

Initial velocity, $(\vec{u}) = (\vec{0})$ (As car starts from rest)

Final velocity, $\vec{v} = v\hat{i}$ along east

$$t = 2\text{s}$$

$$(\vec{v}) = (\vec{u}) + (\vec{a})t$$

$$v\hat{i} = \vec{0} + \vec{a} \times 2 \text{ or } \vec{a} = \frac{v}{2}\hat{i}$$

Force exerted on the car is

$$\vec{F} = m\vec{a} = \frac{mv}{2}\hat{i} = \frac{mv}{2} \text{ eastward}$$

This is because of friction on the tyres exerted by the road.

Short Answer Type Questions

Q.10. A girl riding a bicycle along a straight road with a speed of 5 ms^{-1} throws a stone of mass 0.5 kg which has a speed of 15 ms^{-1} with respect to the ground along her direction of motion. The mass of the girl and bicycle is 50 kg. Does the speed of the bicycle change after the stone is thrown? What is the change in speed, if so?

[NCERT Exemplar Q. No. 16]

Ans. Mass of girl and bicycle = 50 kg
Mass of stone = 0.5 kg

where, u_1 = Initial speed of the cycle, the girl and the stone

v_1 = Final speed of the cycle and the girl

v_2 = Final speed of stone

According to law of conservation of momentum

Initial momentum = Final momentum

$$(m_1 + m_2) u_1 = m_1 v_1 + m_2 v_2$$

$$(50 + 0.5) \times 5 = 50 \times v_1 + 0.5 \times 15$$

$$50 v_1 = 252.5 - 7.5$$

$$= 245.0$$

$$v_1 = \frac{245.0}{50} = 4.9 \text{ m/s}$$

Hence, the speed of cycle decreases by $5 - 4.9 = 0.1 \text{ m/s}$

Q. 11. A person of mass 50 kg stands on a weighing scale on a lift. If the lift is descending with a downward acceleration of 9 ms^{-2} , what would be the reading of the weighing scale?

($g = 10 \text{ ms}^{-2}$) [NCERT Exemplar Q. No. 17]

Ans. If lift is descending, with acceleration a , the apparent weight decreases on weighing scale,

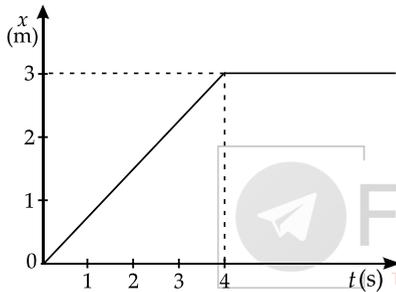
$$W_1 = R = (mg - ma) = m(g - a)$$

Apparent weight due to reaction force,

$$W_1 = 50(10 - 9) = 50 \text{ N}$$

$$\therefore \text{Reading of weighing scale} = \frac{R}{g} = \frac{50}{10} = 5 \text{ kg}$$

Q. 12. The position time graph of a body of mass 2 kg is as given in Fig. What is the impulse on the body at $t = 0 \text{ s}$ and $t = 4 \text{ s}$.



[NCERT Exemplar Q. No. 18]

Ans. From graph, $\tan \theta = \frac{3}{4} = 0.75$.

At $t \geq 4$, slope of graph = 0,

Initial velocity = $v_1 = 0$

Velocity at $t = 0$,

$$v_2 = \frac{3}{4} \text{ m/s}$$

Velocity at $t \geq 4$,

$$v_3 = 0.$$

At $t = 0$, Impulse

$$= mv_2 - mv_1 = 2[0.75 - 0] = 1.50 \text{ kg m/s}$$

At $t = 4$, Impulse

$$= m(v_3 - v_2) = 2[0 - 0.75] = -1.50 \text{ kg m/s}$$

Q. 13. A person driving a car suddenly applies the brakes on seeing a child on the road ahead. If he is not wearing seat belt, he falls forward and hits his head against the steering wheel. Why?

[NCERT Exemplar Q. No. 19]

Ans. When a person applies brakes suddenly, the only retarding force that acts on him, if he is not using a seat belt, comes from the friction exerted by the seat. This is not enough to prevent him from moving forward, due to inertia of motion, when the vehicle is brought to a sudden halt.

Q. 14. The velocity of a body of mass 2kg as a function of t is given by $v(t) = 2t\hat{i} + t^2\hat{j}$. Find the momentum and the force acting on it, at time $t = 2\text{s}$. [AI]

[NCERT Exemplar Q. No. 20]

Ans. As, $v(t) = 2t\hat{i} + t^2\hat{j}$

$$\vec{v} \text{ at } 2\text{sec}, \vec{v}(2) = 2 \times 2\hat{i} + (2)^2\hat{j}$$

$$\vec{v} = 4\hat{i} + 4\hat{j}$$

$$\text{Momentum, } \vec{p} = m\vec{v} = 2(4\hat{i} + 4\hat{j}) = 8\hat{i} + 8\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2t\hat{j}$$

$$(\vec{a})_{t=2} = (2\hat{i} + 4\hat{j}) \text{ m/s}^2$$

$$\vec{F} = m\vec{a} = 2(2\hat{i} + 4\hat{j}) = (4\hat{i} + 8\hat{j}) \text{ N}$$

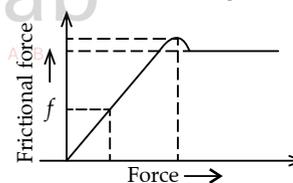
Q. 15. A block placed on a rough horizontal surface is pulled by a horizontal force F . Let f be the force applied by the rough surface on the block. Plot a graph of f versus F .

[NCERT Exemplar Q. No. 21]

Ans. When a small force F_1 is applied on the block, it does not move. At this stage, f is equal to F .

$f = F$ until the block is stationary.

f remains constant if F increases beyond this point and the block starts moving.



Q. 16. Why are porcelain objects wrapped in paper or straw before packing for transportation?

[NCERT Exemplar Q. No. 22]

Ans. Porcelain objects are brittle in nature and can crack with even small jerk on it.

In transportation, the vehicle may need to halt suddenly. To bring a fragile material, like porcelain object to a sudden halt means applying a large force and this is likely to damage the object. If it is wrapped up in straw, the object can travel some distance, as the straw is soft, before coming to a halt. Thus reduces the possibility of damage.

Q. 17. Why does a child feel more pain when she falls down on a hard cement floor, than when she falls down on the soft muddy ground in the garden? [NCERT Exemplar Q. No. 23]

Ans.
$$F = ma = \frac{v - u}{t}$$

The body of the child is brought to a sudden halt when she falls on a cement floor. If it is a mud floor the body travels some distance before it comes to rest, which takes some time. This means the force which brings the child to rest is less for the fall on a

mud floor, as the change in momentum is brought about over a longer period.

Q. 18. A woman throws an object of mass 500 g with a speed of 25 ms⁻¹.

- (a) What is the impulse imparted to the object?
- (b) If the object hits a wall and rebounds with half the original speed, what is the change in momentum of the object?

[NCERT Exemplar Q. No. 24]

Ans. (a) mass of object, $m = 500 \text{ g} = 0.5 \text{ kg}$
 $u = 0, v = 25 \text{ m/s}$

$$\text{Impulse} = mv - mu$$

$$I = \Delta p = 0.5(25 - 0) = 12.5 \text{ N-s}$$

(b) $m = 0.5 \text{ kg}, u = +25 \text{ m/s}$

$$v = \frac{-25}{2} \text{ m/s} - 12.5 \text{ N-s}$$

$$\Delta p = 0.5 \left(\frac{-25}{2} - 25 \right)$$

$$= 0.5[-12.5 - 25] = 0.5 \times (-37.5)$$

$$\Delta p = -18.75 \text{ kg m/s}$$

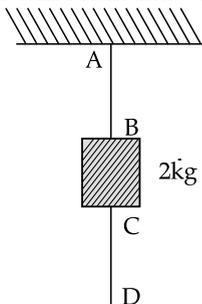
Force is opposite to initial velocity of the object.

Q. 19. Why are mountain roads generally made winding upwards rather than going straight up?

[NCERT Exemplar Q. No. 25]

Ans. On an inclined plane force of friction on a body going upward is $f = \mu R = \mu mg \cos \theta$ is the force of friction, if θ is angle made by the slope. If θ is small, force of friction is high and there is less chance of skidding. The road straight up would have a larger slope and smaller value of friction, therefore there are more chances of skidding.

Q. 20. A mass of 2 kg is suspended with thread AB. Thread CD of the same type is attached to the other end of 2 kg mass. Lower thread is pulled gradually, harder and harder in the downward direction so as to apply force on AB. Which of the threads will break and why?



[NCERT Exemplar Q. No. 26]

Ans. Thread AB will break.

since, force on the upper thread will be equal to sum of the weight of the body and the applied force.

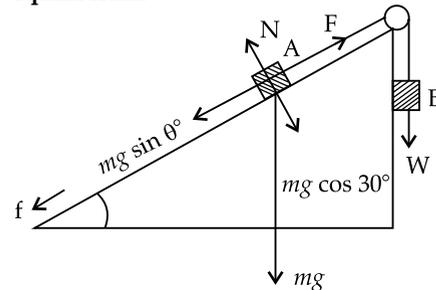
Q. 21. In the above given problem if the lower thread is pulled with a jerk, what happens?

[NCERT Exemplar Q. No. 27]

Ans. Thread CD will break.

If the force is large and sudden, thread CD breaks because as CD is jerked, the pull is not transmitted to AB instantaneously (transmission depends on the elastic properties of the body). Therefore, before the mass moves, CD breaks.

Q. 22. Block A of weight 100 N rests on a frictionless inclined plane of slope angle 30°. A flexible cord attached to A passes over a frictionless pulley and is connected to block B of weight W. Find the weight W for which the system is in equilibrium.



[NCERT Exemplar Q. No. 29]

Ans. $mg \sin 30^\circ = F$

$$\frac{1}{2} mg = F$$

$$F = \frac{1}{2} \times 100 \text{ N}$$

$$= 50 \text{ N}$$

For B (at rest), $W = F = 50 \text{ N}$.

Q. 23. A block of mass M is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is μ and the acceleration due to gravity is g, calculate the minimum force required to be applied by the finger to hold the block against the wall? [NCERT Exemplar Q. No. 30]

Ans. Let F force is applied by finger on a body of mass M,

Under balanced condition,

$$F = N$$

N is the normal reaction of the wall on the block. The minimum upward frictional force needed to ensure that the block does not fall is Mg. The frictional force = μN . Thus, minimum force required,

$$F = \frac{Mg}{\mu}$$

Q. 24. A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400m from the bottom of the cliff. Find the recoil velocity of the gun. (acceleration due to gravity = 10 ms⁻²)

[NCERT Exemplar Q. No. 31]

Ans. Applying 2nd kinematic equation of motion

$$s = ut + \frac{1}{2}at^2$$

Putting, $u = 0, g = 10 \text{ m/s}^2, s = 500 \text{ m}$

$$500 = \frac{10 \times t^2}{2}$$

$$\therefore t = 10\text{s}$$

Horizontal range = $v \times 10$

$$\text{or, } 400 = v \times 10 \therefore v = 40 \text{ m/s}$$

From law of conservation of momentum,

$$m_b u_b + m_g u_g = m_b v_b + m_g v_g$$

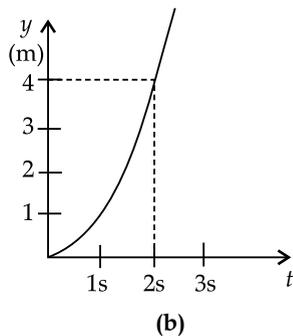
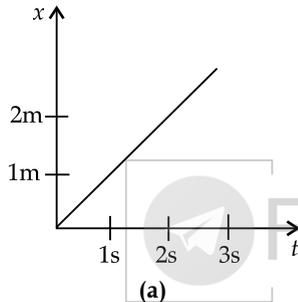
$$m_b \times 0 + m_g \times 0 = 1 \times 40 + 100v_g$$

$$100v_g = -40$$

$$\text{or } v_g = -0.4 \text{ m/s}$$

–ve sign indicates that the recoil velocity is in reverse direction.

Q. 25. Figure shows $(x, t), (y, t)$ diagram of a particle moving in 2- dimensions.



If the particle has a mass of 500 g. find the force (direction and magnitude) acting on the particle.

[NCERT Exemplar Q. No. 32]

Ans. From graph (a)

$$\text{As, } v_x = \frac{dx}{dt} = \frac{2}{2} = 1 \text{ m/s}$$

$$a_x = \frac{dv_x}{dt} = 0$$

from (b), $y = t^2$

$$v_y = \frac{dy}{dt} = 2t$$

$$a_y = \frac{dv_y}{dt} = 2$$

$$F_y = ma_y$$

$$= 0.5 \times 2 = 1 \text{ N (toward Y-axis)}$$

$$F_x = 0.5 \times 0 = 0 \text{ N}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{0^2 + 1^2}$$

$$F = 1 \text{ N (towards Y-axis)}$$

Q. 26. A person in an elevator accelerating upwards with an acceleration of 2 ms^{-2} , tosses a coin vertically upwards with a speed of 20 ms^{-1} . After how much time will the coin fall back into his hand? ($g = 10 \text{ ms}^{-2}$)

[NCERT Exemplar Q. No. 33]

Ans. As, $v = u + at$

$$0 = 20 - 12t$$

$$t = \frac{20}{12} = \frac{5}{3} \text{ s}$$

\therefore Time of ascent = time of descent

\therefore Total time after which coin falls back

$$t = \frac{5}{3} + \frac{5}{3} = \frac{10}{3} \text{ s}$$

$$= 3.33 \text{ s}$$

Long Answer Type Questions

Q. 27. There are three forces \vec{F}_1, \vec{F}_2 and \vec{F}_3 acting on a body, all acting on a point P on the body. The body is found to move with uniform speed.

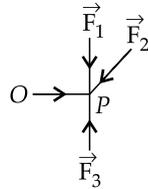
(a) Show that the forces are coplanar.

(b) Show that the torque acting on the body about any point due to these three forces is zero.

[NCERT Exemplar Q. No. 34]

Ans. (a) The body is moving with uniform speed after the action of three forces, \vec{F}_1, \vec{F}_2 and \vec{F}_3 on a point on body.

Since, the body is moving with no acceleration, the sum of the forces is zero $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Let $\vec{F}_1, \vec{F}_2, \vec{F}_3$ be the three forces passing through a point. Let \vec{F}_1 and \vec{F}_2 be in the plane A. Then $\vec{F}_1 + \vec{F}_2$ must be in the plane A since, $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$. \vec{F}_3 is also in the plane A.



(b) Consider the torque of the forces about P. Since all the forces pass through P, the torque is zero. Now consider torque about another point O. Then torque about O is

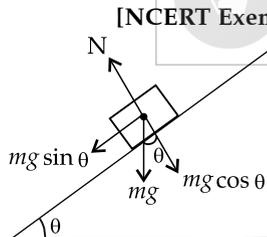
$$\text{Torque} = OP \times (\vec{F}_1 + \vec{F}_2 + \vec{F}_3)$$

As a resultant of \vec{F}_1, \vec{F}_2 and \vec{F}_3 is zero,

$$\text{Since, } \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0,$$

$$\text{Torque} = OP \times \vec{F} = 0.$$

Q. 28. When a body slides down from rest along a smooth inclined plane making an angle of 45° with the horizontal, it takes time T. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time pT , where p is some number greater than 1. Calculate the coefficient of friction between the body and the rough plane.



Ans. For smooth inclined plane
Let s = length of inclined plane
or $mg \sin \theta = ma$
or $a = g \sin \theta$

Applying 2nd kinematic equation of motion

$$s = ut + \frac{1}{2}at^2$$

$$\text{or } s = 0 \times T + \frac{1}{2}(g \sin \theta)T^2 \quad (\because t = T)$$

$$\text{or } s = \frac{1}{2}g \sin \theta T^2$$

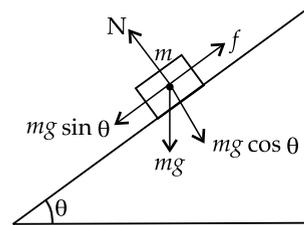
$$\text{or } s = \frac{1}{2\sqrt{2}}gT^2 \quad (\text{i}) \quad (\because \theta = 45^\circ)$$

For rough inclined plane.

$$f = \mu N = \mu mg \cos \theta$$

$$mg \cos \theta - f = ma_1$$

$$\text{or } a_1 = (\sin \theta - \mu \cos \theta)g = \frac{1}{\sqrt{2}}(1 - \mu)g$$



$$\text{Using, } s = ut + \frac{1}{2}at^2$$

$$\text{or } s = 0 \times (pT) + \frac{1}{2} \left\{ \frac{1}{\sqrt{2}}(1 - \mu)g \right\} \times p^2 T^2$$

$$\text{or } s = \frac{1}{2\sqrt{2}}(1 - \mu)gp^2 T^2 \quad (\text{ii})$$

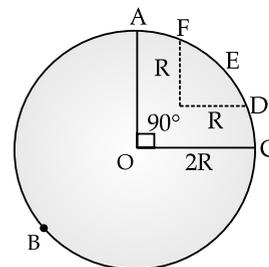
\therefore by (i) & (ii), we get

$$\frac{1}{2\sqrt{2}}gT^2 = \frac{1}{2\sqrt{2}}(1 - \mu)gp^2 T^2$$

$$\text{or } 1 = (1 - \mu)p^2$$

$$\text{or } \mu = \left(1 - \frac{1}{p^2} \right)$$

Q. 29. A racing car travels on a track (without banking) ABCDEFA (as shown in figure). ABC is a circular arc of radius $2R$. CD and FA are straight paths of length R and DEF is a circular arc of radius $R = 100$ m. The coefficient of friction on the road is $\mu = 0.1$. The maximum speed of the car is 50 ms^{-1} . Find the minimum time for completing one round.



[NCERT Exemplar Q. No. 37]

Ans. The centripetal force to keep car in circular motion is provided by frictional force (inward to centre).

For DEF

$$\frac{m v^2}{R} = mg\mu$$

$$v_{\max} = \sqrt{g\mu R} = \sqrt{100} = 10 \text{ ms}^{-1}$$

For ABC

$$\frac{v^2}{R} = g\mu, v = \sqrt{200} = 14.14 \text{ ms}^{-1}$$

$$\text{Time for DEF} = \frac{\pi}{2} \times \frac{100}{10} = 5\pi \text{ s}$$

$$\text{Time for ABC} = \frac{3\pi}{2} \times \frac{200}{14.14} = \frac{300\pi}{14.14} \text{ s}$$

For FA and DC = $2 \times \frac{100}{50} = 4$ s

Total time = $5\pi + \frac{300\pi}{14.14} + 4 = 86.3$ s

Q. 30. The displacement vector of a particle of mass m

is given by $\vec{r}(t) = \hat{i} A \cos \omega t + \hat{j} B \sin \omega t$.

(a) Show that the trajectory is an ellipse.

(b) Show that $F = -m\omega^2 r$.

[NCERT Exemplar Q. No. 38]

Ans. (a) $x = A \cos \omega t, y = B \sin \omega t$

$$\frac{x}{A} = \cos \omega t \quad (1)$$

$$\frac{y}{B} = \sin \omega t \quad (2)$$

Squaring & adding (1), (2)

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$$

which is the equation of ellipse

$$(b) \frac{d\vec{r}}{dt} = \vec{v} = -\hat{i} \omega A \sin \omega t + \hat{j} \omega B \cos \omega t$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\hat{i} \omega^2 A \cos \omega t - \hat{j} \omega^2 B \sin \omega t$$

$$= -\omega^2 (\hat{i} A \cos \omega t + \hat{j} B \sin \omega t) = -\omega^2 \vec{r}$$

$$\therefore \vec{F} = -m\omega^2 \vec{r}$$

Q. 31. A cricket bowler releases the ball in two different ways

(a) giving it only horizontal velocity, and

(b) giving it horizontal velocity and a small downward velocity.

The speed v_s at the time of release is the same. Both are released at a height H from the ground. Which one will have greater speed when the ball hits the ground? Neglect air resistance.

[NCERT Exemplar Q. No. 39]

Ans. For (a) $\frac{1}{2} v_v^2 = gH$ $v_v = \sqrt{2gH}$

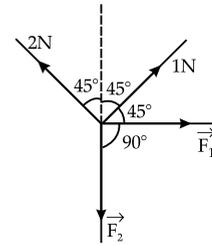
Speed at ground = $\sqrt{v_s^2 + v_v^2} = \sqrt{v_s^2 + 2gH}$

For (b) also $\left[\frac{1}{2} m v_s^2 + mgH \right]$ is the total energy of the ball when it hits the ground.

So, the speed would be the same for both (a) and (b).

Q. 32. There are four forces acting at a point P produced by strings as shown in Fig. which is at rest. Find

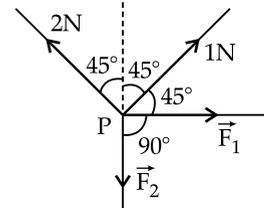
the forces \vec{F}_1 and \vec{F}_2 .



[NCERT Exemplar Q. No. 40]

Ans. Particle is at rest, $a = 0$.

So, resultant force due to all forces will be zero.



Resolving the 2N force:

Its horizontal component is $2 \sin 45^\circ$

Its vertical component is $2 \cos 45^\circ$

Similarly, resolving the 1N force:

Its horizontal component is $1 \sin 45^\circ$

Its vertical component is $1 \cos 45^\circ$

Since, the point is at rest,

Net horizontal component = $F_1 + 1 \sin 45^\circ - 2 \sin 45^\circ = 0$

$$\therefore F_1 = \frac{1}{\sqrt{2}} \text{ N}$$

Net vertical component = $2 \cos 45^\circ + 1 \cos 45^\circ - F_2 = 0$

$$\therefore F_2 = \frac{3}{\sqrt{2}} \text{ N}$$

Q. 33. A rectangular box lies on a rough inclined surface. The coefficient of friction between the surface and the box is μ . Let the mass of the box be m.

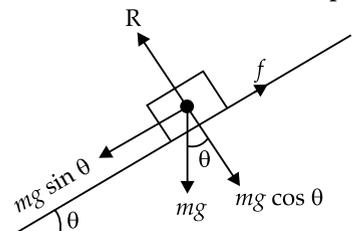
(a) At what angle of inclination θ of the plane to the horizontal will the box just start to slide down the plane?

(b) What is the force acting on the box down the plane, if the angle of inclination of the plane is increased to $\alpha > \theta$?

(c) What is the force needed to be applied upwards along the plane to make the box either remain stationary or just move up with uniform speed?

(d) What is the force needed to be applied upwards along the plane to make the box move up the plane with acceleration a?

[NCERT Exemplar Q. No. 41]



- Ans. (a) $\theta = \tan^{-1}(\mu)$
 (b) If $\alpha > \theta$, angle of inclination of plane with horizontal, it will slide down with force
 $F_1 = (mg \sin \alpha - \mu mg \cos \alpha)$
 (c) To keep box either stationary or just move it up with uniform velocity

$$F_2 = mg \sin \alpha + f = mg (\sin \alpha + \mu \cos \alpha)$$

- (d) Force applied to move box upward with acceleration a ,

$$F_3 = mg (\sin \alpha + \mu \cos \alpha) + ma.$$

- Q. 34.** A helicopter of mass 2000 kg rises with a vertical acceleration 15ms^{-2} . The total mass of the crew and passengers is 500 kg. Give the magnitude and direction of the ($g = 10 \text{ ms}^{-2}$)

- (a) force on the floor of the helicopter on the surrounding air.
 (b) action of the rotor of the helicopter on the surrounding air.
 (c) force on the helicopter due to the surrounding air. [NCERT Exemplar Q. No. 42]

Ans. (a) $F = (500 \times 15) + (500 \times 10) = 7500 + 5000 = 12500 = 12.5 \times 10^3\text{N}$ where F is the upward reaction of the floor and is equal to the force downwards on the floor, by Newton's 3rd law of motion.

- (b) $R - 2500 \times 10 = 2500 \times 15$

$R = 6.52 \times 10^4\text{N}$, action of the air on the system, upwards. The action of the rotor on the surrounding air is $6.25 \times 10^4\text{N}$ downwards.

- (c) Force on the helicopter due to the air = $6.25 \times 10^4\text{N}$ upwards.



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

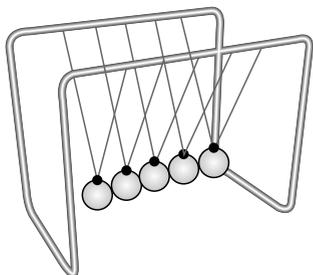
A Objective Questions

- I.** Read the following text and answer any 4 of the following questions on the basis of the same:

Newton's Cradle : The device consists of a row of five metal balls positioned to just barely touch one another suspended from a frame by thin wires.

On a small cradles, the balls are hung from the crossbars by light wire, with the balls at the point of an inverted triangle. This ensures that the balls can only swing in one plane, parallel to the crossbars. If the ball could move on any other plane, it would impart less energy to the other balls in the impact or miss them altogether, and the device wouldn't work as well, if at all.

All the balls are, ideally, exactly of the same size, weight, mass and density. As long as the balls are all the same size and density, they can be as big or as small as you like. The balls must be perfectly aligned at the centre to make the cradle to make the cradle work the best.



When a ball on one end of the cradle is pulled away from the others and then released, it strikes the next ball in the cradle, which remains motionless. But the last ball on the opposite end

of the row is thrown into the air, then swings back to strike the other balls, starting the chain reaction again in reverse.

This device illustrates the three main principles of Physics - conservation of energy, conservation of momentum and friction.

Everything that moves has momentum equal to its mass multiplied by its velocity. Like energy, momentum is also conserved. Momentum is a vector quantity, when 1st ball hits 2nd ball, it's travelling in a specific direction, let's say east to west. This means that its momentum is also moving east to west. Any change in direction of the motion brings a change in the momentum, which cannot happen without the influence of an outside force.

That is why 1st ball doesn't simply bounce off 2nd ball, the momentum carries the energy through all the balls in a westward direction.

It is to remember that the law of conservation only works in a closed system, which is free from any external force. The Newton's cradle is not a closed system. When 5th ball swings out away from the rest of the balls, it is affected by the force of gravity, which brings the ball down.

But, the horizontal line of balls at rest, functions as a closed system, free from any influence of any force other than gravity. It's here, during the small time between the first ball's impact and the 5th ball swinging out, that momentum is conserved.

- Q.1.** Newton's cradle illustrates the three main principles of Physics:

- (A) Law of conservation of energy,
 (B) Law of conservation of momentum and friction
 (C) Law of conservation of mass
 (D) Both (A) and (B)

Ans. Option (D) is correct.

Explanation: This device illustrates the three main principles of Physics—conservation of energy, conservation of momentum and friction.

Q. 2. In Newton's cradle, the balls are hung from the crossbars by light wire, with the balls at the point of an inverted triangle. This ensures

- (A) that the balls can only swing in one plane, parallel to the crossbars
- (B) that the balls can only swing in one plane, perpendicular to the crossbars
- (C) that the balls do not fall down
- (D) that the balls execute simple harmonic motion

Ans. Option (A) is correct.

Explanation: The balls are hung from the crossbars by light wire, with the balls at the point of an inverted triangle. This ensures that the balls can only swing in one plane, parallel to the crossbars.

Q. 3. When the 1st ball at one end of the cradle is pulled away and then released, then

- (A) the 1st ball strikes the 2nd ball in the cradle, which goes into motion. The 2nd ball strikes the 3rd ball which goes into motion and so on
- (B) the 1st ball strikes the 2nd ball in the cradle, which remains motionless. But the last ball on the opposite end of the row is thrown into the air, then swings back to strike the other balls, starting the chain reaction again in reverse
- (C) the 1st ball strikes the 2nd ball in the cradle and no other movement is observed
- (D) the 1st ball executes a simple harmonic motion. Other balls remain at rest

Ans. Option (B) is correct.

Explanation: When a ball on one end of the cradle is pulled away from the others and then released, it strikes the next ball in the cradle, which remains motionless. But the last ball on the opposite end of the row is thrown into the air, then swings back to strike the other balls, starting the chain reaction again in reverse.

Q. 4. What is a closed system?

- (A) System which is at rest
- (B) System which has zero energy
- (C) System which is free from any external force
- (D) System which is free from any friction

Ans. Option (C) is correct.

Explanation: The law of conservation only works in a system, which is free from any external force.

Such a system is called closed system. The Newton's cradle is not a closed system throughout its operation.

Q. 5. When the momentum is conserved in Newton's cradle?

- (A) Throughout the operation
- (B) During the small time between the first ball's impact and the 5th ball swinging out, that momentum is conserved.
- (C) At the time when the 1st ball strikes the 2nd ball
- (D) At the time when the 5th ball swing's out

Ans. Option (B) is correct.

Explanation: Law of conservation only works in a closed system. But the Newton's cradle is not a closed system throughout its operation. When 5th ball swings out away from the rest of the balls, it is affected by the force of gravity, which brings the ball down.

But, the horizontal line of balls at rest, functions as a closed system, free from any influence of any force other than gravity. It's here, during the small time between the first ball's impact and the 5th ball swinging out, that momentum is conserved.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

When a vehicle moves round a level curved road, the force of friction between the tyres and the road provides the necessary centripetal force. If the frictional force is not enough to provide the necessary centripetal force, the vehicle skids. In order to avoid skidding, the outer edge of the road is raised above the level of the inner edge. This is known as banking of curved roads.

A cyclist has to bend slightly towards the centre of the circular track in order to take a safe turn without slipping.

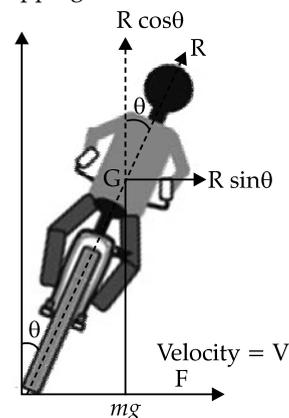


Fig. shows a cyclist taking a turn towards right on a circular path of radius r . m is the mass of the cyclist with the bicycle and v is his velocity. When the cyclist negotiates the curve, he bends inwards from the vertical, by an angle θ .

Let R be the reaction of the ground on the cyclist. R has into two components:

- (i) $R \sin \theta$ - towards the centre of the curve providing necessary centripetal force for circular motion and
- (ii) $R \cos \theta$ - balancing the weight.

Q. 1. When a car travels along a level circular road, then how does the necessary centripetal force develop?

Ans. The necessary centripetal force is developed by the force of friction between the tyres and the road.

Q. 2. What is the condition of skidding?

Ans. For skidding,

Centripetal force $>$ frictional force

Or,
$$\frac{mv^2}{R} > \mu mg$$

Or,
$$\frac{v^2}{rg} > \mu$$

Putting
$$\frac{v^2}{rg} = \tan \theta$$

$\tan \theta > \mu$

This is the condition of skidding

Q. 3. What is banking of curved road?

Ans. To provide necessary centripetal force, the outer edge of the curved road is raised above the level of the inner edge. This is known as banking of curved roads.

Q. 4. A cyclist while negotiating a circular path with speed 30 ms^{-1} found that he has to bend an angle by 30° with vertical to avoid skidding. What is the radius of the circular path?

Ans.
$$\tan \theta = \frac{v^2}{rg}$$

Or,
$$\tan 30^\circ = \frac{30^2}{(r \times 9.8)}$$

Or,
$$\frac{1}{\sqrt{3}} = \frac{900}{(r \times 9.8)}$$

Or,
$$r = \frac{900\sqrt{3}}{9.8}$$

$$\therefore r \approx 159\text{m}$$

Q. 5. State the factors on which the angle of banking depend.

Ans. The angle of banking depends on

- (i) the velocity of the vehicle,
- (ii) the radius ' r ' of the curved road
- (iii) the acceleration due to gravity ' g '.

□□

UNIT-IV WORK, ENERGY AND POWER

Study Time
Maximum Time: 3 hours
Maximum Questions: 99

CHAPTER

5

WORK, ENERGY AND POWER



Syllabus

Work done by a constant force and a variable force; kinetic energy, work-energy theorem, Power; Notion of potential energy, potential energy of a spring, conservative forces; non-conservative forces; motion in a vertical circle; elastic and inelastic collisions in one and two dimensions.

Topic-1 Work and Power

TOPIC - 1

Work and Power

.... P. 99

TOPIC - 2

Energy and Collision

.... P. 102



Revision Notes

FreeKItaAb

T.M.E / FREEKITAAB

- **Work :** Work is done when the body is displaced actually through some distance in the direction of applied force.

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs \cos \theta$$

- Work done is a scalar quantity. However, work done is positive when θ lies between 0 (zero) and $\pi/2$. Work done is negative when θ lies between $\pi/2$ and $3\pi/2$.
- S. I. unit of work is joule (J) and the C.G.S unit of work is erg, where 1 joule = 10^7 erg.
- Work done by a body does not depend on the time taken to complete the work.
- **Internal work or zero work:** The work in which muscles are strained, but work done is not useful, is called internal work. For example, when a person carrying load keeps on standing at the same place, work done is zero, but he gets tired on account of internal work.

Dimensions : $[ML^2 T^{-2}]$

Power : Power of a body is defined as the time rate of doing work by the body. Thus, in power, time taken by the body to complete the work is significant.

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

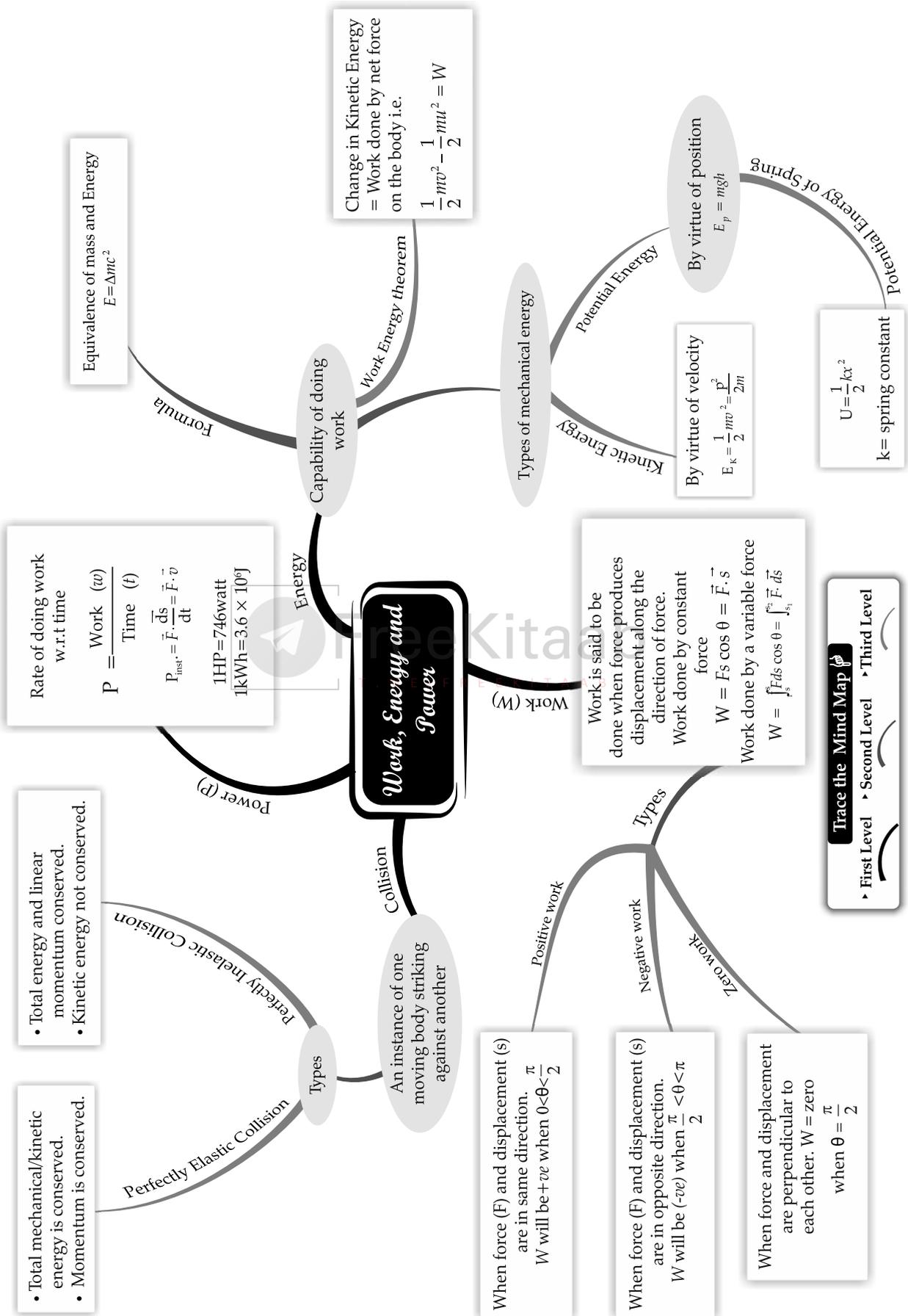
$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

- Here, θ is the angle between force \vec{F} and velocity \vec{v} of the body.
- **Unit :** 1 W = 1 J s⁻¹
- **Dimensions :** $[ML^2 T^{-3}]$
- Power is a scalar quantity.

Scan to know more about this topic



Work done by a force



Key Words

- **Conservative force** is a force if work done by or against the force in moving a body depends only on the initial & final positions of the body and not on the nature of path followed between initial and final positions, e.g., gravitational force, electrostatic force between two electric charges, all central forces, etc.
- **Non-conservative force** is a force if work done by or against the force in moving a body from one position to another depends on the path followed between these two positions. e.g., frictional forces, elastic forces, etc.

Key Formulae

- Work = Force × Displacement in the direction of force
- $$W = \vec{F} \cdot \vec{s}$$

$$= F_s \cos \theta$$
- Maximum work
When $\cos \theta = 0^\circ$
 $W = F_s$
- Minimum Work when $\theta = 90^\circ$
Then $W = F_s \times \cos 90^\circ = 0$
- Work done by variable force :

$$W = \int_{x_A}^{x_B} \vec{F} \cdot d\vec{x}$$

- $$\text{Power} = \frac{\text{Work done}}{\text{Time Taken}}$$
- $$P = \vec{F} \cdot \vec{v}$$
- $$P = Fv \cos \theta.$$

Mnemonics

<p>Concept: Positive work, negative work</p> <p>Fernandez d'souza Ordered donut With noodles</p> <p>Fernandez d'souza was Served donut With Pizza.</p> <p>Interpretation:</p> <p>F - Force</p> <p>d - Displacement</p> <p>o - opposite</p> <p>d - direction</p> <p>w - Work done</p>	<p>n - Negative</p> <p>If Force and displacement are in opposite directions work done is negative.</p> <p>F - Force</p> <p>d - Displacement</p> <p>s - Same</p> <p>d - direction</p> <p>w - Work done</p> <p>p - positive</p> <p>If Force and displacement are in same directions work done is positive</p>
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Very Short Answer Type Questions (1 mark each)

- | | |
|---|--|
| <p>Q. 1. What is the work done by Earth's gravitational force in keeping the Moon in its orbit in a complete revolution ? [A] [U] (DDE)</p> <p>Ans. Zero, because gravitational force is a conservative force and act perpendicular to direction of motion of Moon. 1</p> <p>Q. 2. What do you mean by positive work? [U] (DDE)</p> <p>Ans. Positive work means that force (or its component) is parallel to displacement. 1</p> <p>Q. 3. What do you mean by negative work? [U] (DDE)</p> | <p>Ans. Negative work means that force (or its component) is opposite to displacement. 1</p> <p>Q. 4. A spring is stretched. Is the work done by the stretching force positive or negative ? [A] (DDE)</p> <p>Ans. Positive because the force and the displacement are in the same direction. 1</p> <p>Q. 5. Does the work done in raising a load on to a platform depend upon how fast is it raised ? [A]</p> <p>Ans. No, because the work done is independent of time. 1</p> |
|---|--|



Short Answer Type Questions-I

(2 marks each)

Q. 1. What is conservative force ? [R]

[MSE Chandigarh 2008]

Ans. Refer to Know the Terms. 2

Q. 2. What is instantaneous power ? [R]

Ans. Instantaneous power is defined as the limiting value of the average power when time tends to zero.

$$\text{In general} \quad = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} \quad 1$$

$$P = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta W}{\Delta t} \right)$$

$$\vec{F} \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

where, \vec{v} is instantaneous velocity. 1



Short Answer Type Questions-II

(3 marks each)

Q. 1. Give conditions for no work. [R]

Ans. In physics no work is said to be done, if

(a) The applied force (F) is zero. A body moving with uniform velocity on a smooth surface has some displacement but no external force so in this case work done is zero. 1

(b) The displacement (S) is zero. A labourer standing with a load on his head does no work. ½

(c) The angle between force and displacement (θ) is $\pi/2$ rad or 90° . Then $\cos \theta = \cos 90^\circ = 0$. Thus, work done is also zero. In circular motion, instantaneous work done is always zero because of this reason. 1

(d) The change in kinetic energy (AKE) is zero. ½

Q. 2. Prove that instantaneous power is given by the dot product of force and velocity, i.e., $P = \vec{F} \cdot \vec{v}$ [U]

Ans. Suppose, ΔW be the amount of work done in a small time interval Δt , when P_{av} be the average power, then ½

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \dots(i) \quad \frac{1}{2}$$

When P be the instantaneous power, then by def.

$$P = \lim_{\Delta t \rightarrow 0} P_{av} \\ = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} \quad \frac{1}{2}$$

or
$$P = \frac{dW}{dt} = \frac{d}{dt}(W) \quad \dots(ii)$$

Now
$$W = \vec{F} \cdot \vec{s} \quad \dots(iii) \quad \frac{1}{2}$$

where, F = constant force producing a displacement
∴ From equations (ii) and (iii),

$$P = \frac{d}{dt} (\vec{F} \cdot \vec{s}) \\ = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v} \quad 1$$

Topic-2 Energy & Collision



Revision Notes

➤ **Energy** : Energy of a body is defined as the capacity of the body to do the work. Energy is a scalar quantity. It has the same units and dimensions as those of work. Some practical units of energy and their relation with S.I. unit of energy (joule) are :

(i) 1 calorie = 4.2 J

(ii) 1 kiloWatt hour (kWh) = 3.6×10^6 J

(iii) 1 electron volt (1 eV) = 1.6×10^{-19} J

➤ **Work-Energy Theorem** : According to this principle, work done by net force in displacing a body is equal to change in kinetic energy of the body and i.e.,

$$W = K_f - K_i$$

$$K_f = \text{final K.E.}$$

$$K_i = \text{initial K.E.}$$

➤ **Collisions** : When a body strikes against another body such that there is exchange of energy and linear momentum then the two are said to collide. Collisions are of two types :

Scan to know more about this topic



Work energy theorem

- (i) **Perfectly elastic collision** is that in which there is no change in kinetic energy of the system, i.e.,
 Total *K.E.* before collision = Total *K.E.* after collision.
 e.g., collisions between atomic and subatomic particles are perfectly elastic collisions.
- (ii) **Perfectly inelastic collision** is that in which *K.E.* is not conserved. Here, the bodies stick together after impact.
 Linear momentum is conserved in every collision elastic as well as inelastic, further total energy is also conserved in all such collisions. Kinetic energy alone is not conserved in inelastic collisions.

Key Words

- **Kinetic Energy** is the energy possessed by the body by virtue of its motion. It is always positive.
- **Potential Energy** is the energy possessed by the body by virtue of its position. It can be both negative as well as positive.
- **Gravitational Potential Energy** is the energy possessed by the body by virtue of its position with respect to center of Earth or other body.
- **Potential Energy of spring** is the energy associated with the state of compression or expansion of an elastic spring.
- **Internal Energy** is the energy possessed by the body by virtue of particular configuration of its molecules.
- **Coefficient of Restitution or Coefficient of Resilience** is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by '*e*'.

Key Formulae

- Kinetic Energy (*K.E.*) = $\frac{1}{2} mv^2$,
 where, *m* = mass, *v* = velocity of particles
- Potential Energy (*P.E.*) = mgh
- Velocity (*v*) = $\sqrt{2gh}$
- Force in Spring (*F*) = $-kx$,
 where, *k* = spring constant, *x* = compression.
- **Mass Energy Equivalence :**

$$E = mc^2$$
 where, *m* = mass that disappears
E = energy that appears
c = velocity of light
- **Coefficient of Resilience** $(e) = \frac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$

$$e = \frac{v_2 - v_1}{u_2 - u_1}$$
 for perfectly elastic collision, $e = 1$
 for perfectly inelastic collision, $e = 0$
- **Elastic collision in 1- Dimension :**

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

$$v_2 = \frac{2m_1 u_1}{m_1 + m_2} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) u_2$$
- **Inelastic Collision in 1-Dimension :**

$$v = \frac{m_1 u_1}{m_1 + m_2} \text{ if } u_2 = 0$$

$$\text{Loss in K.E.} = \frac{m_1 m_2 u_1^2}{2(m_1 + m_2)}$$





Very Short Answer Type Questions

(1 mark each)

Q. 1. What is mass energy equivalence? [R] (DDE)

Ans. The relation between mass of a particle m and its equivalent energy (E) is given by $E = mc^2$ where, $c =$ velocity of light in vacuum. 1

Q. 2. What is the source of the kinetic energy of the falling rain drops? [R]

Ans. It is the gravitational potential energy which is being converted into kinetic energy. 1

Q. 3. What type of energy is stored in the spring of watch? [U]

Ans. Potential energy. 1

Q. 4. When an arrow is shot, where from the arrow acquires its K.E.? [A]

Ans. It is the potential energy of the bend bow which is converted into K.E. of arrow. 1

Q. 5. Can P.E. of an object be negative? [A]

Ans. Yes, it can be negative when forces involved are attractive. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. What is the relation between linear momentum and KE? [A1] [U] (DDE)

Ans. We know that, KE of a particle,

$$K = \frac{1}{2}mv^2$$

where m is the mass of particle and v is the velocity

$$K = \frac{1}{2} \frac{mv^2 \times m}{m} \quad 1$$

$$K = \frac{1}{2} \frac{(mv)^2}{m}$$

$$K = \frac{p^2}{2m} \quad \{\because p = mv\}$$

$$\therefore p = \sqrt{2mK} \quad 1$$

Q. 2. Give conditions for elastic collision. [R]

Ans. In an elastic collision,

(a) Total momentum is conserved, i.e., total final momentum is equal to the total initial momentum.

(b) Total mechanical energy is conserved, i.e., total final energy is equal to the total initial energy.

(c) Total kinetic energy is conserved, i.e., total final kinetic energy is equal to the total initial kinetic energy.

(d) All the forces are of conservative nature, i.e., work done does not depend upon the actual paths.

(½ mark each)

Q. 3. Body B_1 collides with body B_2 of same mass but at rest, what will happen to them if collision is elastic. [A]

Ans. When body (B_2) is at rest, we get

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

and

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 \quad 1$$

Putting $m_1 = m_2 = m$, we get $v_1 = 0$ and $v_2 = u_1$, i.e., body B_1 comes to rest whereas body B_2 moves with the velocity of body B_1 . 1

Q. 4. Is kinetic energy a scalar or a vector? Give its S.I. unit and dimensional formula. [U]

Ans. Kinetic energy is a scalar. ½

S.I. unit of kinetic energy is joule (J) ½

Dimensional formula of kinetic energy is $[ML^2T^{-2}]$. 1

Q. 5. A bullet weighing 10 g is fired with a velocity of 800 ms^{-1} . After passing through a mud wall 1 m thick, its velocity decreases to 100 ms^{-1} . Find the average resistance offered by the mud wall. [A] [A1]

Ans. Using $F \cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ½

where $m = 10 \text{ g} = \frac{10}{1000} \text{ kg}$

$$= 0.01 \text{ kg}$$

and $v = 100 \text{ ms}^{-1}, u = 800 \text{ ms}^{-1}$. ½

and $s = 1 \text{ m}$,

we get $F = \frac{1}{2} \times 0.01 \times (100^2 - 800^2)$

$$= -3150 \text{ N}. \quad 1$$



Short Answer Type Questions-II

(3 marks each)

Q. 1. State and derive work- energy relationship. [U] [KVS 2013]

OR

Derive a relationship between kinetic energy and work. [KVS 2008]

Ans. It states that change in kinetic energy of a body is equal to work done and vice versa. Let a constant

force \vec{F} be applied to a body moving with initial

velocity \vec{u} , so that its velocity becomes \vec{v} along the direction of force when s is its displacement.

Using Newton's second law of motion we get magnitude of force $F = ma$ and from equation of motion, we get $v^2 - u^2 = 2as$, where a is the acceleration of the body. **1**

Multiplying both sides by $m/2$, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

$$\text{i.e., } \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs = W$$

$$\text{i.e., } K.E._{(f)} - K.E._{(i)} = W \quad \mathbf{1}$$

where $K.E._{(f)}$ is final kinetic energy and $K.E._{(i)}$ is initial kinetic energy.

Thus work done on a body by a net force is equal to the change in kinetic energy of the body. **1**

Q. 2. A block initially at rest breaks into two parts of masses in the ratio 2 : 3. The velocity of smaller part is $(8\hat{i} + 6\hat{j})$ m/s. Find the velocity of bigger part. [A] [NCT 2011]

Ans. Let mass of the block = m

$$\text{After breaking, } m_1 = \frac{2}{5}m \text{ and } m_2 = \frac{3}{5}m$$

$$\text{Linear momentum} = m_1 \vec{v}_1 + m_2 \vec{v}_2 \quad \mathbf{1/2}$$

According to law of conservation of momentum

$$\text{or } m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0 \quad \mathbf{1}$$

\vec{v}_1 = velocity of smaller part,

\vec{v}_2 = velocity of bigger part

$$\text{or } \frac{2}{5}m(8\hat{i} + 6\hat{j}) + \frac{3}{5}m(\vec{v}_2) = 0 \quad \mathbf{1/2}$$

$$\begin{aligned} \text{or } \frac{3}{5}m\vec{v}_2 &= -\frac{1}{5}m(16\hat{i} + 12\hat{j}) \\ \vec{v}_2 &= -\left(\frac{16}{3}\hat{i} + 4\hat{j}\right) \quad \mathbf{1} \end{aligned}$$

Q. 3. Prove work energy theorem for a variable force.

[U] [NCT 2008, 09; MSE Chandigarh 2008, 09]

Ans. Using the relation for kinetic energy,

$$K.E. = \frac{1}{2}mv^2$$

we get rate of change of K.E. with respect to time as

$$\begin{aligned} \frac{d}{dt}(K.E.) &= \frac{d}{dt}\left[\frac{1}{2}mv^2\right] \\ &= \frac{1}{2} \times m \times 2 \times v \times \frac{dv}{dt} \\ &= mv \frac{dv}{dt} \end{aligned}$$

$$\text{But } \frac{mdv}{dt} = ma = F \quad \mathbf{1}$$

where a is acceleration and F is force.

$$\therefore \frac{d}{dt}K.E. = Fv = F \frac{dx}{dt}$$

$$\text{or } d(K.E.) = Fdx \quad \mathbf{1}$$

Integrating between the initial and final energies, i.e., $K.E._i$ and $K.E._f$ and also position, i.e., x_i and x_f respectively, we get

$$\int_{K.E._i}^{K.E._f} d(K.E.) = \int_{x_i}^{x_f} Fdx$$

$$\therefore K.E._{(f)} - K.E._{(i)} = W$$

The work energy theorem is thus verified for variable force. **1**



Commonly Made Error

► Students commit error while finding the derivatives and the integrals of functions to prove the required result.



Answering Tip

► Students should learn about the basics of derivatives and integrals and their use to get the desired result.

Long Answer Type Questions

(5 marks each)

Q. 1. What are the various types of equilibrium? [U]

Ans. Different types of equilibrium are—

Stable :

- (i) When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position. It is said to be in stable equilibrium position.
- (ii) Potential energy is minimum.

(iii) $\frac{d^2U}{dx^2} = \text{positive}$

- (iv) Example: A marble placed at the bottom of a hemispherical bowl.

Unstable :

- (i) When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium position.
- (ii) Potential energy is maximum

(iii) $\frac{d^2U}{dx^2} = \text{negative}$

(iv) Example: A marble balanced on top of a hemispherical bowl.

Neutral :

(i) When a particle is slightly displaced from a position it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium

(ii) Potential is constant

(iii) $\frac{d^2U}{dx^2} = 0$

(iv) Example: A marble placed on a horizontal table.

Q. 2. Derive a relation for an inelastic collision in one dimension. □

Ans.

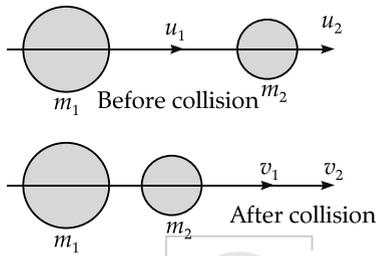


Figure shows two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 respectively, along a single axis. They collide involving some loss of kinetic energy. Therefore, the collision is inelastic. Let v_1 and v_2 be the velocities of the two bodies after collision.

As the two bodies or one system, is closed and isolated, we can write the law of conservation of linear momentum for the two body system as :

$$\begin{aligned} \text{Total momentum before the collision } (p_i) \\ &= \text{Total momentum after the collision } (p_f) \\ m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \end{aligned}$$

In a perfectly inelastic collision, the body of mass m_2 happens to be initially at rest ($u_2 = 0$). After the

collision, the two bodies move together with a common velocity v . As the total linear momentum of the system cannot change, therefore, $P_i = P_f$, i.e., $m_1u_1 + m_2u_2 = (m_1 + m_2)v$
or $m_1u_1 = (m_1 + m_2)v$ ($u_2 = 0$)
or $v = \frac{m_1u_1}{m_1 + m_2}$..(ii) 1

Knowing m_1, m_2 and u_1 , we can calculate the final velocity v . As the mass ratio $\frac{m_1}{m_1 + m_2} < 1, v < u_1$.

Total K.E. before collision, 1

$$E_1 = \frac{1}{2} m_1u_1^2 \quad \frac{1}{2}$$

Total K.E. after collision,

$$E_2 = \frac{1}{2} (m_1 + m_2)v^2$$

$$E_2 = \frac{1}{2} (m_1 + m_2) \left(\frac{m_1u_1}{m_1 + m_2} \right)^2$$

Using (ii)

or $E_2 = \frac{m_1^2u_1^2}{2(m_1 + m_2)}$ 1/2

$$\begin{aligned} \text{Difference in K.E.} &= E_1 - E_2 \\ &= \frac{1}{2} m_1u_1^2 - \frac{m_1^2u_1^2}{2(m_1 + m_2)} \\ &= \frac{m_1^2u_1^2 + m_1m_2u_1^2 - m_1^2u_1^2}{2(m_1 + m_2)} \\ &= \frac{m_1m_2u_1^2}{2(m_1 + m_2)} \end{aligned}$$

The difference being positive it may be concluded that there is loss in kinetic energy. 1

Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. A cyclist comes to a skidding stop in 20 m. During this process, the force on the cycle due to the road is 100 N and is directly opposed to the motion. Work done by the road on the cycle is

- (A) -2000 J (B) 2000 J
(C) 1000 J (D) 100 J

Ans. Option (A) is correct.

Explanation: $W = Fs \cos 180^\circ = 100 \times 20 \times (-1) = -2000 \text{ J}$

Q. 2. A ball is dropped from a height of 1 m. If the

coefficient of restitution between the surface and ball is 0.6, the ball rebounds to a height of

- (A) 0.6 m (B) 0.36 m
(C) 3.6m (D) 1m

Ans. Option (B) is correct.

Explanation: The velocity of the ball when it reaches the ground $= v_0 = \sqrt{2gH}$

or, $v_0 = \sqrt{2g \times 1}$

$\therefore v_0 = \sqrt{2g}$

The ball rebounds with a velocity v'

$$v' = ev_0 = e\sqrt{2g}$$

Maximum height travelled by the ball starting with a velocity v' is $h = \frac{v'^2}{2g}$

or,
$$h = \frac{e^2 \times 2g}{2g}$$

$\therefore h = e^2 = 0.36 \text{ m}$

Q. 3. A body of mass 100 g falls from a height of 10 m. Its increase in kinetic energy is

- (A) 9800 J (B) 9.8 J
(C) 980 J (D) 100 J

Ans. Option (B) is correct.

Explanation: Loss of $PE = \text{Gain in } KE$

$$\text{Loss in } PE = mgh = \left(\frac{100}{1000}\right) \times 9.8 \times 10 = 9.8 \text{ J}$$

So, increase in $KE = 9.8 \text{ J}$

Q. 4. 1 joule = 1 _____

- (A) Nm (B) Nm^2
(C) N^2m^2 (D) N^2m

Ans. Option (A) is correct.

Explanation: 1 Joule = 1 Newton \times 1 meter = 1 Nm

Q. 5. If momentum rises by 20%, kinetic energy rises by

- (A) 44% (B) 40%
(C) 88% (D) 20%

Ans. Option (A) is correct.

Explanation: Initially, $KE = \frac{p^2}{2m}$

Finally, $(KE)' = \frac{p'^2}{2m}$

Putting, $p' = 1.20p$

$$(KE)' = \frac{(1.20p)^2}{2m}$$

or, $(KE)' = \frac{1.44p^2}{2m}$

or, $(KE)' = 1.44 (KE)$

So, KE increases by 44%

C Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is true

Q. 1. Assertion (A): Mass and energy are not conserved separately. They are conserved as a single entity called mass-energy.

Reason (R): Mass and energy are inter-convertible.

Ans. Option (A) is correct.

Explanation: According to Einstein's theory, mass and energy are related as $E=mc^2$

where, $E = \text{Energy}$

$m = \text{mass}$

$c = \text{speed of light}$

So, mass and energy are not conserved separately. They are conserved as a single entity.

Hence, assertion and reason both are true and the reason explains the assertion.

Q. 2. Assertion (A): The kinetic energy of the body of mass 2 kg and momentum of 2 Ns is 1 J.

Reason (R): The relation between kinetic energy and linear momentum of an object is given by

$$K = \frac{p^2}{2m}$$

Ans. Option (C) is correct.

Explanation: $K = \frac{p^2}{2m}$

or, $K = \frac{2^2}{2 \times 2}$

$\therefore K = 1 \text{ J}$

So, the assertion is true. But the reason is false.

Q. 3. Assertion (A): A spring stores potential energy when compressed as well as when stretched.

Reason (R): Potential energy is stored in a body by deformation of its shape.

Ans. Option (A) is correct.

Explanation: When a spring is compressed or stretched its shape is deformed. Hence, elastic potential is stored in it.

NCERT Corner

Q. 1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative :

- (a) Work done by a man in lifting a bucket out of well by means of a rope tied to the bucket.
(b) Work done by gravitational force in the above case.
(c) Work done by friction on a body sliding down an inclined plane.

(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

(e) Work done by the resistive force of air on a vibrating pendulum is bringing it to rest.

Ans. As, $W = Fs \cos \theta$

(a) $\theta = 0^\circ$, work is positive.

(b) $\theta = 180^\circ$, work is negative.

- (c) As force of friction act opposite to direction of motion, work is negative.
- (d) As applied force acts in the direction of motion, work is positive.
- (e) The resistive force of air acts opposite to direction of the motion, hence work is negative.

Q. 2. A body of mass 2kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the

- (a) Work done by the applied force in 10 s,
- (b) Work done by friction in 10 s,
- (c) Work done by the net force on the body in 10 s,
- (d) Change in kinetic energy of the body in 10 s, and interpret your results.

Ans. Given : $m = 2\text{kg}$, $u = 0$, $F = 7\text{N}$, $\mu = 0.1$, $t = 10\text{s}$

$$\text{Acceleration} = a_1 = \frac{F}{m} = \frac{7}{2} = 3.5\text{ms}^{-2}$$

$$\begin{aligned} \text{Frictional force} = f &= \mu R = \mu mg \\ &= 0.1 \times 2 \times 9.8 \\ &= 1.96\text{ N.} \end{aligned}$$

Retardation-

$$a_2 = \frac{-f}{m} = \frac{-1.96}{2} = -0.98\text{ ms}^{-2}$$

$$\begin{aligned} \text{Net acceleration, } a &= a_1 + a_2 = 3.5 - 0.98 \\ &= 2.52\text{ ms}^{-2} \end{aligned}$$

Distance, in 10 seconds

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ &= 0 + \frac{1}{2} \times 2.52 \times (10)^2 \\ &= 126\text{m} \end{aligned}$$

(a) Work done (applied force) = $F \cdot s = 7 \times 126 = 882\text{J}$.

(b) Work done (frictional force) = $-f \cdot s$
 $= -1.96 \times 126$
 $= -247\text{ J}$

(c) work done (net force) = Net force \times s
 Net force, $F = 7 - 1.96 = 5.04$
 Work done = $5.04 \times 126 = 635\text{ J}$.

(d) Velocity at the end of 10 second.
 $v = u + at = 0 + 2.52 \times 10 = 25.2\text{ m/s}$.
 Initial $K.E. = 0$

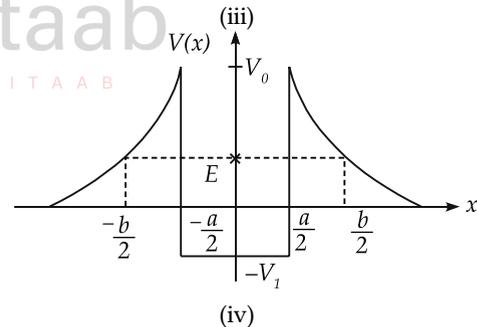
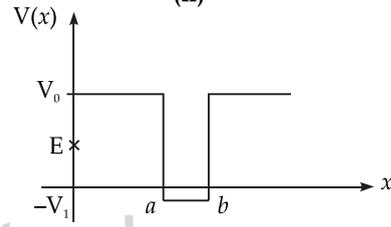
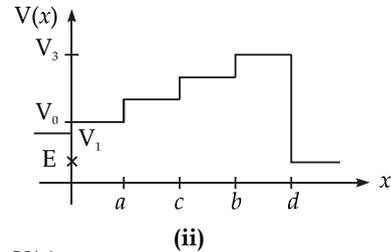
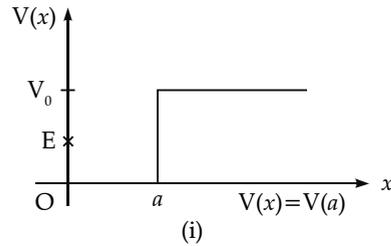
$$\text{Final } KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times (25.2)^2 = 635\text{J}$$

Change in $K.E. = 635 - 0 = 635\text{J}$.

$\therefore K.E.$'s change is equal to work done by net force.

Q. 3. Given below are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the

minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.



Ans. It is clear that the total energy of the body is given by

$$E = K.E. + P.E.$$

or
$$K.E. = E - P.E. = \frac{1}{2}mv^2$$

Hence, $K.E.$ of the body can never be negative. Thus, $P.E.$ cannot be greater than E .

(i) In the region between $x = 0$ and $x = a$, $P.E.$ is zero, so $K.E.$ is positive. In the region $x > a$, the $P.E.$ (V_0) has a value greater than E . So, $K.E.$ will be negative in the region. Hence, the particle can't be present in the region $x > a$.

The minimum total energy that the particle can have in this case is zero.

(ii) Here, in all the regions, i.e., $-\infty < x < a$, $a < x < b$, $b < x < c$, $c < x < d$ and $d < x < \infty$, the $P.E.$ is greater than the total energy. Hence, the particle cannot be present in the region $-\infty < x < \infty$.

The minimum total energy that the particle can have in this case is V_1 .

- (iii) In region $x < a$ and $x > b$, the P.E. is V_0 which is greater than the total energy of the particle. So, K.E. will be negative in this region. Thus, the particle cannot be present in the region $x < a$ and $x > b$. In the region between $x > a$ and $x < b$, the P.E. is negative this clears that there is a positive value of K.E. Hence, the particle can be present in the region between $x > a$ and $x < b$.

The minimum total energy that the particle can have in this case is $-V_1$.

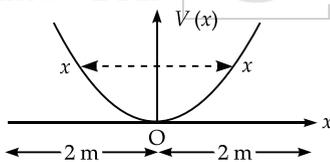
- (iv) In this case, the P.E. of the particle is more than the total energy (E) in the regions $-\frac{b}{2} < x < -\frac{a}{2}$

$$\text{and } \frac{a}{2} < x < \frac{b}{2}.$$

Hence, K.E. of the particle will be negative in these regions, thus it will not be present in the regions $-\frac{b}{2} < x < -\frac{a}{2}$ and $\frac{a}{2} < x < \frac{b}{2}$.

The minimum total energy that the particle can have in this case is $-V_1$.

- Q. 4.** The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5 \text{ Nm}^{-1}$, the graph of $V(x)$ versus x is shown in figure. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2 \text{ m}$.



- Ans.** Total energy,

$$E = K.E. + P.E. \\ = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where, m = mass of particle

k = force constant = 0.5 Nm^{-1}

x = displacement, v = velocity of particle

$$\text{or } 1 = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}x^2$$

$$\text{or } x^2 + 2mv^2 = 4$$

Then particle turns back at x when $v = 0$

$$\therefore x^2 + 2m(0)^2 = 4$$

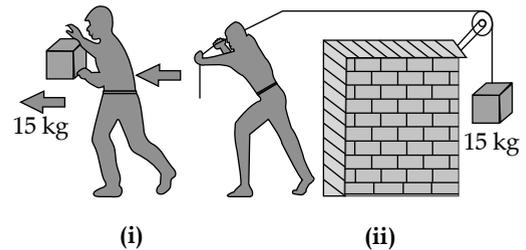
$$\text{or } x = \pm 2 \text{ m}.$$

- Q. 5.** Answer the following:

- (a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?
- (b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the

gravitational force over every complete orbit of the comet is zero. Why?

- (c) An artificial satellite orbiting the Earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the Earth?
- (d) In Figure (i) the man walks 2 m carrying a mass of 15 kg on his hands. In Figure (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



- Ans.** (a) Rocket

The burning of the casing of a rocket in flight (due to friction) results in the reduction of the mass of the rocket.

According to the conservation of energy:

$$\text{Total energy} = \text{Potential energy} + \text{Kinetic energy} \\ = mgs + \frac{1}{2}mv^2$$

The reduction in the rocket's mass causes a drop in the total energy. Therefore, the heat energy required for the burning is obtained from the rocket.

- (b) Gravitational force is a conservative force. Since the work done by a conservative force over a closed path is zero, the work done by the gravitational force over every complete orbit of a comet is zero.
- (c) When an artificial satellite, orbiting around Earth, moves closer to Earth, its potential energy decreases because of the reduction in the height. Since, the total energy of the system remains constant, the reduction in P.E. results in an increase in K.E. Hence, the velocity of the satellite increases. However, due to atmospheric friction, the total energy of the satellite decreases by a small amount.
- (d) Work done in figure (i)
Mass, $m = 15 \text{ kg}$
Displacement, $s = 2 \text{ m}$
Work done, $W = Fs \cos\theta$
Where, θ = Angle between force and displacement
So, Work done = $mgs \cos\theta = 15 \times 2 \times 9.8 \cos 90^\circ = 0$
Work done in figure (ii)
Mass, $m = 15 \text{ kg}$
Displacement, $s = 2 \text{ m}$
Here, the direction of the force applied on the rope and the direction of the displacement of the rope are same.

Therefore, the angle between them, $\theta = 0^\circ$

Since, $\cos 0^\circ = 1$

Work done, $W = F_s \cos\theta = mgs$

$$= 15 \times 9.8 \times 2 = 294 \text{ J}$$

Hence, more work is done in the figure (ii).

Q. 6. Underline the correct alternative:

- (a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.
- (b) Work done by a body against friction always results in a loss of its kinetic/potential energy.
- (c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.
- (d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Ans. (a) Decreases

A conservative force does a positive work on a body when it displaces the body in the direction of force. As a result, the body advances toward the center of force. It decreases the separation between the two, thereby decreasing the potential energy of the body.

- (b) Kinetic energy
The work done against the direction of friction reduces the velocity of a body. Hence, there is a loss of kinetic energy of the body.
- (c) External force
Internal forces, irrespective of their direction, cannot produce any change in the total momentum of a body. Hence, the total momentum of a many-particle system is proportional to the external forces acting on the system.
- (d) Total linear momentum
The total linear momentum always remains conserved whether it is an elastic collision or an inelastic collision.

Q. 7. State if each of the following statements is true or false. Give reasons for your answer.

- (a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.
- (b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.
- (c) Work done in the motion of a body over a closed loop is zero for every force in nature.
- (d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Ans. (a) False, it is total momentum and total energy which are conserved.

(b) False, force on the system can change total energy of the system.

(c) False, in the case of non-conservative forces like friction, the work done in a closed loop is not zero.

(d) True, there is at least some loss of kinetic energy in an inelastic collision.

Q. 8. Answer carefully, with reasons :

- (a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact) ?
- (b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?
- (c) What are the answers to (a) and (b) for an inelastic collision ?
- (d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ?

Ans. (a) No, the total kinetic energy is not conserved during the given elastic collision due to a part of the kinetic energy is used in deforming the balls in that short interval for which they are in contact during collision and gets converted into potential energy. In an elastic collision, the K.E. before and after collision is same.

(b) Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

(c) In an inelastic collision, total K.E. is not conserved during collision and even after collision. The total linear momentum is however conserved after collision.

(d) The collision is elastic because involved forces are conservative.

Q. 9. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it in time t is proportional to:

- (i) $t^{1/2}$, (ii) t , (iii) $t^{3/2}$, (iv) t^2

Ans. (ii) Suppose, m = mass of the body

Given :

a = acceleration produced in the body

v = velocity of the body

P = Power delivered to the body in a time t

$$a = \frac{dv}{dt} = \frac{F}{m}$$

$$v = at = \frac{F}{m}t$$

$$P = Fv = F \times \frac{F}{m}t$$

$$= \frac{F^2}{m^2} \times mt = \left(\frac{F}{m}\right)^2 m \times t$$

or $P = a^2 mt$

Since, a and m are constants, therefore

$$P = \text{constant} \times t$$

or $P \propto t$

Q. 10. A body is moving uni-directionally under the influence of a source of constant power. Its displacement in time t is proportional to:

- (i) $t^{1/2}$, (ii) t , (iii) $t^{3/2}$, (iv) t^2

Ans. (iii) Suppose constant power acts on the body of the mass m for a time t to give it a velocity v

$$K.E. = \text{work done} = \text{power} \times \text{time}$$

$$\text{or } \frac{1}{2}mv^2 = Pt$$

$$\text{or } v = \sqrt{\frac{2Pt}{m}} \dots(i)$$

$$\text{We know that } v = \frac{dx}{dt}$$

$$\text{or } dx = v dt$$

If x be the displacement of the body, then

$$\begin{aligned} x &= \int dx = \int v dt = \sqrt{\frac{2P}{m}} \int t^{1/2} dt \\ &= \sqrt{\frac{2P}{m}} \left(\frac{t^{1/2+1}}{\frac{1}{2}+1} \right) \\ &= \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2} = \frac{2}{3} \sqrt{\frac{2P}{m}} t^{3/2} \end{aligned}$$

Here, P = constant and m is also constant for a body, so x = constant $\times t^{3/2}$ or x is directly proportional to $t^{3/2}$.

Q. 11. A body constrained to move along Z-axis of a co-ordinate system is subjected to a constant force $\vec{F} = (-\hat{i} + 2\hat{j} + 3\hat{k})$ N. What is the work done by this force, in moving the body over a distance of 4 m along the Z-axis?

Ans. Here,

$$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{s} = 4\hat{k}$$

($\because \vec{s}$ = 4 m distance is along Z-axis).

$$W = ?$$

$$W = \vec{F} \cdot \vec{s}$$

$$\begin{aligned} W &= \left(-\hat{i} + 2\hat{j} + 3\hat{k} \right) \cdot \left(4\hat{k} \right) \\ &= 12\hat{k} \cdot \hat{k} = 12J \quad \left(\because \hat{k} \cdot \hat{k} = 1 \right) \end{aligned}$$

Q. 12. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds, (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J).

Ans. Mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg

Mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Kinetic energy of the electron, $E_{Ke} = 10 \text{ keV} = 10^4 \text{ eV}$

$$= 10^4 \times 1.60 \times 10^{-19}$$

$$= 1.60 \times 10^{-15} \text{ J}$$

Kinetic energy of the proton, $E_{Kp} = 100 \text{ keV} = 10^5 \text{ eV} = 1.60 \times 10^{-14} \text{ J}$

For the velocity of an electron V_e , its kinetic energy is given by the relation:

$$E_{Ke} = (1/2) m v_e^2$$

$$\therefore V_e = (2E_{Ke}/m)^{1/2}$$

$$= (2 \times 1.60 \times 10^{-15} / 9.11 \times 10^{-31})^{1/2} = 5.93 \times 10^7 \text{ m/s}$$

For the velocity of a proton V_p , its kinetic energy is given by the relation:

$$E_{Kp} = (1/2) m v_p^2$$

$$v_p = (2 \times 1.6 \times 10^{-14} / 1.67 \times 10^{-27})^{1/2} = 4.38 \times 10^6 \text{ m/s}$$

Hence, the electron is moving faster than the proton.

$$\text{The ratio of their speeds } \frac{V_e}{V_p} = 5.93 \times 10^7 / 4.38 \times 10^6$$

$$= 13.54 : 1$$

Electron is faster; Ratio of speeds is 13.54 : 1

Q. 13. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height. It attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m/s?

Ans. Given, $r = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

Using formula,

$$P.E. = mgh$$

$$\text{where, } m = \frac{4}{3} \pi r^3 \rho$$

$$m = \frac{4}{3} \times \frac{22}{7} \times (2 \times 10^{-3})^3 \times 10^3$$

$$= \frac{32 \times 22}{21} \times 10^{-6} \text{ kg}$$

and distance moved,

$$h = \frac{d}{2} = 250 \text{ m}$$

\therefore Work done during each half = P.E.

$$= \frac{32 \times 22 \times 10^{-6}}{21} \times 9.8 \times 250$$

$$= 0.082 \text{ J.}$$

$$K.E. \text{ on reaching the ground} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times \left(\frac{32 \times 22}{21} \right) \times 10^{-6} \times (10)^2 = 0.00168$$

Work done by resistive force

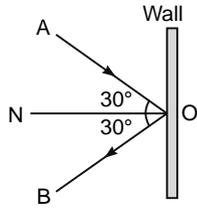
$$= K.E. - \text{work done by gravitational force}$$

$$= 0.00168 - 2 \times 0.082$$

$$= -0.163 \text{ J}$$

Q. 14. A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with normal and rebounds with same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Ans. Since, the wall is too heavy, the recoiling molecule produces no velocity in the wall.



When m is mass of the gas molecule and M is mass of wall, the total K.E. after collision,

$$E_2 = \frac{1}{2} m (200)^2 + \frac{1}{2} M (0)^2$$

or $E_2 = 2 \times 10^4 \text{ mJ}$ $\frac{1}{2}$
which is equal to the K.E. of the molecule before collision

$$[E_1 = \frac{1}{2} m(200)^2 = 2 \times 10^4 \text{ mJ}]$$

Hence, the collision is elastic.

- Q. 15.** A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Ans. Volume of the tank, $V = 30 \text{ m}^3$

Time of operation, $t = 15 \text{ min} = 15 \times 60 = 900 \text{ s}$

Height of the tank, $h = 40 \text{ m}$

Efficiency of the pump, $\eta = 30\%$

Density of water, $\rho = 10^3 \text{ kg/m}^3$

Mass of water, $m = \rho V = 30 \times 10^3 \text{ kg}$

Output power can be obtained as:

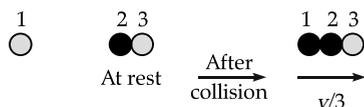
$$P_0 = \text{Work done} / \text{Time} = mgh/t = 30 \times 10^3 \times 9.8 \times 40 / 900 = 13.067 \times 10^3 \text{ W}$$

For input power P_i , and efficiency η , is given by the relation:

$$\eta = P_0 / P_i = \frac{30}{100}$$

$$P_i = 13.067 \times 10^3 \times \frac{100}{30} = 0.436 \times 10^5 \text{ W} = 43.6 \text{ kW}$$

- Q. 16.** Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v . If the collision is elastic, which of the following is a possible result after collision?



Ans. Suppose, $m =$ mass of each ball bearing
Before collision, total K.E. of the system

$$= \frac{1}{2} mv^2 + 0 = \frac{1}{2} mv^2 \quad \dots(1)$$

After collision, K.E. of the system is given by

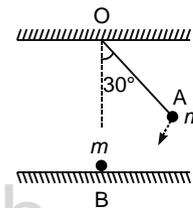
case (i) $E_1 = \frac{1}{2} 2m \left(\frac{v}{2}\right)^2 = \frac{1}{4} mv^2 \quad \dots(2)$

case (ii) $E_2 = \frac{1}{2} mv^2 \quad \dots(3)$

case (iii) $E_3 = \frac{1}{2} (3m) \left(\frac{v}{3}\right)^2 = \frac{1}{6} mv^2 \quad \dots(4)$

Hence, from above equations, we see that the K.E. is conserved only in case (ii), so, case (ii) is the only possible result after collision.

- Q. 17.** The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure given below. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



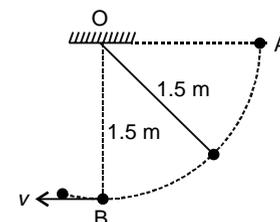
Ans. It is quite clear that in perfectly elastic head on collision, when two equal masses collide with each other, then they exchange their speeds.

In the present case, bob A is moving with certain speed and bob B is at rest. So after collision, bob A comes to rest and the bob B starts moving with the speed of the bob A. The bob A transfers whole of its momentum to bob B and so, bob A, will not rise at all after the collision.

Note : The bob shall not rise because when two bodies of same mass undergo an elastic collision, their velocities are interchanged and ball A rests and ball B moves.

- Q. 18.** The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lower most point B, given that it dissipates 5% of its initial energy against air resistance.

Ans.



Given : $h = 1.5 \text{ m}$, $v = ?$, Energy dissipated = 5%.
Taking B as the lowest position of the bob, its potential energy at B is zero. At the horizontal position A, total potential energy of the bob

is mgh . In going from A to B, P.E. of the bob is converted into K.E.

Energy converted = 95% (mgh)

When v is velocity acquired at B, then

$$K.E. = \frac{95}{100} mgh$$

or $\frac{1}{2}mv^2 = \frac{95}{100}mgh$

$$v = \sqrt{\frac{95}{100} \times 2gh}$$

or $v = \sqrt{\frac{19}{20} \times 2 \times 9.8 \times 1.5}$

or $v = 5.285 \text{ ms}^{-1}$
 $\approx 5.3 \text{ ms}^{-1}$

Q. 19. A trolley of 300 kg carrying a sand bag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolleys floor at the rate of 0.05 kg s^{-1} . What is the speed of the trolley after the entire sand bag is empty?

Ans. As the trolley carrying the sand bag moving uniformly, therefore, external force on system = zero.

When the sand leaks out, it does not lead to application of any external force on the trolley. Hence, the speed the trolley shall not change.

So, speed will remain same

Speed = 27 km/h.

Q. 20. A body of mass 0.5kg travels in a straight line with velocity $v = ax^{\frac{3}{2}}$ where $a = 5\text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2 \text{ m}$ is

- (A) 1.5 J (B) 50 J
(C) 10 J (D) 100 J

Ans. Option (B) is correct

Explanation:

$$v = ax^{3/2}$$

At $x = 0, v(0) = 0$

At $x = 2, v(2) = a \times 2^{3/2} = 5 \times 2^{3/2}$

Now, Work done = $KE_f - KE_i$

or, Work done = $\frac{1}{2} m [v(2)]^2 - \frac{1}{2} m [v(0)]^2$

or, Work done = $\frac{1}{2} m [v(2)]^2$

or, Work done = $\frac{1}{2} \times 0.5 \times [5 \times 2^{3/2}]^2$

\therefore Work done = 50 J

Q. 21. The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ? (b) What is the kinetic energy of the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30 \text{ m}^2$, $v = 36 \text{ km/h}$ and the density of air is 1.2 kg m^{-3} . What is the electrical power produced?

Ans. Area of the circle swept by the windmill = A

Velocity of the wind = v

Density of air = ρ

(a) Volume of the wind flowing through the windmill per sec = Av

Mass of the wind flowing through the windmill per sec = ρAv

Mass m , of the wind flowing through the windmill in time $t = \rho Avt$

(b) Kinetic energy of air = $(1/2)mv^2$
 $= (1/2)(\rho Avt)v^2 = (1/2)\rho Av^3t$

(c) Area of the circle swept by the windmill = $A = 30 \text{ m}^2$
Velocity of the wind = $v = 36 \text{ km/h}$

Density of air, $\rho = 1.2 \text{ kg m}^{-3}$

Electric energy produced = 25% of the wind energy

$= (25/100) \times$ Kinetic energy of air

$= (1/8)\rho Av^3t$

Electrical power = Electrical energy / Time

$= (1/8)\rho Av^3t / t$

$= (1/8)\rho Av^3$

$= (1/8) \times 1.2 \times 30 \times (10)^3$

$= 4.5 \text{ kW}$

Q. 22. A person trying to loose weight (dieter) lifts a 10 kg mass 0.5 m 1000 times. Assuming that the potential energy lost each time she lowers the mass is dissipated.

(a) How much work does she do against the gravitational force ?

(b) Fat supplies $3.8 \times 10^7 \text{ J}$ of energy per kilogram, which is converted to mechanical energy with a 20 % efficiency rate.

How much fat will the dieter use up ?

Ans. Mass $m = 10 \text{ kg}$

Height $h = 0.5$

Number of times the mass lifted (n) = 1000

(a) Work done against gravitational force

$$= n \times mgh$$

$$= 1000 \times 10 \times 9.8 \times 0.5$$

$$= 49000 \text{ J.}$$

(b) Energy supplied by fat per kilogram

$$= 3.8 \times 10^7 \text{ J}$$

Mechanical energy supplied by fat per kilogram

= 20% of total energy supplied by fat

$$= \frac{20}{100} \times 3.8 \times 10^7$$

$$= 0.76 \times 10^7 \text{ J/kg}$$

Fat used up by the dieter

$$= \frac{1}{0.76 \times 10^7} \times 49000$$

$$= 6.45 \times 10^{-3} \text{ kg.}$$

Q. 23. A large family uses 8 kW of power.

(a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square

metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ?

- (b) Compare the area of that of the roof of a typical house.

Which value is denoted in the question ?

Ans. Power used by a family (P) = 8 kW

- (a) Solar energy incident on horizontal surface per square metre = 200 W
Electrical energy obtained from solar energy per unit area

$$= 200 \times \frac{20}{100} \text{ W}$$

$$= 40 \text{ W}$$

∴ Area needed to supply 8 kW.

$$= \frac{8 \text{ kW}}{40 \text{ W}} = \frac{8000}{40} = 200 \text{ m}^2.$$

- (b) This area is comparable to the roof of a large house of 14 m × 14 m = 196 m²

Value : We should use solar energy commercially as it is renewable source of energy.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

- Q. 1. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because,
- (A) the two magnetic forces are equal and opposite, so they produce no net effect.
 (B) the magnetic forces do not work on each particle.
 (C) the magnetic forces do equal and opposite (but non-zero) work on each particle.
 (D) the magnetic forces are necessarily negligible.

Ans. Option (B) is correct.

Explanation: As the magnetic field due to motion of electron and proton act in a direction perpendicular to the direction of motion, no work is done by the forces. This is why one ignores the magnetic force of one particle on another.

- Q. 2. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments: one in which the charged particle is also a proton and in another, a positron. In the same time t , the work done on the two moving charged particles is
- (A) same as the same force law is involved in the two experiments.
 (B) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.
 (C) more for the case of a positron, as the positron moves away a larger distance.
 (D) same as the work done by charged particle on the stationary proton.

Ans. Option (C) is correct.

Explanation: Force between two protons = force between a proton and a positron. Because of having much lighter weight than proton, positron moves away a larger distance compared to proton.

As work done = force × displacement, therefore in the same time t , work done in case of positron is more than that of proton.

- Q. 3. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is
- (A) constant and equal to mg in magnitude.
 (B) constant and greater than mg in magnitude.
 (C) variable but always greater than mg .
 (D) at first greater than mg , and later becomes equal to mg .

Ans. Option (D) is correct.

Explanation: In the whole process, the man exerts a variable force (F) on the ground to set his body in motion. This force is in addition to the force required to support his weight (mg). Once the man is in standing position, F become zero.

- Q. 4. A bicyclist comes to a skidding stop in 10m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is
- (A) + 2000J
 (B) -200J
 (C) zero
 (D) -20,000J

Ans. Option (C) is correct.

Explanation: Just because road does not move at all. So, the work done by the cycle on the road must be zero.

- Q. 5. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?
- (A) Kinetic energy.
 (B) Potential energy.
 (C) Total mechanical energy.
 (D) Total linear momentum.

Ans. Option (C) is correct.

Q. 6. During inelastic collision between two bodies, which of the following quantities always remain conserved?

- (A) Total kinetic energy.
- (B) Total mechanical energy.
- (C) Total linear momentum.
- (D) Speed of each body.

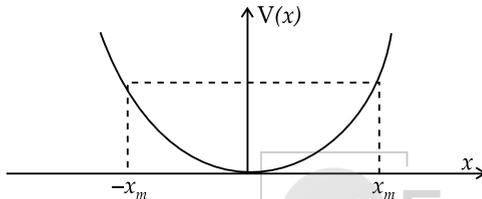
Ans. Option (C) is correct.

Q. 7. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$

where k is the force constant of the oscillator. For $k = 0.5\text{N/m}$. The graph of $V(x)$ versus x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the P.E. and K.E., respectively of the particle at $x = +x_m$, then which of the following is correct?

[NCERT Exemplar Q. No. 8]

- (A) $V = 0, K = E$
- (B) $V = E, K = 0$
- (C) $V < E, K = 0$
- (D) $V = 0, K < E$.



Ans. Option (B) is correct.

Explanation: At any instant of time, the total energy of an oscillator is the sum of kinetic energy and potential energy.

Total energy $E = U + K$

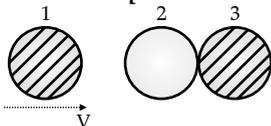
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

At $x = \pm x_m$, the particle turns back

\therefore its velocity at this point is zero, i.e., $v = 0$

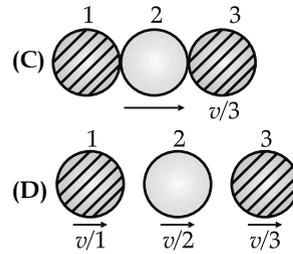
$\therefore K = 0 \quad \therefore E = V$

Q. 8. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in Fig. [NCERT Exemplar Q. No. 9]



If the collision is elastic, which of the following is a possible result after collision?

- (A)
- (B)



Ans. Option (B) is correct.

Explanation: Let m be the mass of each ball bearing.

Total kinetic energy of the system before collision,

$$\begin{aligned} &= \frac{1}{2}mv^2 + 0 \\ &= \frac{1}{2}mv^2 \end{aligned}$$

In (A), K of the system after collision,

$$K_1 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

In (B), K of the system after collision,

$$K_2 = \frac{1}{2}(m)(v)^2 = \frac{1}{2}mv^2$$

In (C), K of the system after collision,

$$K_3 = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$$

In (D), K of the system after collision,

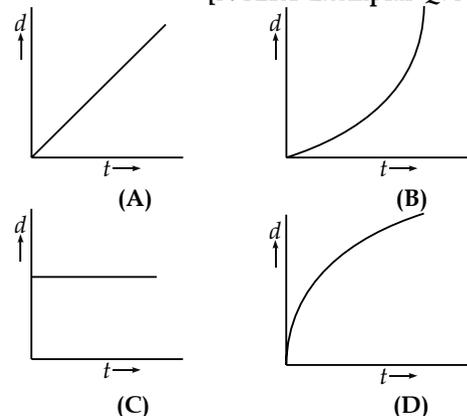
$$\begin{aligned} K_4 &= \frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{3}\right)^2 \\ &= \frac{49}{72}mv^2 \end{aligned}$$

K is only conserved in (B)

\therefore (B) is the only possibility

Q. 9. A body is moving uni-directionally under the influence of a source of constant power supplying energy. Which of the diagrams shown below correctly shows the displacement – time curve for its motion?

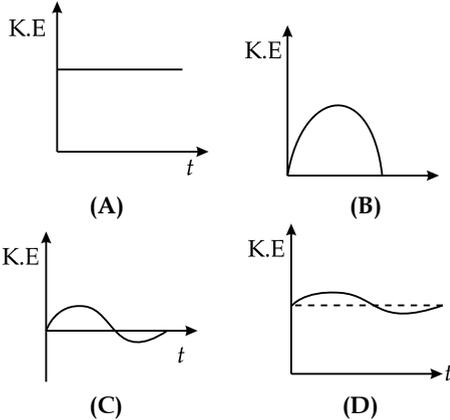
[NCERT Exemplar Q. No. 11]



Ans. Option (B) is correct.

Explanation: For constant power, displacement $d \propto t^{3/2}$

Q. 10. Which of the diagrams shown below most closely shows the variation in kinetic energy of the Earth as it moves once around the sun in its elliptical orbit? [NCERT Exemplar Q. No. 12]



Ans. Option (D) is correct.

Explanation: As Earth moves once around the sun in its elliptical orbit, its kinetic energy is maximum when it is farthest from the sun. As kinetic energy is never zero during its motion, \therefore Option (D) is right.

Q. 11. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be [NCERT Exemplar Q. No. 14]

- (A) $250 \pi^2$ (B) $100 \pi^2$
(C) $5 \pi^2$ (D) 0

Ans. Option (A) is correct.

Explanation: Given mass, $m = 5\text{kg}$,
Radius, $R = 1\text{m}$

$$v = 300\text{rpm} = \frac{300}{60}\text{rps} = 5\text{rps}$$

The angular speed,
 $\omega = 2\pi v = 2\pi \times 5 = 10\pi \text{ rad/s}$

The linear speed is
 $v = \omega R = (10\pi)(\text{rad/s})(1\text{m})$
 $= 10\pi \text{ m/s}$

$$K.E. = K = \frac{1}{2}mv^2$$

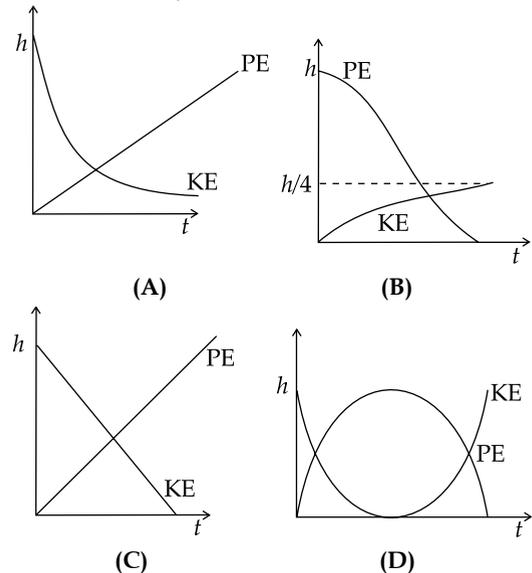
$$K = \frac{1}{2} \times (5\text{kg})(10\pi \text{ m/s})^2$$

$$= 250\pi^2 \text{ J}$$

Q. 12. A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in figure correctly the change in kinetic

and potential energy of the drop during its fall up to the ground?

[A I] [NCERT Exemplar Q. No. 15]



Ans. Option (B) is correct.

Explanation: At h , PE of raindrop is maximum and $KE=0$. As raindrop falls its PE goes on decreasing and KE goes on increasing up to point $\frac{h}{4}$ above the ground. At this stage, rain drop has acquired near terminal velocity (=constant). \therefore At this stage, KE tends to be constant. PE becomes zero when raindrop falls on the ground. Hence, (B) is most appropriate.

Q. 13. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 m/s, at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s^2 , the kinetic energy of the shotput when it just reaches the ground will be [A I] [NCERT Exemplar Q. No. 16]

- (A) 2.5 J (B) 5.0 J
(C) 52.5 J (D) 155.0 J

Ans. Option (D) is correct.

Explanation: Initial K.E. $= \frac{1}{2}mv^2$
 $= \frac{1}{2}(10\text{kg}) \times (1\text{m/s})^2$
 $= 5 \text{ J}$

Initial PE at height 1.5m
 $= mgh$
 $= (10\text{kg})(10\text{m/s}^2)(1.5\text{m})$
 $= 150 \text{ J}$

Total initial energy
 $= 155 \text{ J}$

Q. 14. A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held

firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001 s, the force that the batsman had to apply to hold the bat firmly at its place would be [NCERT Exemplar Q. No. 18]

- (A) 10.5 N (B) 21.0 N
(C) 1.05×10^4 N (D) 2.1×10^4 N

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} F &= \frac{\Delta p}{t} = \frac{mv - (-mv)}{t} \\ &= \frac{2mv}{t} \\ &= \frac{2 \times 150 \times 10^{-3} \text{ kg} \times 35 \text{ m/s}}{0.001 \text{ s}} \\ &= 10500 \text{ N} \\ &= 1.05 \times 10^4 \text{ N} \end{aligned}$$

Very Short Answer Type Questions

Q. 15. A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy? [NCERT Exemplar Q. No. 22]

Ans. No work is done. Force of friction on body is due to the tendency of block M to slide down over inclined plane.

As there is no work done so there is no dissipation of energy.

Q. 16. Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case? [NCERT Exemplar Q. No. 23]

- Ans.** (i) Power is required to decrease the velocity due to free fall.
(ii) Limit on number of passengers is to check the too much increase of speed.

Q. 17. A body is being raised to a height h from the surface of Earth. What is the sign of work done by (a) applied force (b) gravitational force? [NCERT Exemplar Q. No. 24]

- Ans.** (a) Positive
Work done by applied force,
 $W_A = \text{Force} \times \text{displacement} \times \cos \theta$
or $W_A = mgh \cos 0^\circ = mgh$ { $\because \theta = 0^\circ; F = mg$ }
 $W_A \rightarrow$ positive
(b) Negative. Work done by gravitational force
 $W_g = mg \times \text{displacement} \times \cos \theta$
 $= mgh \cos 180^\circ$ { $\because \theta = 180^\circ$ }
 $= -mgh \Rightarrow W_g \rightarrow$ negative

Q. 18. A body falls towards Earth in air. Will its total mechanical energy be conserved during the fall? Justify. [NCERT Exemplar Q. No. 26]

Ans. No, the mechanical energy is not conserved, Because resistive force of air also acts on the body which is a non-conservative force. So, the gain in KE would be smaller than the loss in PE.

Q. 19. A body is moved along a closed loop. Is the work done in moving the body necessarily zero? If not, state the condition under which work done over a closed path is always zero. [NCERT Exemplar Q. No. 27]

Ans. No. Work done by a body moving along a closed loop is necessarily zero, only if all the forces acting on the system are conservative.

Q. 20. In an elastic collision of two billiard balls, which of the following quantities remain conserved in the short time of collision of the balls (i.e., when they are in contact).
(a) Kinetic energy.
(b) Total linear momentum?

Given reason for your answer in each case. [NCERT Exemplar Q. No. 28]

Ans. (b) Total linear momentum remains conserved.
While balls are in contact, there may be deformation which means certain portion of KE is converted into PE. So, kinetic energy is not conserved. Momentum is always conserved.

Q. 21. Calculate the power of a crane in watts, which lifts a mass of 100 kg to a height of 10 m in 20s. [NCERT Exemplar Q. No. 29]

Ans. $F = mg = 100 \times 9.8 \text{ N}$ [$m = 100 \text{ kg}, g = 9.8 \text{ m/s}^2$]
Power $P = \frac{mgh}{t}$
 $= \frac{100 \times 9.8 \times 10}{20} \text{ W}$
 $= 490 \text{ W}$

Q. 22. The average work done by a human heart while it beats once is 0.5 J. Calculate the power used by heart if it beats 72 times in a minute. [NCERT Exemplar Q. No. 31]

Ans. Work done in 72 beats by heart = $0.5 \times 72 \text{ J}$
 $= 36 \text{ J}$

$$\begin{aligned} \text{Power} &= \frac{\text{Work done}}{\text{Time}} \\ &= \frac{36 \text{ J}}{60 \text{ s}} \\ &= 0.6 \text{ Watt} \\ &= 0.6 \text{ W} \end{aligned}$$

Q. 23. Give example of a situation in which an applied force does not result in a change in kinetic energy.

[NCERT Exemplar Q. No. 31]

Ans. When an object rotates in a circular path, the centripetal force is perpendicular to the displacement. Hence, no work is done. So, the force does not result in change in kinetic energy.

Q. 24. Two bodies of unequal mass are moving in the same direction with equal kinetic energy. The two bodies are brought to rest by applying retarding force of same magnitude. How would the distance moved by them before coming to rest compare? [NCERT Exemplar Q. No. 32]

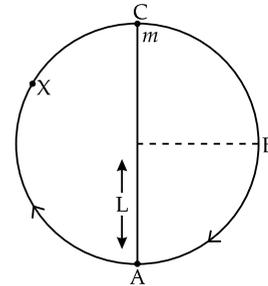
Ans. By work energy theorem, work done is equal to change in K.E.

Both bodies had same K.E. and hence same amount of work is needed to be done. Since, force applied is same, they would come to rest within the same distance.

Q. 25. A bob of mass m suspended by a light string of length L is whirled into a vertical circle as shown in Fig. What will be the trajectory of the particle if the string is cut at

- (a) point B?
- (b) point C?
- (c) point X?

[AI] [NCERT Exemplar Q. No. 33]

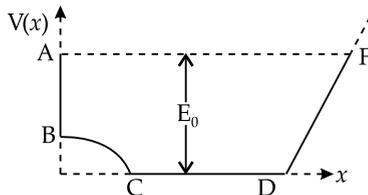


- Ans.** (a) If the string is cut at point B, the trajectory is straight line vertically downward.
 (b) At point C, parabolic path with vertex at C
 (c) At point X, it is parabolic path with vertex higher than X.

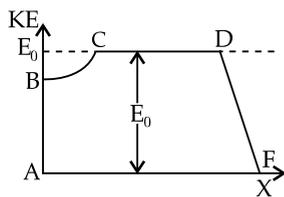
Short Answer Type Questions

Q. 26. A graph of potential energy $V(x)$ versus x is shown in Fig. A particle of energy E_0 is executing motion in it. Draw graph of velocity and kinetic energy versus x for one complete cycle AFA.

[NCERT Exemplar Q. No. 34]



Ans.



$$KE = TE - PE$$

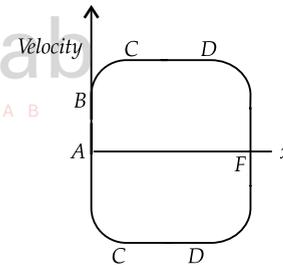
or,

$$KE = E_0 - PE$$

Computation of KE from the given graph:

Point	PE	KE	Remark
A	E_0	0	
B	Less than E_0	Positive (but not maximum)	Nature is reverse parabolic.
C	0	E_0 (maximum)	
D	0	E_0 (maximum)	
F	E_0	0	

(a) Kinetic energy versus x .



(b) Velocity versus x .

$$KE = \frac{1}{2} mv^2$$

\therefore

$$v = \sqrt{\frac{2KE}{m}}$$

Computation of velocity:

Point	KE	Velocity
A	0	0
B	Positive (but not maximum)	Positive (but not maximum)
C	E_0 (maximum)	Maximum
D	E_0 (maximum)	Maximum
F	0	0

Q. 27. A ball of mass m , moving with a speed $2v_0$, collides in-elastically ($e > 0$) with an identical ball at rest. Show that

- (a) For head-on collision, both the balls move forward.

(b) For a general collision, the angle between the two velocities of scattered balls is less than 90° . [NCERT Exemplar Q. No. 35]

Ans. (a) For head on collision:

Let v_1, v_2 be the velocities of two balls after collision.

From Conservation of momentum

$$\Rightarrow 2mv_0 = mv_1 + mv_2$$

$$\text{or } 2v_0 = v_1 + v_2$$

$$\text{and } e = \frac{v_2 - v_1}{2v_0} \Rightarrow v_2 = v_1 + 2v_0e$$

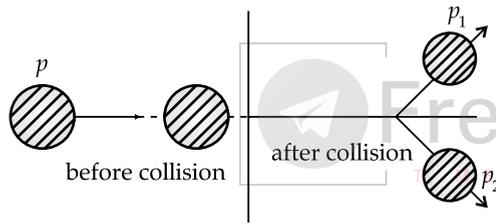
$$\therefore 2v_1 = 2v_0 - 2ev_0$$

$$\therefore v_1 = v_0(1 - e)$$

Since, $e < 1 \Rightarrow v_1$ has the same sign as v_0 , therefore the ball moves along v_1 i.e., in forward direction after collision.

(b) From conservation of momentum $\Rightarrow p = p_1 + p_2$

$$\text{But KE is lost } \Rightarrow \frac{p^2}{2m} > \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$



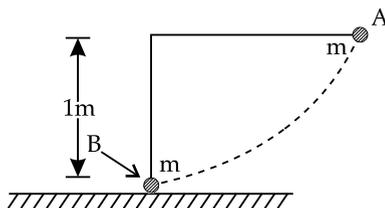
$$\Rightarrow \therefore p^2 > p_1^2 + p_2^2$$

Thus p, p_1 and p_2 are related as shown in the figure.

θ is acute (less than 90°)

Q. 28. The bob A of a pendulum released from horizontal to the vertical hits another bob B of the same mass at rest on a table as shown in Fig.

[A1] [NCERT Exemplar Q. No. 37]



If the length of the pendulum is 1 m, calculate

(a) The height to which bob A will rise after collision.

(b) The speed with which bob B starts moving.

Neglect the size of the bobs and assume the collision to be elastic.

Ans. (a) When ball A strikes to an identical ball at rest then ball A transfer its entire momentum to the ball B on the table and does not rise at all.

$$(b) \frac{1}{2}mv^2 = mgh$$

$$\text{or } v = \sqrt{2gh},$$

$$\text{here, } g = 9.8 \text{ m/s}^2,$$

$$h = 1\text{m}$$

Putting values we get

$$v = 4.43\text{m/s}$$

Q. 29. A raindrop of mass 1.00 g falling from a height of 1 km hits the ground with a speed of 50 ms^{-1} . Calculate

(a) the loss of P.E. of the drop.

(b) the gain in K.E. of the drop.

(c) Is the gain in K.E. equal to loss of P.E.? If not why.

Take $g = 10 \text{ ms}^{-2}$ [NCERT Exemplar Q. No. 38]

Ans. Given mass of rain drop, $(m) = 0.001 \text{ kg}$

$$= 1.0 \times 10^{-3} \text{ kg}$$

$$\text{Height, } h = 1 \text{ km} = 1000 \text{ m}$$

$$\text{Speed, } v = 50 \text{ m/s, } u = 0.$$

$$(a) \text{ Loss of } PE = mgh = 1 \times 10^{-3} \times 10 \times 10^3 = 10 \text{ J}$$

$$(b) \text{ Gain in } KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^{-3} \times 2500 = 1.25 \text{ J}$$

(c) No, because a part of PE is used up in doing work against the viscous drag of air.

Q. 30. Suppose the average mass of raindrops is $3.0 \times 10^{-5} \text{ kg}$ and their average terminal velocity 9 m s^{-1} . Calculate the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.

[NCERT Exemplar Q. No. 40]

Ans. Given : $m = 3.0 \times 10^{-5} \text{ kg},$

$$\rho = 1.0 \times 10^3 \text{ kg/m}^3,$$

$$v = 9 \text{ m/s}$$

$$A = 1\text{m}^2$$

$$h = 100 \text{ cm} \Rightarrow V = 1\text{m}^3$$

$$M = \rho V = 10^3 \text{ kg},$$

Energy transferred by rain,

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 10^3 \times (9)^2 = 4.05 \times 10^4 \text{ J}.$$

Q. 31. An engine is attached to a wagon through a shock absorber of length 1.5m. The system with a total mass of 50,000 kg is moving with a speed of 36 km/hr when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber

gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, calculate the spring constant. [NCERT Exemplar Q. No. 41]

Ans. $m = 50,000 \text{ kg}$,
 $v = 36 \times \frac{5}{18} \text{ m/s} = 10 \text{ m/s}$
 $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 10^4 \times 10^2 \text{ J}$
 $= 2.5 \times 10^6 \text{ J}$

90% of KE of wagon lost due to friction and only 10% of this is stored in the spring.

$$\frac{1}{2}kx^2 = 10\% \text{ of } 2.5 \times 10^6 \text{ J} = 2.5 \times 10^5 \text{ J}$$

Here, $x = 1 \text{ m}$

so, $k = \frac{2 \times 2.5 \times 10^5}{(1)^2} \text{ N/m}$
 $k = 5.0 \times 10^5 \text{ N/m}$

Q. 32. An adult weighing 600N raises the centre of gravity of his body by 0.25 m while taking each step of 1 m length in jogging. If he jogs for 6 km, calculate the energy utilised by him in jogging assuming that there is no energy loss due to friction of ground and air. Assuming that the body of the adult is capable of converting 10% of energy intake in the form of food, calculate the energy equivalents of food that would be required to compensate energy utilised for jogging. [NCERT Exemplar Q. No. 42]

Ans. $mg = 600 \text{ N}$, $g = 10 \text{ m/s}^2$, $h = 0.25 \text{ m}$

No. of steps in 6 km = $\frac{6000 \text{ m}}{1 \text{ m / step}} = 6000 \text{ steps}$

In 6 km there are 6000 steps.

$\therefore E = 6000(mg)h$
 $= 6000 \times 600 \times 0.25 \text{ J}$
 $= 9 \times 10^5 \text{ J}$

This is 10% of intake energy

\therefore Intake energy = $10E = 9 \times 10^6 \text{ J}$.

Q. 33. On complete combustion a litre of petrol gives off heat equivalent to $3 \times 10^7 \text{ J}$. In a test drive a car weighing 1200 kg, including the mass of driver, runs 15 km per litre while moving with a uniform speed on a straight track. Assuming that friction offered by the road surface and air to be uniform, calculate the force of friction acting on the car during the test drive, if the efficiency of the car engine were 0.5.

[NCERT Exemplar Q. No. 43]

Ans. \therefore Energy given by car in 1 litre petrol
 $= 0.5 \times 3 \times 10^7 \text{ J}$
 $= 1.5 \times 10^7 \text{ Joule}$

With 0.5 efficiency, 1 litre generates $1.5 \times 10^7 \text{ J}$, which is used for 15 km drive.

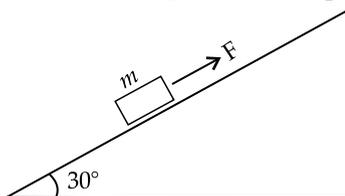
$Fd = 1.5 \times 10^7 \text{ J}$, where $d = 1.5 \times 10^4 \text{ m}$
 Force of friction $F = \frac{1.5 \times 10^7 \text{ J}}{1.5 \times 10^4 \text{ m}}$
 $F = 10^3 \text{ N}$

Long Answer Type Questions

Q. 34. A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined surface. The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline, calculate [AI]

- work done against gravity
- work done against force of friction
- increase in potential energy
- increase in kinetic energy
- work done by applied force.

[NCERT Exemplar Q. No. 44]



Ans. $m = 1 \text{ kg}$, $\theta = 30^\circ$, $\cos 30^\circ = 0.866$, $\sin 30^\circ = 0.5$
 $F = 10 \text{ N}$, $\mu = 0.1$

Distance $d = 10 \text{ m}$

(a) $W_g = mg \sin \theta \times d = 1 \times 10 \times 0.5 \times 10 = 50 \text{ J}$
 (b) $W_f = \mu mg \cos \theta \times d = 0.1 \times 10 \times 0.866 \times 10 = 8.66 \text{ J}$

(c) $\Delta U = mgh = 1 \times 10 \times 5 = 50 \text{ J}$

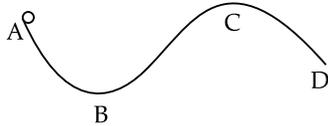
(d) $a = \{F - (mg \sin 30^\circ + \mu mg \cos 30^\circ)\}$
 $a = \frac{[10.0 - (5.0 + 0.87)]}{1.0} \text{ m/s}^2$
 $= 4.13 \text{ m/s}^2$

Apply 3rd Kinematic equation of motion,
 $v^2 - u^2 = 2ad$

Change in KE, $\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mad = 41.3 \text{ J}$

(e) Work done = Force \times displacement
 $= 10 \times 10 \text{ J}$
 $= 100 \text{ J}$

Q. 35. A curved surface is shown in Fig. The portion BCD is free of friction. There are three spherical balls of identical radii and masses. Balls are released from rest one by one from A which is at a slightly greater height than C.



With the surface AB, ball 1 has large enough friction to cause rolling down without slipping; ball 2 has a small friction and ball 3 has a negligible friction.

- (a) For which balls is total mechanical energy conserved?
- (b) Which ball(s) can reach D?
- (c) For balls which do not reach D, which of the balls can reach back A?

[NCERT Exemplar Q. No. 45]

- Ans.** (a) Force of friction is zero and negligible for ball 1 & 3 respectively, so, energy is conserved for balls 1 and 3.
- (b) Ball 1 acquires rotational energy, ball 2 loses energy by friction. They cannot cross at C. Ball 3 can cross over.
- (c) Ball 3 have negligible friction & crosses C so, ball can not reach at A.
Ball 1, 2 turn back before reaching C. Because of loss energy, ball 2 cannot reach back to A. Ball 1 has a rotational motion in “wrong” sense when it reaches B. It cannot roll back to A, because of kinetic friction.



COMPETENCY BASED QUESTIONS

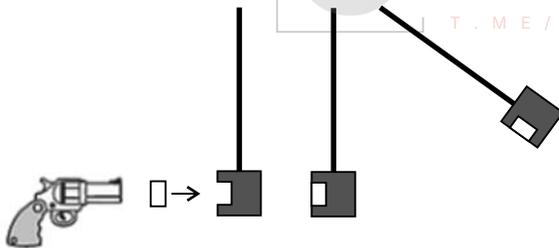
These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

The ballistic pendulum was invented in 1742 by English mathematician Benjamin Robins.

A Ballistic Pendulum is a device for measuring a bullet's momentum and speed by employing perfectly inelastic collision.



A large wooden block suspended by two cords serves as the pendulum bob. When a bullet is fired into the bob, it gets embedded in the bob and its momentum is transferred to the bob.

The bullet's momentum and velocity can be determined from the amplitude of the pendulum swing. The velocity of the bullet, in turn, can be derived from its calculated momentum.

After collision, if the pendulum reaches a height h , then from principle of conservation of mechanical energy

$$\frac{1}{2}(m + M)v_p^2 = (m + M)gh$$

where, m = mass of bullet, M = mass of the bob
 v_p = velocity of the bob-bullet combination

$$\therefore v_p = \sqrt{2gh}$$

Now, Momentum before collision

$$= \text{Momentum after collision}$$

$$mv_B = (m + M)v_p$$

where, v_B = velocity of bullet

$$v_B = \frac{m + M}{m} \sqrt{2gh}$$

The ballistic pendulum used to be a common tool for the determination of the muzzle velocity of bullets as a measure of the performance of firearms and ammunition (Nowadays, the ballistic pendulum has been replaced by the ballistic chronograph, an electronic device).

Q. 1. In ballistic pendulum the collision is

- (A) Elastic
- (B) Perfectly inelastic
- (C) Inelastic
- (D) Partly elastic, partly inelastic

Ans. Option (B) is correct.

Explanation: A large wooden block suspended by two cords serves as the pendulum bob. When a bullet is fired into the bob, it gets embedded in the bob and its momentum is transferred to the bob. Hence, the collision is perfectly inelastic.

Q. 2. Which two principles of Physics are applied to find the velocity of the bullet?

- (A) conservation of mechanical energy and conservation of momentum
- (B) conservation of mechanical energy and conservation of mass
- (C) conservation of mass and conservation of momentum
- (D) conservation of mechanical energy, conservation of momentum and conservation of mass

Ans. Option (A) is correct.

Explanation: Principle of conservation of mechanical energy, an expression for the bob-bullet combination after collision is derived. Then the principle of conservation of momentum is applied to find the velocity of the bullet before collision.

Q. 3. The ballistic pendulum was invented by a

- (A) Chemist (B) Physicist
(C) Mathematician (D) Warrior

Ans. Option (C) is correct.

Explanation: The ballistic pendulum was invented in 1742 by English mathematician Benjamin Robins.

Q. 4. Ballistic pendulum has been replaced by

- (A) Seismograph (B) Chronograph
(C) Gyrograph (D) Tachograph

Ans. Option (B) is correct.

Explanation: The ballistic pendulum has now been replaced by the ballistic chronograph, an electronic device.

Q. 5. A ballistic pendulum of 1 kg is fired with a bullet of mass 1 g. If the pendulum rises 2 cm, find the velocity of the bullet.

- (A) 12.65 m/s (B) 6330 m/s
(C) 0.633 m/s (D) 633 m/s

Ans. Option (D) is correct.

Explanation: $v_B = \frac{m + M}{m} \sqrt{2gh}$

Putting,

$m = 1 \text{ g} = 0.001 \text{ kg}$

$M = 1 \text{ kg}$

$g = 10 \text{ m/s}^2$

$h = 2 \text{ cm} = 0.02 \text{ m}$

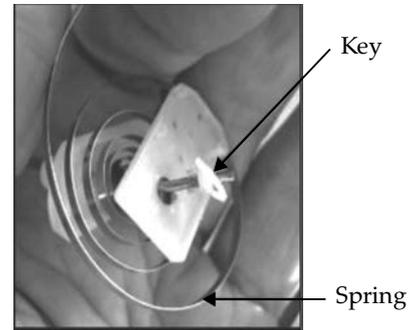
$v_B = \frac{0.001 + 1}{0.001} \sqrt{2 \times 10 \times 0.02} = 633 \text{ m/s}$

II. Read the following text and answer the following questions on the basis of the same:

Clockwork refers to the inner workings of mechanical clock or watch (where it is known as "movement") and different types of toys which work using a series of gears driven by a spring. Clockwork device is completely mechanical and its essential parts are:

- A key (or crown) which you wind to add energy
- A spiral spring in which the energy is stored
- A set of gears through which the spring's energy is released. The gears control how quickly (or slowly) a clockwork machine can do things. Such as in mechanical clock / watch the mechanism is the set of hands that sweep around the dial to tell the time. In a clockwork car toy, the gears drive the wheels.

Winding the clockwork with the key means tightening a sturdy metal spring, called the mainspring. It is the process of storing potential energy. Clockwork springs are usually twists of thick steel, so tightening them (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.



While the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower. In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at 1/60 of that speed, and the hour hand at 1/3600 of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

Q. 1. What is the meaning of "movement" of old age mechanical clocks?

- (A) The gears which move the hands of the clock
(B) A spring and combination of gears which move the hands of the clock
(C) The hands of the clock
(D) The pendulum of the clock

Ans. Option (B) is correct.

Explanation: Movement refers to the inner workings of mechanical clock using a series of gears driven by a spring.

Q. 2. What type of energy is stored in the spring while winding it?

- (A) Kinetic (B) Potential
(C) Both (A) and (B) (D) Heat

Ans. Option (B) is correct.

Explanation: Winding the spring means tightening a sturdy metal spring. It is the process of storing potential energy (forcing the spring to occupy a much smaller space) is actually quite hard work. With each turn of the key, fingers do work and potential energy is stored in the spring.

Q. 3. When the spring of a clockwork uncoils

- (A) Kinetic energy is converted into potential energy
(B) Potential energy is converted into kinetic energy
(C) Potential energy is converted into heat, light and sound energy
(D) Kinetic energy is converted into heat, light and sound energy

Ans. Option (B) is correct.

Explanation: When the spring uncoils, the potential energy is converted into kinetic energy through gears, cams, cranks and shafts which allow wheels to move faster or slower.

Q. 4. In clockwork devices, transform the speed of a rotating to drive wheels slower or faster.

- (A) Gear, shaft (B) Shaft, gear
(C) Spring, gear (D) Shaft, spring

Ans. Option (A) is correct.

Explanation: In an ancient clock, gears transform the speed of a rotating shaft so that it drives the second hand at one speed, the minute hand at 1/60 of that speed, and the hour hand at 1/3600 of that speed. Clockwork toy cars often use gears to make themselves race along at surprising speed.

Q. 5. More energy is stored in a spring if the

- (A) Spring is larger, harder and wound for a shorter time
(B) Spring is smaller, harder and wound for a shorter time
(C) Spring is larger, harder and wound for a longer time
(D) Spring is larger, softer and wound for a shorter time

Ans. Option (C) is correct.

Explanation: With each turn of the key, fingers do work and potential energy is stored in the spring. The amount of energy stored depends on the size and tension of the spring. Harder a spring is to turn and longer it is wound, the more energy it stores.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

In all collisions, total linear momentum is conserve. The total KE of the system is not necessarily conserved. If there is no loss of KE during a collision it is called an elastic collision. If there is a loss of KE during a collision it is called inelastic collision. Impact and deformation during collision may generate heat and sound. Part of the initial kinetic energy is transformed into other forms of energy. When two bodies stick together after a collision, the collision is perfectly inelastic. If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a one-dimensional collision, or head-on collision. Most of the collisions in daily life are inelastic in nature.

Q. 1. What type of collision is it when a car crashes against a tree and stops?

Ans. Inelastic collision since, two bodies stick together after a collision.

Q. 2. A and B are two identical balls . A is moving along the positive x direction with speed v while B is moving along negative x direction with same speed. If the collision is perfectly elastic, what will be the impulse received by the ball B?

Ans. The collision is perfectly elastic. So, ball B moves

in the backward direction i.e., along positive x-axis with the same speed as before collision. The change in velocity becomes $v - (-v) = 2v$
∴ Change in momentum = $m(2v) = 2mv$ (where m is mass of each ball)

So, the impulse received by B is $2mv$.

Q. 3. Two bodies A and B moving towards each other and collide inelastically. The velocities of A and B before collision were 2 m/s and 4 m/s respectively. The common velocity after collision becomes 1 m/s in the direction of A. Find the ratio of K.E. of the two bodies before collision.

Ans. From momentum conservation,

$$2 \times m_1 - 4 \times m_2 = 1 \times (m_1 + m_2)$$

or, $m_1 = 5m_2$

$$\text{Now, the ratio of kinetic energy} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2}$$

$$= \frac{5m_2 \times 2^2}{m_2 \times 4^2}$$

$$= \frac{20}{16}$$

$$= 1.25$$

Q. 4. Define coefficient of restitution. What is its magnitude?

Ans. Coefficient of restitution

$$= \frac{\text{Relative speed between two objects after collision}}{\text{Relative speed between the objects before collision}}$$

$e = 0$, for perfectly inelastic collision

$0 < e < 1$, for real-world inelastic collision.

$e = 1$, for a perfectly elastic collision.

Q. 5. A sphere of mass m moving with a constant velocity u collides another stationary sphere of the same mass. If e is the coefficient of restitution, then find the ratio of the velocities of two spheres after collision in terms of e.

Ans. From momentum conservation,

$$mu = mv_1 + mv_2$$

$$\therefore u = v_1 + v_2 \quad \dots(i)$$

By definition,

$$\text{The coefficient of restitution} = e = \frac{v_2 - v_1}{u_2 - u_1}$$

$$\text{or, } e = \frac{v_2 - v_1}{u}$$

$$\therefore eu = v_2 - v_1 \quad \dots(ii)$$

Solving equations (i) and (ii)

$$v_1 = \frac{u(1-e)}{2}$$

$$v_2 = \frac{u(1+e)}{2}$$

$$\therefore \frac{v_1}{v_2} = \frac{(1-e)}{(1+e)}$$

UNIT-V MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

Study Time
Maximum Time: 3 hours
Maximum Questions: 80

CHAPTER 6

SYSTEM OF PARTICLES AND ROTATIONAL MOTION



Syllabus

Centre of mass of a two-particle system, momentum conservation and centre of mass motion; Centre of mass of a rigid body, centre of mass of a uniform rod; Moment of a force, torque, angular momentum, law of conservation of angular momentum and its applications; Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions; Moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects (no derivation).



Topic-1

Centre of Mass & Motion of Rotational Particles

TOPIC - 1
 Centre of Mass & Motion of Rotational Particles P. 124

TOPIC - 2
 Moment of Inertia & Radius of Gyration P. 131

Revision Notes

- **Kinds of Motion of Rigid Body**
 - (i) **Pure Translational Motion** : All the particles of body are moving together with same velocity at particular instant of time. eg., A car moving is a straight line.
 - (ii) **Pure Rotational Motion** : A rigid body rotates about a fixed axis. Every particle of the body moves in a circle which lies in a plane perpendicular to axis and has its centre on the axis. eg., A potter's wheel.
 - (iii) **Combination of Translational and Rotational Motion** : The motion of rigid body, which is not pivoted or fixed in some way is either pure translation motion or a combination of translation and rotation. eg., A vehicle's wheel.
- **Center of Mass of a two particle system** : Position vector of centre of mass of a two particle system is such that the product of total mass of the system and position vector of centre of mass is equal to sum of the products of masses of two particles and their respective position vectors.
- **Momentum Conservation** : Total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

$$\vec{p} = M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots + m_n\vec{v}_n$$

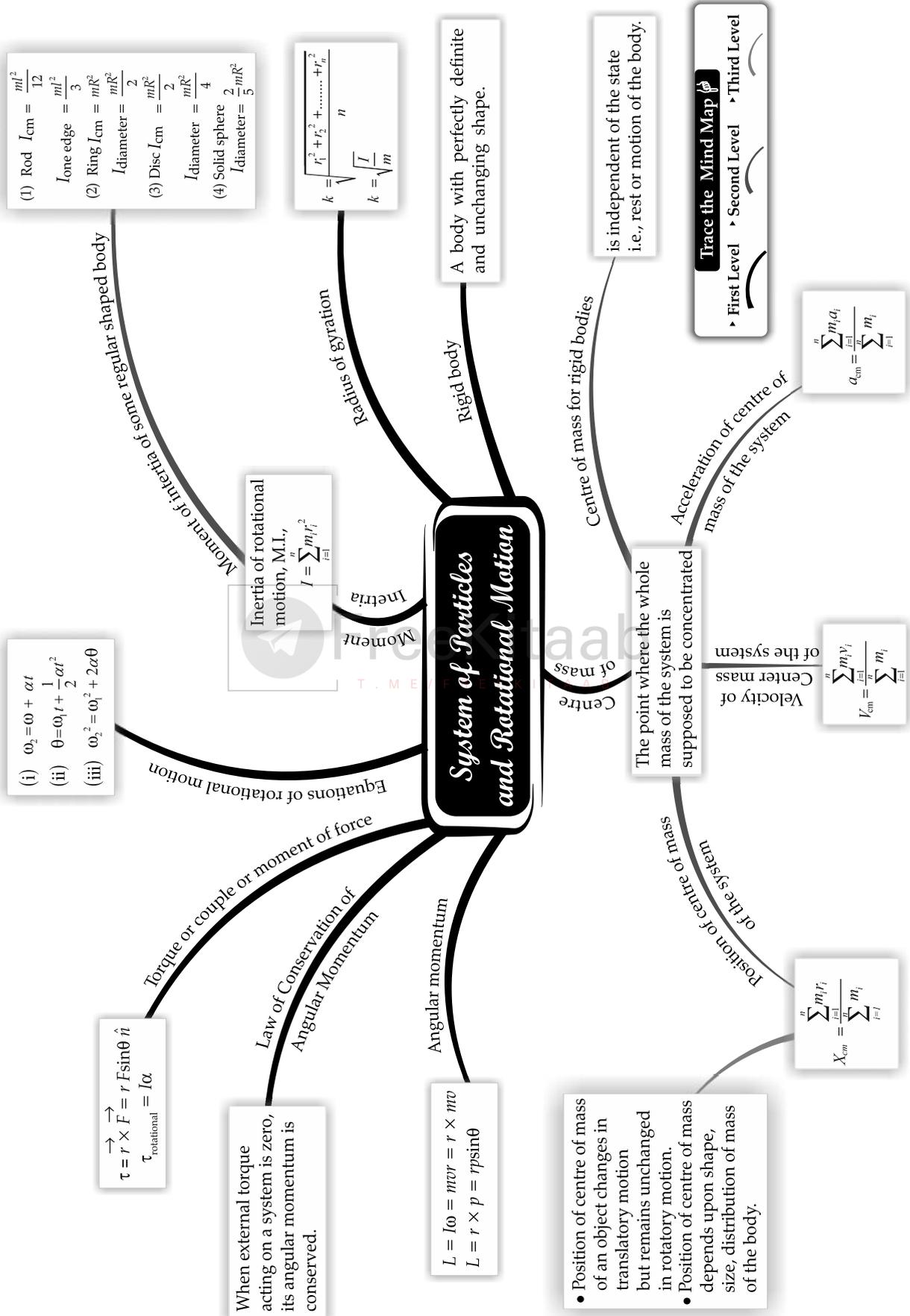
Differentiating it,

$$\frac{d\vec{p}}{dt} = M\frac{d\vec{v}}{dt} = M\vec{a} = \vec{F}_{ext}$$

Scan to know more about this topic



Examples: Centre of mass



This is Newton's II law.

For isolated system, $\vec{F}_{\text{ext}} = \vec{0}$.

$$\therefore \frac{d\vec{p}}{dt} = F_{\text{ext}} = 0 \text{ or } \vec{p} = \text{constant}$$

$$\therefore M\vec{v} = \text{Constant.}$$

- **Moment of Force or Torque :** Torque due to a force is moment of force and measures the turning effect to the force about the axis of rotation. The general expression for torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- **Angular Momentum and its Conservation :** Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about the axis. It is equal to the product of linear momentum of the particle and the perpendicular distance of the line of action of linear momentum from the axis of rotation. It is the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

$$\begin{aligned} \vec{L} &= \vec{r} \times \vec{p} = rp \sin \phi \\ &= \vec{d} \times \vec{p} \end{aligned}$$

where, $d = r \sin \phi =$ perpendicular distance of line of action of \vec{p} from the axis. Angular momentum is a vector quantity, whose direction is given by **right handed screw rule**.

• $\vec{L} \perp \vec{d}$ and $\vec{L} \perp \vec{p}$

- **Rate of change of angular momentum is torque**, i.e., $\vec{\tau} = d\vec{L}/dt$.

As
$$\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{\tau}_{\text{ext}}$$

for isolated system $\vec{\tau}_{\text{ext}} = \vec{0}$.

$$\therefore \vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt} = \vec{0}$$

So,
$$\vec{L} = \text{constant.}$$

- **Equilibrium of Rigid Bodies :**

1st Condition : A rigid body is said to be in translational equilibrium, if it remains at rest or moving with a constant velocity in a particular direction. For this, the net external force or the vector sum of all external forces acting on the body must be zero i.e.,

$$\sum \vec{\tau}_i = \vec{0}$$

Translational Static Equilibrium is of 3 types :

- (i) Stable Equilibrium
- (ii) Unstable Equilibrium
- (iii) Neutral Equilibrium

2nd Condition : A rigid body is said to be in rotational equilibrium, if the body does not rotate or rotates with constant angular velocity. For this, the net external torque or the vector sum of all the torques acting on the body must be zero i.e.,

$$\sum \vec{\tau}_i = \vec{0}$$

- **Principle of Moments**

According to principle of moments, body will be in rotational equilibrium if algebraic sum of the moments of all forces acting on the body, about a fixed point is zero.



Key Words

- A **rigid body** is defined as a system of particles, in which distance between any two particles does not change under the influence of external forces, where, size and shape of the body will remain unaffected under the effect of external forces.
- **The centre of mass** of a body is a point where the whole mass of the body is supposed to be concentrated. If all the forces acting on the body were applied on the centre of mass, the nature of motion of the body shall remain unaffected.
- **Centre of gravity** of a body is a point where the weight of the body acts and total gravitational torque on the body is zero.

Key Formulae

- Position vector of centre of mass of n particles system

$$\vec{r} = \frac{\sum_{i=1}^n m_i \vec{r}_i}{M}, \text{ where, } M \text{ is the total mass to the system i.e., } M = \sum_{i=1}^n m_i$$

- For two particle system

$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

- Coordinates of centre of mass

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$z = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

- Velocity of C.M. of a system of two particles is

$$\vec{v}_{\text{cm}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

- Angular Momentum

$$\vec{L} = \vec{r} \times m\vec{v}$$

- Equations of Rotational Motion :

(a) $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$

(b) $v = r\omega$

$$\omega = 2\pi v = \frac{2\pi}{T}$$

(c) $a = r\alpha$

(d) Centripetal acceleration = $\frac{v^2}{r} = r\omega^2$

- Torque :

- (i) Work done by torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$dW = \tau(d\theta)$$

- (ii) Power of torque

$$P = \frac{dW}{dt} = \tau \left(\frac{d\theta}{dt} \right)$$

$$P = \tau\omega$$



Mnemonics

Concept: Position of centre of mass of different objects.

Mnemonics: **R S** Das in a **C**inema hall met **Ch**iranjeet **m**all **a**rea and **R**am **C**handran behind **C**entral door.

Interpretation:

R - Ring
S - Sphere
D - Disc
C - Centre

} Ring, sphere, disc have centres of mass at their respective centre.

C - Cylinder
m - mid-point
a - Axis

} Cylinders have centres of mass at the mid-point on the respective axis.

R - Rectangular lamina
c - cube
c - cross-point
d - diagonal

} Rectangular lamina, cube have centres of mass at the cross point of respective diagonals.



Very Short Answer Type Questions

(1 mark each)

- Q. 1. What is the rotational analogue of force ?** [R]
Ans. Rotational analogue of force is torque. 1
- Q. 2. What is rotatory motion ?** [R]
Ans. When a body rotates about a fixed axis, the motion of the body is said to be rotatory motion. 1
- Q. 3. What do you mean by moment of a force ?** [R]
Ans. Moment of a force about a given point is the product of the force and the perpendicular distance of the point from the axis of rotation. 1

- Q. 4. What is the other name for angular momentum ?** [R]
Ans. Moment of momentum. 1
- Q. 5. Why do we prefer to use a wrench with a long arm ?** [U]
Ans. The turning effect of a force is $\tau = r \times F$. When arm of the wrench is long, r is larger. So, smaller force is required to produce the same turning effect. 1



Short Answer Type Questions-I

(2 marks each)

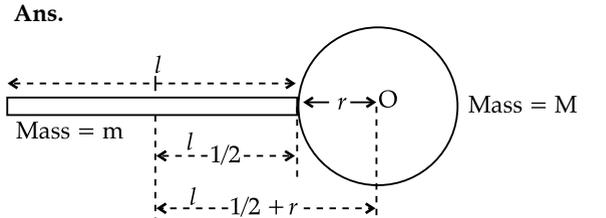
- Q. 1. Define torque. What is its physical significance ?** [R]
Ans. It is defined as the turning effect of a force about the axis of rotation or it is the moment of force about the axis of rotation,
 i.e., $\tau = Fd$
 where F = Force applied on a body, d is the \perp distance of the line of action of the force from the axis of rotation. 1
- Mathematically in vector form, $\vec{\tau}$ may be expressed as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

 i.e., it is cross product of the position vector \vec{r} and force \vec{F}
 i.e., Torque = force \times lever arm. 1

- where $\Sigma F_i r_i$ is the algebraic sum of moment of forces and $\Delta\theta$ is the angle through which a body is rotated, 1
 $\therefore W = \text{Total torque} \times \text{Angular displacement}$ 1
- Q. 4. Two solid wooden balls roll down two different inclined planes of different inclination but same height. Comment upon their speed and time of descent ?** [A]
Ans. It will reach the bottom with same speed in each case because speed depends upon height and not in inclination.
 It will take longer to roll down on the plane with smaller inclination. 2
- Q. 5. A thin rod of length l and mass m has a disc which attached to one of its ends such that rod and disc are coplanar. Find C.M. from the centre of disc, if mass of the disc is M and radius is r .** [A]

- Q. 2. Two equal and opposite forces act on a rigid body. Under what conditions will the body (a) rotate, (b) not rotate ?** [A]
Ans. (a) Two equal and opposite forces acting on a rigid body such that their lines of action don't coincide constitute a couple. This couple produces the turning effect on the body. Hence, the rigid body will rotate. 1
(b) If the two equal and opposite forces act in such a way that their lines of action coincide, then the body will not rotate. 1



- Q. 3. Give an expression for work done in rotational motion in terms of torque.** [U]
Ans. Work done in rotational motion,

$$W = \Sigma F_i r_i \times \Delta\theta,$$

Here

$$x_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

$$x_{CM} = \frac{m(0) + m\left(r + \frac{l}{2}\right)}{m + M}$$

$$x_{CM} = \frac{m(2r + l)}{2(M + m)} \quad 2$$



Short Answer Type Questions-II

(3 marks each)

- Q. 1. Deduce the expression for position vector of centre of mass for n particle system?** [U] (DDE)
Ans. If a system contains n particles of masses $m_1, m_2, m_3, \dots, m_n$ whose position vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ respectively, then position vector of centre of mass

- $$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$
- Q. 2. A system consisting of two objects has a total momentum of $(18 \text{ kgm/s})\hat{i}$ and its center of mass has the velocity of $(3 \text{ m/s})\hat{i}$. One of the object has the mass 4 kg and velocity $(1.5 \text{ m/s})\hat{i}$. Find the mass and velocity of the other objects.**

Ans. Given,
 Total momentum = $(18 \text{ kgm/s})\hat{i}$,
 Velocity of center of mass = $(3 \text{ m/s})\hat{i}$,
 Mass of one object = 4 kg ,
 Velocity of this object = $(1.5 \text{ m/s})\hat{i}$
 Let m be the mass of other object and v be the velocity.

Now we know total momentum = Total mass \times velocity of center of mass
 $(18 \text{ kgm/s})\hat{i} = (m+4)(3 \text{ m/s})\hat{i}$
 or $m = 2 \text{ kg}$

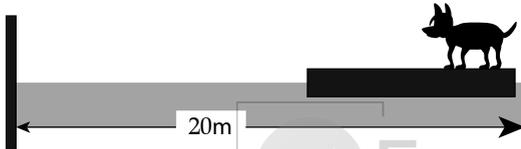
$$\text{Now, } v_{\text{cm}} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$\text{Or, } 3\hat{i} = (4 \times 1.5\hat{i} + 2v_2)/6$$

$$\text{So, } 18\hat{i} = 6\hat{i} + 2v_2$$

$$\text{Or, } v_2 = 6\hat{i} \text{ m/s} \quad 3$$

Q. 3. A dog of mass 10 kg is standing on a flat 10 m long boat so that it is 20 meters from the shore. It walks 8 m on the boat towards the shore and then stops. The mass of the boat is 40 kg and friction between the boat and the water surface is negligible. How far is the dog from the shore now?



Ans. We consider boat and dog as a system. Initially, centre of mass of the system is at rest. Since, no external force is acting on the system, hence centre of mass of the system will remain stationary. Let initially distance of the centre of mass of the boat from the shore be $x_{1\text{cm}}$ m.

$$\text{Then, } x_{1\text{cm}} = \left(\frac{40 \times x + 10 \times 20}{40 + 10} \right) \text{m}$$

Here, $x_{1\text{cm}}$ = distance of the C.M. of the system from the shore.

Since, dog moves towards the shore, for the centre of mass of the system to be at rest, the boat has to move away from the shore. Let distance moved by the boat be ' x '. Then,

$$x_{2\text{cm}} = \left(\frac{40(x+x') + 10(20-8+x')}{40+10} \right)$$

$$\text{As } x_{1\text{cm}} = x_{2\text{cm}}$$

$$\Rightarrow \left(\frac{40x + 200}{50} \right) = \left(\frac{40(x+x') + 10(12+x')}{50} \right)$$

$$\Rightarrow 50x' = 80 \Rightarrow x' = 1.6 \text{ m.}$$

Hence, distance of dog from the shore is $(20 - 8 + 1.6)\text{m} = 13.6 \text{ m}$ 3



Commonly Made Error

► Student fail to consider that the Centre of mass of the system will remain constant and hence could not obtain desired results.



Answering Tip

► Student should do enough practice in solving numerical problems related to centre of mass.

Long Answer Type Questions

(5 marks each)

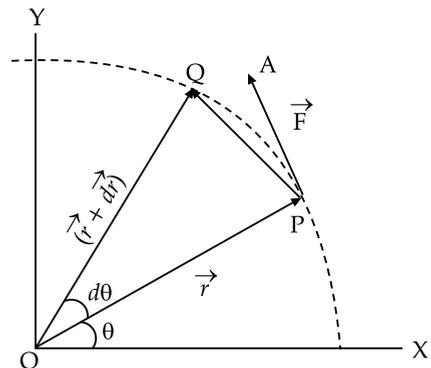
Q. 1. Explain the concept of angular momentum. Also derive an expression for angular momentum in cartesian co-ordinate. U

or

Find the relation between torque and angular momentum. [NCT 2010]

Ans. The concept of angular momentum :

- (i) The angular momentum of a particle with respect to a point gives an idea of the strength of its rotational tendency about that point. ½
- (ii) The magnitude of the angular momentum is defined in terms of mass and velocity of the particle and its distance from the reference point, i.e., $L = mvr$. ½
- (iii) The vector concept of the angular momentum is useful. Its direction is the axial direction given by right hand rule. The direction of \vec{L} is \perp to the plane containing \vec{r} and \vec{v} .
 Expression for angular momentum in cartesian co-ordinates :



Let us consider a particle of mass m rotating in x - y plane.

P be its position at any instant. Its position vector is \vec{r} .

$$\vec{OP} = \vec{r}$$

$$\angle XOP = \theta$$

In time dt the particle reaches Q under the action

of force $\vec{F} = \vec{PA}$

$$\vec{OQ} = \vec{r} + \vec{dr}$$

$$\angle POQ = d\theta$$

Now $\vec{OP} + \vec{PQ} = \vec{OQ}$

Or, $\vec{PQ} = \vec{OQ} - \vec{OP}$

Or, $\vec{PQ} = \vec{r} + \vec{dr} - \vec{r}$

$\therefore \vec{PQ} = \vec{dr}$

Work done to rotate the particle from P to Q is dW

$$dW = \vec{F} \cdot \vec{dr}$$

or, $dW = (F_x \hat{i} + F_y \hat{j})(dx \hat{i} + dy \hat{j})$

or, $dW = F_x dx + F_y dy$

or, $dW = F_x (-y d\theta) + F_y (x d\theta)$

or, $dW = x F_y d\theta - y F_x d\theta$

or, $dW = (x F_y - y F_x) d\theta$

$\therefore dW = \tau d\theta$

Where $\tau = (x F_y - y F_x) \dots(1) \frac{1}{2}$

$$\tau = F_x dx + F_y dy \dots(i)$$

According to Newton's 2nd law of motion

$$F_x = \frac{dp_x}{dt} = \frac{d}{dt}(mv_x) \\ = \frac{mdv_x}{dt} \dots \frac{1}{2}$$

$$F_y = \frac{dp_y}{dt} = \frac{d}{dt}(mv_y) \\ = \frac{mdv_y}{dt} \dots 1$$

Substituting in (1), we get

$$\tau = x \frac{mdv_y}{dt} - y \frac{mdv_x}{dt}$$

or $\tau = m \left[x \frac{dv_y}{dt} - y \frac{dv_x}{dt} \right] \dots(2) 1$

Now, Differentiating $(xv_y - yv_x)$,

$$\frac{d}{dt}(xv_y - yv_x) = x \frac{dv_y}{dt} + v_y \frac{dx}{dt} - y \frac{dv_x}{dt} - v_x \frac{dy}{dt} \\ = x \frac{dv_y}{dt} + v_y v_x - y \frac{dv_x}{dt} - v_x v_y \\ \left[\because \frac{dx}{dt} = v_x, \frac{dy}{dt} = v_y \right] \\ = x \frac{dv_y}{dt} - y \frac{dv_x}{dt} \dots(3)$$

Substituting (3) in (2)

we get, $\tau = m \frac{d}{dt}(xv_y - yv_x) \dots 1$

$$\tau = \frac{d}{dt}(xmv_y - ymv_x)$$

As $mv_y = p_y$

and $mv_x = p_x$

$$\tau = \frac{d}{dt}(xp_y - yp_x) \dots(4)$$

$$xp_y - yp_x = L$$

or $\tau = \frac{dL}{dt} \dots 1$

Q.2. Derive an expression for torque in polar coordinates. [MSE Chandigarh 2009]

Ans. Suppose the line of action of force F makes an angle α with x -axis

$$F_x = F \cos \alpha \dots(1)$$

$$F_y = F \sin \alpha \dots(2) 1$$

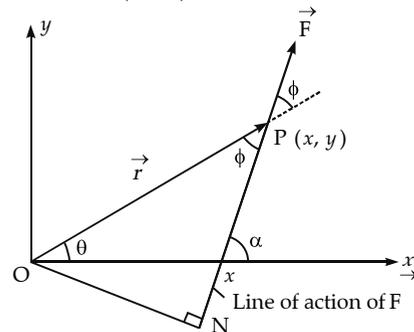
If x, y are the co-ordinates of the point P where $\vec{OP} = \vec{r}$ and $\angle XOP = \theta$.

Then $x = r \cos \theta$

$$y = r \sin \theta \dots(3) \frac{1}{2}$$

Substituting those values in $\tau = (x F_y - y F_x)$,

$$\tau = (r \cos \theta) F \sin \alpha - (r \sin \theta) (F \cos \alpha) \\ = rF [\sin \alpha \cos \theta - \cos \alpha \sin \theta] \dots \frac{1}{2} \\ \tau = rF \sin(\alpha - \theta) \dots(4) \frac{1}{2}$$



Let ϕ be the angle which the line of action of \vec{F} make, with the position vector $\vec{OP} = \vec{r}$

As clear from figure

$$\theta + \phi = \alpha$$

or $\phi = \alpha - \theta \dots(5) 1$

Putting in (4)

$$\tau = r F \sin \phi \dots(6) 1$$

Equation (6) is the expression for torque in polar co-ordinates. $\frac{1}{2}$

Topic-2 Moment of Inertia & Radius of Gyration



Revision Notes

- **Principle of Conservation of Angular Momentum** : According to this principle, when no external torque acts on a system of particles, then the total angular momentum of system always remains a constant. i.e.,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots + \vec{L}_n = \text{constant.}$$

- **Laws of Rotational Motion**

I Law : A body continues to be in a state of rest or in a state of uniform rotation about a given axis unless an external torque is applied on the body.

II Law : The rate of change of angular momentum of a body about a given axis is directly proportional to external torque applied on the body.

III Law : When a rigid body A exerts a torque on another rigid body B in contact with it, then the body B would exert an equal and opposite torque on the body A.

- **Moment of inertia** of a body about a given axis is the property by virtue of which, the body opposes any change in its state of rest or state of uniform rotation about that axis. For a single particle, moment of inertia (I) is equal to product of mass (m) of the particle and square of perpendicular distance (r) of the particle from the axis of rotation.,

i.e.,

$$I = mr^2.$$

Moment of inertia is a scalar quantity, whose unit is kgm^2 . It plays the same role in rotational motion as is played by the mass in linear motion.

- **Radius of gyration** of a body about a given axis is the distance (K) of a point from the given axis, where if whole mass of the body is concentrated, the body would have the same moment of inertia, as it has with the actual distribution of mass. E E K I T A A B
- **Kinetic energy of rotation** of a body is the energy possessed by body on account of its rotation about a given axis.

Scan to know more about this topic



Moment of inertia

Scan to know more about this topic



Radius of gyration



Key Formulae

- **Moment of Inertia—**

(i) **Circular ring** : (perpendicular to plane, at centre) $I = MR^2$

(ii) **Circular disc** : (perpendicular to plane, at centre) $I = \frac{1}{2}MR^2$

(iii) **Angular disc (or ring) —**

$$(R = \text{outer radius, } r = \text{inner radius}) I = \frac{1}{2}(R^2 + r^2)$$

(iv) **Thin rod** : (axis perpendicular to its length at mid point) $I = \frac{Ml^2}{12}$, Where l is the length of the rod.

(v) **Solid cylinder** : (along axis of cylinder) $I = \frac{1}{2}MR^2$

(vi) **Hollow cylinder** : $I = MR^2$
about its long axis of symmetry.

(vii) **Solid sphere** : (about its diameter.) $I = \frac{2}{5}MR^2$

(viii) **Hollow sphere (or thin spherical shell)** : (about its diameter) $I = \frac{2}{3}MR^2$

(ix) **Solid cylinder (or ring)** : (about central axis) $I = \frac{1}{2}MR^2$

(x) (a) Solid cylinder (about axis through its CM) : $I = M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$

(b) Hollow cylinder : $I = M\left(\frac{l^2}{12} + \frac{R^2}{2}\right)$

about an axis passing through C.G. & perpendicular to its own axis

(xi) Uniform rectangular lamina (or thin slab)— about an axis passing through C.G. & perpendicular to its plane.

$$I = M\left(\frac{l^2 + b^2}{12}\right)$$

Where, l = length, b = breadth

(xii) Elliptical disc :

$$I = \frac{M}{4}(a^2 + b^2)$$

about an axis passing through its C.G. & perpendicular to its plane.

(a = semi-major axis, b = semi-minor axis.)

(xiii) Uniform cone : about an axis joining the vertex to centre of its base.

$$I = \frac{3}{10}MR^2$$

(xiv) Triangular lamina—

$$I_1 = M \times h^2/6 \text{ (about the base as axis)}$$

$$I_2 = \frac{b^2}{6} \cdot M \text{ (about the height as axis)}$$

$$I_3 = \frac{Mb^2h^2}{6(b^2 + h^2)} \text{ (about the hypotenuse as axis)}$$

➤ Kinetic energy of rotation :

$$K.E. = \frac{1}{2}I\omega^2,$$

➤ Angular Momentum—

$$L = I\omega$$

➤ Relation between Angular Momentum & Torque :

$$\tau = I\alpha, \tau = dL/dt$$

➤ From Principle of Conservation of Angular momentum :

$$\frac{I_1}{I_2} = \frac{\tau_1}{\tau_2}$$

➤ Radius of Gyration—

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$$

Here, r_1, r_2, \dots, r_n = perpendicular distance of particles from axis of rotation

n = total no. of particles.



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is rotational analogue of mass of a body? \mathbb{R}

Ans. Rotational analogue of mass is moment of inertia. 1

Q. 2. What are the factors on which moment of inertia of a body depend? \mathbb{U}

Ans. Moment of inertia of a body depends on position and orientation of the axis of rotation. It also depends on the distribution of mass of the body about the given axis. 1

- Q. 3. Is radius of gyration of a body a constant quantity?** U
- Ans.** No, radius of gyration of a body depends on axis of rotation and also on distribution of mass of the body about this axis. 1
- Q. 4. How does the M.I. change with speed of rotation?** U

Ans. M.I. is not affected by speed of rotation of the body. 1

- Q. 5. The moment of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their angular momenta are equal. Which one has greater Kinetic Energy?** AI A (DDE)

Ans. $\because K = \frac{L^2}{2I}$ or, $K \propto \frac{1}{I} \therefore K_B > K_A$ 1



Short Answer Type Questions-I

(2 marks each)

- Q. 1. What is the turning effect of force called? On what factors does it depend?** U (DDE)
- Ans.** Turning effect of force is called torque. Factors on which it depends are
- (i) Magnitude of force,
(ii) Perpendicular distance of force vector from axis of rotation. 2
- Q. 2. Why the speed of whirl wind in a Tornado is alarmingly high?** U (DDE)
- Ans.** In whirl wind, air from wide regions get concentrated in a small space, so I is considerably less. Since, $I\omega = \text{constant}$ so ω is so high. 2
- Q. 3. There are two spheres of same mass and radius one is solid and the other is hollow. Which of them has a larger moment of inertia about its diameter?** A
- Ans.** The hollow sphere shall have greater M.I., as its entire mass is concentrated at the boundary of the sphere at maximum distance from the axis. 2
- Q. 4. Using the expression for power and K.E. of rotation motion, derive the relation $\tau = I\alpha$.** U
- Ans.** We know that power is given by

$$P = \tau\omega \quad \dots(1) \frac{1}{2}$$

Also we know that,

$$K.E. = \frac{1}{2} I\omega^2 \quad \dots(2) \frac{1}{2}$$

As

$P = \text{rate of change of energy}$
 $= \text{rate of change of K.E.}$

$$= \frac{d}{dt} \left(\frac{1}{2} I\omega^2 \right)$$

$$= \frac{1}{2} \times I \times 2\omega \frac{d\omega}{dt}$$

$$= I\omega\alpha \quad \left(\because \frac{d\omega}{dt} = \alpha \right) \dots(3)$$

\therefore From equation (1) and (3), we get

$$P = \tau\omega$$

Since

$$\tau = I\alpha$$

1

- Q. 5. What shall be the effect on the length of the day if the polar ice caps of earth melt?** A

Ans. Melting of polar ice caps will increase the radius of gyration and hence, M.I. also increase. In order to conserve angular momentum, the angular velocity ω shall decrease.

So, the length of the day $\left(T = \frac{2\pi}{\omega} \right)$ shall increase.

1+1



Long Answer Type Questions

(5 marks each)

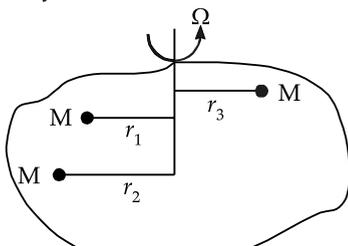
- Q. 1. Derive an expression for K.E. of rotation.**

AI U [MSE Chandigarh 2008; NCT 2008]

- Ans. Kinetic energy of rotation :**

K.E. of rotation of a body is the energy possessed by the body on account of its rotation about a given axis. 1/2

In figure, we have shown a rigid body rotating in xy plane about z -axis with a uniform angular velocity ω .



1/2

Let the body consists of particles of masses $m_1, m_2, m_3, \dots, m_n$ at distance $r_1, r_2, r_3, \dots, r_n$ respectively. Let the linear velocity of different particles are $v_1, v_2, v_3, \dots, v_n$. 1/2

$$v_1 = r_1\omega,$$

$$v_2 = r_2\omega,$$

$$v_3 = r_3\omega, \dots$$

1

K.E. of particles of mass m_1 is

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m (r_1\omega)^2 = \frac{1}{2} m_1 r_1^2 \omega^2 \quad \frac{1}{2}$$

Similarly, K.E. of other particles of the body are :

$$\frac{1}{2} m_2 r_2^2 \omega^2, \frac{1}{2} m_3 r_3^2 \omega^2 \dots$$

\therefore K.E. of rotation of the body

$$\begin{aligned}
 &= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 + \dots \\
 &= \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots) \omega^2 \\
 &= \frac{1}{2} \left(\sum_{i=1}^{i=n} m_i r_i^2 \right) \omega^2 \quad \dots(1) \\
 &= \frac{1}{2} I \omega^2
 \end{aligned}$$

i.e., K.E. of rotation = $\frac{1}{2} I \omega^2$

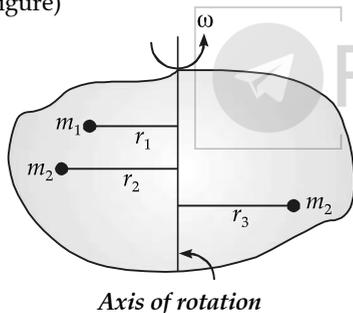
where, $I = \sum_{i=1}^{i=n} m_i r_i^2$ 1

Q. 2. Derive the relation between torque and moment of inertia. [A] [I] [U]

Ans. Relation between torque and Moment of inertia :

Consider a rigid body rotating about a given axis with a uniform angular acceleration α , under the action of a torque. $\frac{1}{2}$

Let the body consist of particles of masses $m_1, m_2, m_3, \dots, m_n$ at perpendicular distance $r_1, r_2, r_3, \dots, r_n$ respectively from the axis of rotation. (as shown in figure)



As the body is rigid, angular acceleration α of all the particles of the body is the same. However, linear accelerations of the particles depend on their distance from the axis. If $a_1, a_2, a_3, \dots, a_n$ are the respective linear accelerations of the particles, then,

$$a_1 = r_1 \alpha, a_2 = r_2 \alpha, a_3 = r_3 \alpha \dots \quad 1$$

Force on particle of mass m is

$$f_1 = m_1 a_1 = m_1 r_1 \alpha$$

Moment of this force about the axis of rotation

$$f_1 \times r_1 = (m_1 r_1 \alpha) \times r_1 = m_1 r_1^2 \alpha \quad 1$$

Similarly, moment of forces on other particles about the axis of rotation are $m_2 r_2^2 \alpha, m_3 r_3^2 \alpha, \dots, m_n r_n^2 \alpha$.

\therefore Torque acting on the body,

$$\tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \dots + m_n r_n^2 \alpha$$

$$\tau = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \alpha$$

So,
$$\tau = \left(\sum_{i=1}^{i=n} m_i r_i^2 \right) \alpha$$

$$\tau = I \alpha \quad 1$$

where $I = \sum_{i=1}^{i=n} m_i r_i^2$ = moment of inertia of the body about the given axis of rotation.

$$\vec{\tau} = I \vec{\alpha} \quad 1$$

Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. On which of the following factors moment of inertia of an object does not depend?

- (A) Axis of rotation
- (B) Angular velocity
- (C) Distribution of mass
- (D) Mass of an object

Ans. Option (B) is correct.

Q. 2. When consideration of moment of inertia of a body becomes important?

- (A) When the motion is linear
- (B) When the motion is rotational
- (C) When the motion is along a curved path
- (D) In all of the above cases

Ans. Option (B) is correct.

Q. 3. Which of the following have the same moment of inertia?

- (A) (i) Moment of inertia of a thin circular ring of radius R about an axis perpendicular to the plane, through the centre.
(ii) Moment of inertia of a hollow cylinder of radius R about the axis of the cylinder.
- (B) (i) Moment of inertia of a thin circular ring of radius R about an axis perpendicular to the plane, through the centre.
(ii) Moment of inertia of a solid cylinder of radius R about the axis of the cylinder.
- (C) (i) Moment of inertia of a thin circular ring of radius R about an axis perpendicular to the plane, through the centre.
(ii) Moment of inertia of a solid sphere of radius R about the diameter of the sphere.

- (D) (i) Moment of inertia of a solid cylinder of radius R about the axis of the cylinder.
(ii) Moment of inertia of a hollow cylinder of radius R about the axis of the cylinder.

Ans. Option (A) is correct.

Explanation: In both the cases moment of inertia is MR^2

Q. 4. For a vector product of two vectors \vec{a} and \vec{b}

which of the following statements is correct?

- (A) magnitude of resultant vector = $ab \sin \theta$ where, a and b are magnitudes of a and b and θ is the angle between the two vectors.
(B) Resultant vector is perpendicular to the plane containing \vec{a} and \vec{b} .
(C) if we take a right handed screw with its head lying in the plane of \vec{a} and \vec{b} and the screw perpendicular to this plane, and if we turn the head in the direction from \vec{a} to \vec{b} , then the tip of the screw advances in the direction of resultant vector.
(D) All of the above

Ans. Option (D) is correct.

Q. 5. Moment of inertia of a solid sphere about its diameter is $\frac{2}{5} Mr^2$. Its radius of gyration is

- (A) $\frac{2}{5} r$ (B) $\frac{5}{2} r$
(C) $\sqrt{\frac{2}{5}} r$ (D) $\sqrt{\frac{5}{2}} r$

Ans. Option (C) is correct.

Explanation: Moment of inertia = $I = Mk^2$
 k is the radius of gyration.

In this case,

$$\frac{2}{5} Mr^2 = Mk^2$$

or, $k^2 = \frac{2}{5} r^2$

$\therefore k = \sqrt{\frac{2}{5}} r$

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is true

Q. 1. Assertion (A): A person standing on a rotating platform suddenly stretches his arms, the platform slows down.

Reason (R): A person by stretching his arms increases the moment of inertia and decreases angular velocity.

Ans. Option (A) is correct.

Explanation: From law of conservation of angular momentum,

$$I_1 \omega_1 = I_2 \omega_2$$

When the person stretches his arms, his moment of inertia increases i.e., $I_2 > I_1$.

So, $\omega_2 < \omega_1$.

So, the assertion and reason both are true and the reason explains the assertion.

Q. 2. Assertion (A): Centre of mass of an object is a point where the whole mass of the body is assumed to be concentrated.

Reason (R): Centre of mass of an object cannot lie where there is no mass.

Ans. Option (C) is correct.

Explanation: Centre of mass of an object is a point where the whole mass of the body is assumed to be concentrated. The assertion is true.

It may lie even where there is no mass e.g., a ring, a curved banana, a hollow sphere etc.

So, the reason is false.

Q. 3. Assertion (A): The size and the shape of the rigid body remain unaffected under the effect of external forces.

Reason (R): The distance between two particles remains constant in a rigid body

Ans. Option (A) is correct.

Explanation: A rigid body is defined as a body with a definite and unchanging shape. Ideally external forces cannot deform a rigid body. Hence, the distances between all pairs of particles of such a body do not change. So, the assertion and reason both are true. The reason also explains the assertion.



NCERT Corner

Q.1. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

Ans. In all four cases (i),(ii),(iii),(iv), centre of mass is situated at their respective geometrical centres. No, it is not necessary that centre of mass of a

body lie inside it because in some cases eg., ring, hollow sphere, a hollow cylinder, etc. centre of mass lies outside.

Q.2. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 \AA ($1 \text{ \AA} = 10^{-10} \text{ m}$). Find the approximate location of the C.M. of the molecule, given that a chlorine atom is about

35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus ?

Ans. Let us choose the nucleus of the hydrogen atom as the origin for measuring distance.

Mass of hydrogen atom,

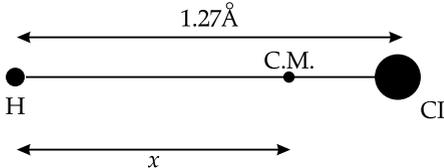
$$m_1 = 1 \text{ unit (say)}$$

Mass of chlorine atom,

$$m_2 = 35.5 \text{ unit (say)}$$

Now, $x_1 = 0$

and $x_2 = 1.27 \text{ \AA}$
 $= 1.27 \times 10^{-10} \text{ m}$



Distance of C.M. of HCl molecule from the origin is given by

$$x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{(1 \times 0) + (35.5 \times 1.27 \times 10^{-10})}{(1 + 35.5)}$$

$$= 1.235 \times 10^{-10} \text{ m}$$

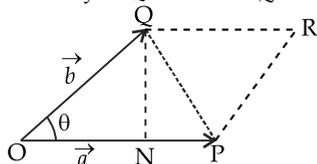
$$x = 1.235 \text{ \AA}$$

Q.3. A child sits stationary at one end of a long trolley moving uniformly with a speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of C.M. (Centre of Mass) of the (trolley + child) system ?

Ans. Trolley and the child make the system, speed of the C.M. of this system remains same irrespective of the movement of the child inside the trolley in any manner because C.M. is affected by external forces and not by the internal forces.

Q.4. Show that the area of the triangle contained between the vectors \vec{a} and \vec{b} is one-half of the magnitude of $\vec{a} \times \vec{b}$.

Ans. Let \vec{a} be represented by \vec{OP} and \vec{b} be represented by \vec{OQ} . Let $\angle POQ = \theta$



Completed the parallelogram OPRQ. Joined PQ. QN drawn perpendicular on OP.

In ΔQNO , $\sin \theta = \frac{QN}{OQ} = \frac{QN}{b}$

$$QN = b \sin \theta$$

Now, by definition,

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

$$= (OP)(QN)$$

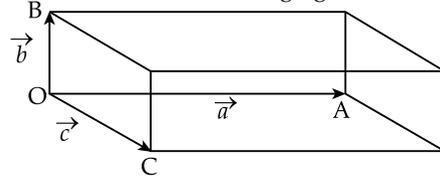
$$= \frac{2(OP)(QN)}{2}$$

$$= 2 \times \text{area of } \Delta OPQ$$

$$\therefore \text{Area of } \Delta OPQ = \frac{1}{2} |\vec{a} \times \vec{b}| \quad \text{Proved.}$$

Q.5. Show that $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors, \vec{a} , \vec{b} and \vec{c} .

Ans. A parallelepiped with origin O and sides a , b , and c is shown in the following figure.



Volume of the given parallelepiped = abc

$$\vec{OA} = \vec{a}$$

$$\vec{OB} = \vec{b}$$

$$\vec{OC} = \vec{c}$$

Let \hat{n} be a unit vector perpendicular to both b and c . Hence, \hat{n} and \vec{a} have the same direction.

$$\therefore \vec{b} \times \vec{c} = bc \sin \theta \hat{n}$$

$$= bc \sin 90^\circ n = bc \hat{n}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (bc \hat{n})$$

$$= abc \cos \theta \quad \because (|\hat{n}| = 1)$$

$$= abc \cos 0^\circ$$

$$= abc$$

$$= \text{Volume of the parallelepiped}$$

Q.6. Find the components along the x , y , z axes of the angular momentum of a particle, whose position vector is \vec{r} with components x , y , and z and momentum is \vec{p} with components p_x , p_y , p_z . Show that if the particle moves only in the x - y plane the angular momentum has only a z -component.

Ans. Angular momentum l of a particle-

$$\vec{l} = \vec{r} \times \vec{p} \quad \left[\begin{array}{l} \vec{r} = \text{position vector} \\ \vec{p} = \text{momentum} \end{array} \right]$$

$$\vec{r} = [x\hat{i} + y\hat{j} + z\hat{k}]$$

Where x , y , z are components of \vec{r} and

$$\vec{p} = [p_x\hat{i} + p_y\hat{j} + p_z\hat{k}]$$

$$\therefore \vec{l} = \vec{r} \times \vec{p}$$

$$= [x\hat{i} + y\hat{j} + z\hat{k}] \times [p_x\hat{i} + p_y\hat{j} + p_z\hat{k}]$$

$$(l_x\hat{i} + l_y\hat{j} + l_z\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= (yp_z - zp_y)\hat{i} + (zp_x - xp_z)\hat{j} + (xp_y - yp_x)\hat{k}$$

We conclude that-

$$l_x = yp_z - zp_y, \quad l_y = zp_x - xp_z, \quad l_z = xp_y - yp_x$$

If given particle moves only in x - y plane,
Then $z=0, p_z=0,$

hence, $l_x=0, l_y=0, l_z=(xp_y - yp_x)\hat{k}$

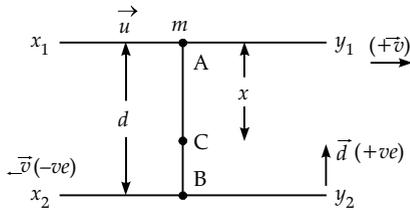
(which is only z -component of l)

Therefore, particle moves only in x - y plane, the angular momentum has only z -component.

Q.7. Two particles, each of mass m and speed v , travel in opposite directions along parallel lines separated by a distance d . Show that the angular momentum vector of the two particle system is the same whatever be the point about which the angular momentum is taken.

Ans. Angular momentum vector of the two particles system about any point A on x_1y_1 is

$$\vec{L}_A = \vec{d} \times (m\vec{v})$$



Similarly, angular momentum vector of the two particle system about any point B on x_2y_2 is

$$\vec{L}_B = \vec{d} \times (m\vec{v})$$

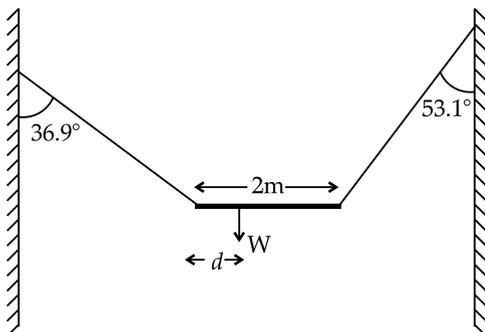
Let us consider any other point C on AB, where $AC = x$.

∴ Angular momentum vector of the two particle system about C at a distance x from B is

$$\begin{aligned} \vec{L}_C &= \vec{x} \times m\vec{v} - (d-x) \times (-m\vec{v}) \\ &= \vec{d} \times (m\vec{v}) \end{aligned}$$

So, $\vec{L}_A = \vec{L}_B = \vec{L}_C$ **Proved**

Q.8. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



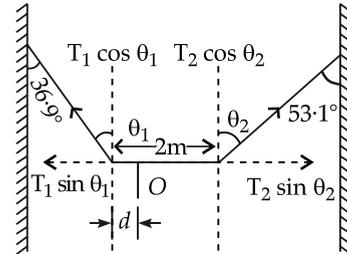
Ans. Given: $\theta_1 = 36.9^\circ, \theta_2 = 53.1^\circ$

Let T_1 & T_2 be tensions in two strings. For equilibrium condition,

$$T_1 \sin \theta_1 = T_2 \sin \theta_2$$

$$\frac{T_1}{T_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{\sin 53.1^\circ}{\sin 36.9^\circ}$$

$$\frac{T_1}{T_2} = \frac{0.7407}{0.5477} = 1.3523 \tag{1}$$



Let O be the position of centre of gravity of rod from left & at distance d .

For rotational equilibrium of rod about O, the moment of vertical forces must be equal and opposite :

$$T_1 \cos \theta_1 \times d = T_2 \cos \theta_2 (2 - d)$$

$$\text{or } \frac{T_1}{T_2} \times \frac{\cos \theta_1}{\cos \theta_2} = \frac{2 - d}{d}$$

From (1)

$$1.3317 \times \frac{\cos 36.9^\circ}{\cos 53.1^\circ} = \frac{2 - d}{d}$$

$$1.3317 \times \frac{0.7996}{0.6004} = \frac{2}{d} - 1$$

$$\frac{2}{d} = \frac{1.3317 \times 0.7996}{0.6004} + 1$$

$$d = 0.7211\text{m}$$

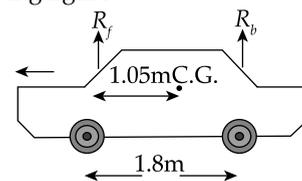
Q.9. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel. **AI**

Ans. Mass of the car, $m = 1800$ kg

Distance between the front and back axles, $d = 1.8$ m

Distance between the C.G. (centre of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure.



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$$\begin{aligned} R_f + R_b &= mg \\ &= 1800 \times 9.8 \\ &= 17640 \text{ N} \quad \dots(i) \end{aligned}$$

For rotational equilibrium, on taking the torque about the C.G., we have:

$$\begin{aligned} R_f(1.05) &= R_b(1.8-1.05) \\ R_f \times (1.05) &= R_b \times 0.75 \\ \frac{R_f}{R_b} &= \frac{0.75}{1.05} = \frac{5}{7} \\ \frac{R_b}{R_f} &= \frac{7}{5} \\ R_b &= 1.4R_f \quad \dots(ii) \end{aligned}$$

Solving equations (i) and (ii), we get
 $1.4R_f + R_f = 17640$

$$R_f = \frac{17640}{2.4} = 7350 \text{ N}$$

$\therefore R_b = 17640 - 7350 = 10290 \text{ N}$
Therefore, the force exerted on each front wheel
 $= \frac{7350}{2} = 3675 \text{ N}$, and

The force exerted on each back wheel
 $= \frac{10290}{2} = 5145 \text{ N}$

Q.10. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time? **[A1]**
[NCERT TB Q. No. 11]

Ans. Moment of inertia of the cylinder

$$I_1 = MR^2$$

and Moment of inertia of the sphere

$$I_2 = \frac{2}{5} MR^2$$

Angular acceleration of cylinder

$$\alpha_1 = \frac{\tau}{I_1} = \frac{\tau}{MR^2}$$

Angular acceleration of sphere

$$\alpha_2 = \frac{\tau}{I_2} = \frac{\tau}{\frac{2}{5}MR^2}$$

$$= 2.5 \frac{\tau}{MR^2}$$

$$= 2.5 \alpha_1$$

From $\omega = \omega_0 + \alpha t$, we find that for given ω_0 and t , $\omega_2 > \omega_1$ i.e., angular speed of solid sphere will be greater than angular speed of hollow cylinder.

Q.11. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s⁻¹. The radius of the cylinder is 0.25m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

[NCERT TB Q. No. 12]

Ans. As $(K.E.)_{\text{rot}} = \frac{1}{2} I\omega^2$

Here, $I = \frac{1}{2} MR^2 = \frac{20}{2} \times (0.25)^2$
 $= 0.625 \text{ kg-m}^2$
 $\omega = 100 \text{ rad s}^{-1}$,

we get $(K.E.)_{\text{rot}} = \frac{1}{2} \times 0.625 \times (100)^2$
 $= 3125 \text{ J}$

Angular momentum,

$$\begin{aligned} L &= I\omega \\ &= 0.625 \times 100 \\ &= 62.5 \text{ kgm}^2\text{s}^{-1}. \end{aligned}$$

Q.12. (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction.

(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

[NCERT TB Q. No. 13]

Ans. Given: $\omega_1 = 40 \text{ rev/min.}, I_2 = \frac{2}{5} I_1$

(a) As no external torque is acting on system

So, $L = \text{constant} = I\omega$

$$I_1\omega_1 = I_2\omega_2$$

$$\text{or } \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{5}{2} \times 40 = 100 \text{ rpm}$$

(b) $\frac{K.E_f}{K.E_i} = \frac{\frac{1}{2} I_2 \omega_2^2}{\frac{1}{2} I_1 \omega_1^2} = \left(\frac{I_2}{I_1} \right) \left(\frac{\omega_2}{\omega_1} \right)^2$
 $= \frac{2}{5} \times \left(\frac{100}{40} \right)^2$
 $= 2.5$

Therefore, $K.E_{\text{final}} = 2.5 K.E_{\text{initial}}$

This increase in K.E. is because of fact that child spends his internal energy in folding his hands which is converting into its kinetic energy.

Q.13. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping. **[NCERT TB Q. No. 14]**

Ans. Here, $M = 3 \text{ kg}$,

$$R = 40 \text{ cm} = 0.4 \text{ m.}$$

Moment of inertia of the hollow cylinder about its axis

$$\begin{aligned} I &= MR^2 \\ &= 3 \times (0.4)^2 \text{ kgm}^2 \\ &= 0.48 \text{ kg-m}^2 \end{aligned}$$

When force of 30 N is applied over the rope wound on the cylinder, the torque will act on the cylinder. It is given by

$$\tau = FR = 30 \times 0.4 = 12 \text{ Nm.}$$

If α is angular acceleration produced, then,

$$\tau = I\alpha$$

or
$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}.$$

The linear acceleration of the rope

$$a = R\alpha = 0.4 \times 25 = 10 \text{ ms}^{-2}$$

Q.14. To maintain a rotor at a uniform angular speed of 200 rad s^{-1} , an engine needs to transmit a torque of 180 Nm . What is the power required by the engine? Assume that the engine is 100% efficient.

[NCERT TB Q. No. 15]

Ans. Here, $\tau = 180 \text{ Nm}$, $\omega = 200 \text{ rads}^{-1}$.

As

$$P = \tau\omega$$

\therefore

$$P = 180 \times 200 = 36000 \text{ W} = 36 \text{ kW}$$

Hence, the power required by the engine is 36 kW.

Q.15. From a uniform disk of radius R , a circular section of radius $R/2$ is cut out. The centre of the hole is at $R/2$, from the centre of the original disc. Locate the centre of mass of the resulting flat body.

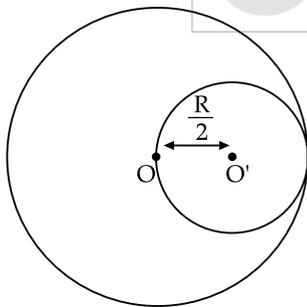
[NCERT TB Q. No. 16]

Ans. Suppose mass per unit area of the disc = m

\therefore Mass of portion removed from the disc,

$$M' = \pi (R/2)^2 \times m$$

$$= \frac{\pi R^2}{4} m = \frac{M}{4}, [M = \text{mass of disc}]$$



In figure, mass M is concentrated at O and mass M' is concentrated at O' , where $OO' = R/2$.

After the circular disc of mass M' is removed, the remaining portion can be considered as a system

of two masses M at O and $M' = \frac{M}{4}$ at O' . If x is the

distance of centre of mass of the remaining part,

then,
$$x = \frac{M \times 0 - M' \times R/2}{M + M'}$$

$$= \frac{-M}{4} \times \frac{R}{2}$$

$$= \frac{M - \frac{M}{4}}{4}$$

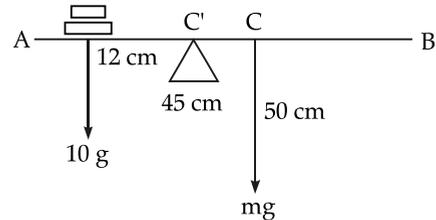
$$= \frac{-MR}{8} \times \frac{4}{3M}$$

$$= \frac{-R}{6}$$

Negative sign shows that P is to the left of O.

Q.16. A meter stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of meter stick? [NCERT TB Q. No. 17]

Ans. Let m be the mass of the stick concentrated at C, the 50 cm mark



For equilibrium

$$10 \text{ g}(45 - 12) = mg(50 - 45)$$

$$10g \times 33 = mg \times 5$$

$$m = \frac{10 \times 33}{5}$$

$$= 66 \text{ g}$$

Q. 17. The oxygen molecule has a mass of $5.30 \times 10^{-26} \text{ kg}$ and a moment of inertia of $1.94 \times 10^{-46} \text{ kg m}^2$ about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

[NCERT TB Q. No. 20]

Ans. Mass of an oxygen molecule, $m = 5.30 \times 10^{-26} \text{ kg}$

Moment of inertia, $I = 1.94 \times 10^{-46} \text{ kg m}^2$

Velocity of the oxygen molecule, $v = 500 \text{ m/s}$

The separation between the two atoms of the oxygen molecule = $2r$

Mass of each oxygen atom = $\frac{m}{2}$

Hence, moment of inertia I , is calculated as:

$$\left(\frac{m}{2}\right)r^2 + \left(\frac{m}{2}\right)r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}}$$

$$r = \sqrt{\frac{1.94 \times 10^{-46}}{5.30 \times 10^{-26}}} = 0.60 \times 10^{-10} \text{ m}$$

It is given that:

$$KE_{rot} = \frac{2}{3} KE_{trans}$$

$$\frac{1}{2} I \omega^2 = \frac{2}{3} \times \frac{1}{2} \times mv^2$$

$$mr^2 \omega^2 = \frac{2}{3} mv^2$$

$$\omega = \sqrt{\frac{2}{3}} \frac{v}{r}$$

$$= \sqrt{\frac{2}{3}} \times \frac{500}{0.6 \times 10^{-10}}$$

$$= 6.80 \times 10^{12} \text{ rad / s}$$



NCERT Exemplar (Only Important Questions)

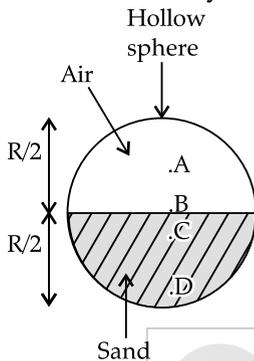
Multiple Choice Questions

Q. 1. For which of the following does the centre of mass lie outside the body?

- (A) A pencil
- (B) A shotput
- (C) A dice
- (D) A bangle

Ans. Option (D) is correct.

Q. 2. Which of the following point is the likely position of the centre of mass of the system shown in Fig. ?



- (A) A
- (C) C

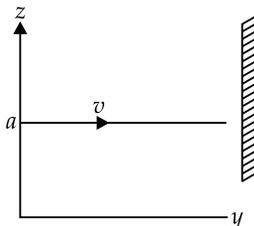
- (B) B
- (D) D

Ans. Option (C) is correct.

Q. 3. A particle of mass m is moving in yz -plane with a uniform velocity v with its trajectory running parallel to $+ve$ y -axis and intersecting z -axis at $z=a$. The change in its angular momentum about the origin as it bounces elastically from a wall at $y=\text{constant}$ is



- (A) $mva\hat{e}_x$
- (B) $2mva\hat{e}_x$
- (C) $ymv\hat{e}_x$
- (D) $2ymv\hat{e}_x$



Ans. Option (B) is correct.

Explanation: Initial velocity, $\vec{V}_i = v\hat{e}_y$

After reflection, final velocity $\vec{V}_f = -v\hat{e}_y$

The trajectory is at constant distance a on z -axis and as particle moves along y -axis, its y component changes

Now, position vector, $\vec{r} = y\hat{e}_y + a\hat{e}_z$

Hence, change in angular momentum-

$$\vec{r} \times m(\vec{V}_f - \vec{V}_i) = 2mva\hat{e}_x$$



Commonly Made Error

- Students fail to take into consideration the term 'bounces elastically' while solving the numerical leading to wrong results.



Answering Tip

- Student should read the question carefully before answering. Moreover, the negative sign of the final component of velocity should be kept in mind.

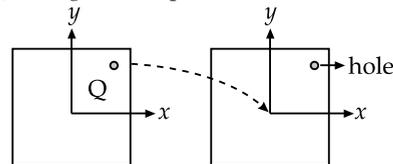
Q. 4. When a disc rotates with uniform angular velocity, which of the following is not true?

- (A) The sense of rotation remains same.
- (B) The orientation of the axis of rotation remains same.
- (C) The speed of rotation is non-zero and remains same.
- (D) The angular acceleration is non-zero and remains same.

Ans. Option (D) is correct.

Q. 5. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the z -axis is then

- (A) increased
- (B) decreased
- (C) the same
- (D) changed in unpredictable manner.



Ans. Option (B) is correct.

Explanation: The small cut piece comes closer to Z axis.

Q. 6. In problem 5, the CM of the plate is now in the following quadrant of x - y plane,

- (A) I
- (B) II
- (C) III
- (D) IV

Ans. Option (C) is correct.

Q. 7. A merry-go-round, made of a ring-like platform of radius R and mass M , is revolving with angular speed ω . A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round).

The speed of the round afterwards is

[NCERT Exemplar Q. No. 8]

- (A) 2ω (B) ω (C) $\frac{\omega}{2}$ (D) 0

Ans. Option (A) is correct.

Explanation: As no external torque acts on the system, angular momentum should be considered i.e., $I\omega = \text{constant}$ (1)

from equation (1), $I_1\omega_1 = I_2\omega_2$

[ω_1 & ω_2 are angular velocities before and after jumping]

$$\begin{aligned} \therefore I &= mr^2, m_1 = 2M, m_2 = M \\ r_1 &= r_2 = R, \therefore m_1 r_1^2 \omega_1 = m_2 r_2^2 \omega_2 \\ \text{or } 2MR^2 \omega_1 &= MR^2 \omega_2 \\ \text{or } \omega_2 &= 2\omega_1 \end{aligned}$$



Very Short Answer Type Questions

Q. 8. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

[A1] [NCERT Exemplar Q. No. 15]

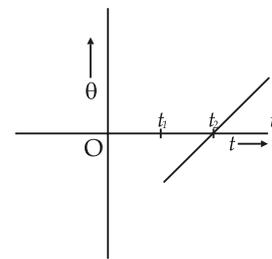
Ans. Moment of inertia of a given body, $I = \sum m_i r_i^2$ (sum of M.I. of each constituent particles).

All mass in a cylinder lies at distance R from axis of symmetry but most of the mass of a solid sphere lies at a smaller distance than R .

That's why solid sphere have smaller moment of inertia than a hollow cylinder.

Q. 9. The variation of angular position θ , of a point on a rotating rigid body, with time t is shown

in Fig. Is the body rotating clock-wise or anti-clockwise? [NCERT Exemplar Q. No. 16]



Ans. In graph, slope of $\theta - t$ is positive, which indicates anti-clockwise rotation.



Short Answer Type Questions-I

Q. 10. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?

[NCERT Exemplar Q. No. 19]

Ans. No, it is not necessarily zero.

Given: $\sum_i \vec{F}_i \neq 0$

Sum of torques about a certain point A,

$$\sum_i \vec{r}_i \times \vec{F}_i = 0$$

Sum of torques about any other point B,

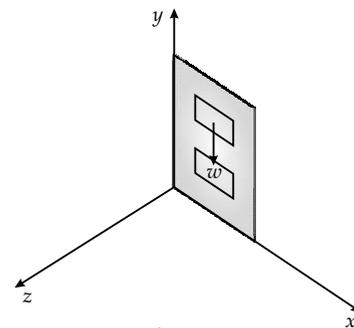
$$\sum_i (\vec{r}_i - \vec{a}) \times \vec{F}_i = \sum_i \vec{r}_i \times \vec{F}_i - \vec{a} \times \sum_i \vec{F}_i$$

\vec{a} and $\sum_i \vec{F}_i$ may or may not be zero.

\therefore torque is not necessarily zero about any other point.

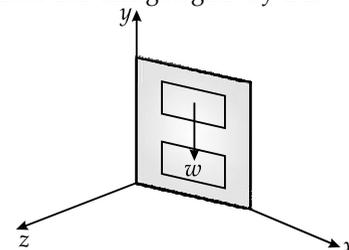
Q. 11. A door is hinged at one end and is free to rotate about a vertical axis. Does its weight cause any torque about this axis?

Give reason for your answer.



[NCERT Exemplar Q. No. 21]

Ans. Let us consider the given diagram, where weight of the door acts along negative y-axis.



A force can produce torque only along a direction normal to itself as $\tau = r \times f$. So, when the door is in the xy -plane, the torque produced can only be

$\pm z$ direction, never about an axis passing through y direction.

Hence, the weight will not produce any torque about y-axis.

Q. 12. $(n-1)$ equal point masses each of mass m are placed at the vertices of a regular n -polygon. The vacant vertex has a position vector \vec{a} with respect to the centre of the polygon. Find the position vector of centre of mass.

[NCERT Exemplar Q. No. 22]

Ans. The centre of mass of a regular n -polygon lies at its geometric centre.

Let \vec{b} is the position vector of the centre of mass of regular n -polygon.

From question, $(n-1)$ equal point masses each of mass m are placed at the $(n-1)$ vertices of a regular n -polygon,

$$\text{Then, } \vec{r}_{\text{cm}} = \frac{(n-1)m\vec{b} + m\vec{a}}{(n-1)m + m}$$

Now, mass m is placed at n th remaining vertex,

$$\text{then } \vec{r}_{\text{cm}} = \vec{0}$$

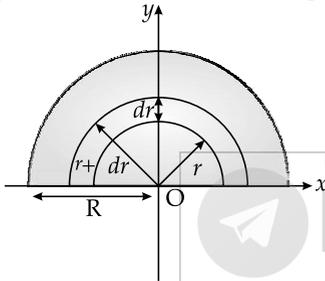
$$\frac{(n-1)m\vec{b} + m\vec{a}}{(n-1)m + m} = 0$$

$$\text{or } \vec{b} = \frac{-m\vec{a}}{(n-1)m} = \frac{-\vec{a}}{(n-1)}$$

Negative sign indicates that c.m lies other side from n th vertex geometrical centre of n -polygon.

👁 Long Answer Type Questions

Q. 13. Find the centre of mass of a uniform (a) half-disc, (b) quarter-disc. [NCERT Exemplar Q. No. 23]



Ans. Let the mass of half disc be M .

$$\text{Area of half disc} = \frac{\pi R^2}{2}$$

$$\text{Mass per unit area, } m = \frac{2M}{\pi R^2}$$

(a) The half disc can be divided into a large number of semi-circular strips having mass dm , thickness dr , and radii varies from $0 \rightarrow R$

Surface area of semicircular strip of radius r and thickness dr is

$$= \frac{\pi}{2} [(r+dr)^2 - r^2]$$

$$= \frac{\pi}{2} [r^2 + dr^2 + 2rdr - r^2]$$

Since, (dr^2) is very small,

$$\therefore = \frac{\pi}{2} (2r \times dr)$$

$$= \pi r dr$$

\therefore Mass of the strip,

$$dm = \pi r dr \times \frac{2M}{\pi R^2}$$

$$= \frac{2M}{R^2} r dr$$

Let (x, y) be the coordinates of c. m. of this strip,

$$\text{So, } (x, y) = \left(0, \frac{2r}{\pi}\right)$$

$$\text{Thus, } x = 0, y = \frac{2r}{\pi}$$

Let x_{cm} and y_{cm} be the co-ordinates of the centre of mass of semicircular strip. Then

$$x = x_{\text{cm}} = \frac{1}{M} \int_0^R x dm = \int_0^R 0 dm = 0$$

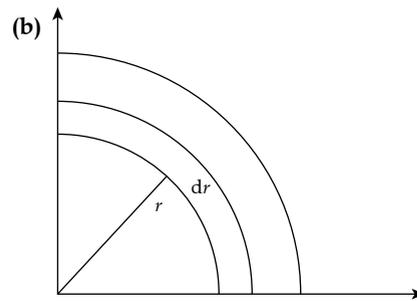
$$y = y_{\text{cm}} = \frac{1}{M} \int_0^R y dm$$

$$= \frac{1}{M} \int_0^R \frac{2r}{\pi} \times \frac{2M}{R^2} r dr = \frac{1}{M} \cdot \frac{4M}{\pi R^2} \int_0^R r^2 dr$$

$$= \frac{4}{\pi R^2} \left[\frac{r^3}{3} \right]_0^R = \frac{4}{3\pi R^2} R^3$$

$$\therefore y_{\text{cm}} = \frac{4R}{3\pi}$$

So, centre of mass of uniform half disc = $\left(0, \frac{4R}{3\pi}\right)$



$$\text{Mass per unit area of quarter disc} = \frac{M}{\frac{\pi R^2}{4}} = \frac{4M}{\pi R^2}$$

Using symmetry,

$$\text{For half disc, } y_{\text{cm}} = \frac{4R}{3\pi}$$

Similarly, for half disc, along x-axis centre of mass,

$$\text{at } x = \frac{4R}{3\pi}$$

So, the centre of mass of quarter disc is=
 $\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$



Commonly Made Error

- ▶ Student can not find the centre of mass of a rigid body.



Answering Tip

- ▶ Student should first find the mass of a small strip of the rigid body and then integrate over the whole body in order to calculate the centre of mass.



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

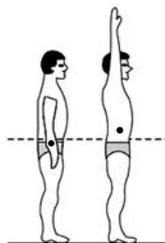
Read the following text and answer any 4 of the following questions based on the same:

- I. In a uniform gravitational field the centre of mass coincide with the centre of gravity. But these two points do not always coincide, however. For example, the Moon's centre of mass is very close to its geometric centre (it is not exact because the Moon is not a perfect uniform sphere), but its centre of gravity is slightly displaced towards Earth because of the stronger gravitational force on the Moon's near side facing the earth. If an object does not have a uniform weight distribution then the centre of mass will be closer to where most of the weight is located. For example, the centre of mass for a hammer is located close to where the head connects to the handle.

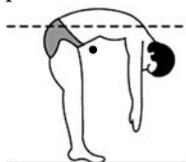
The centre of mass can be located at an empty point in space, such as the centre of a hollow ball. The centre of mass can even be completely outside of an object, such as for a donut or a curved banana.



Standing upright, an adult human's centre of gravity is located roughly at the center of their torso. The centre of gravity rises a few inches when with rising arms.

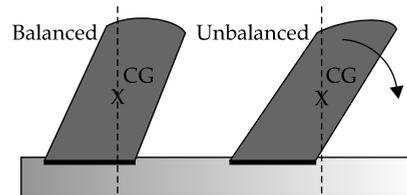


The center of gravity can even be at a point outside the body, such as when bent over in an inverted-U pose.



An object is in balanced position if its center of mass is above its base of support. For the two cylinders below, the left cylinder's center of gravity is above the base of support so the upward

support force from the base is aligned with the downward force of gravity. For the cylinder on the right the center of gravity is not above the base of support so these two forces cannot align and instead create a torque that rotates the object, tipping it over.



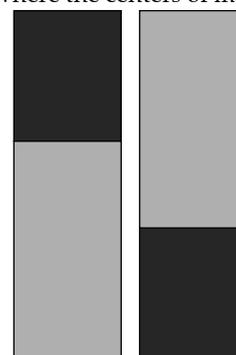
- Q. 1. Does the centre of mass not coincide with the centre of gravity of a body?

- (A) No. These two points do not coincide when the body is placed in high viscous medium
- (B) No. These two points do not coincide when the body is placed in a strong magnetic field
- (C) No. These two points do not coincide when the body is placed in a non-uniform gravitational field
- (D) Yes. These two points always coincide

Ans. Option (C) is correct.

Explanation: In a uniform gravitational field the centre of mass coincide with the center of gravity.

- Q. 2. Two similar blocks are shown below. The first block is top-heavy and the second block is bottom heavy. Where the centers of mass will be located?



(1)

(2)

- (A) At the centre of both the blocks
- (B) Above the geometrical centre for block 1 and below the geometrical centre for block 2
- (C) Above the geometrical centre for block 2 and below the geometrical centre for block 1
- (D) Above the geometrical centre for both the blocks

Ans. Option (B) is correct.

Explanation: If an object does not have a uniform weight distribution then the center of mass will be closer to where most of the weight is located.

Q. 3. Identical blocks are connected as shown. Where the center of mass is expected to be located?

- (A) At point 1 (B) At point 2
- (C) At point 3 (D) At point 4

Ans. Option (D) is correct.

Explanation: Since, all the 3 blocks are identical, the centre of mass will be below the support but above the line joining the blocks B and C.

Q. 4. The leaning tower of Pisa does not fall since

- (A) Its centre of mass is within its base
- (B) Its center of mass is at the foot of the perpendicular dropped from the top.
- (C) Its center of mass coincides with its centre of gravity.
- (D) Its base is well concreted

Ans. Option (A) is correct.

Explanation: The Leaning Tower of Pisa does not fall because its center of mass has been carefully kept within its base.

Today, despite the inclination of about 4° (reduced from over 5°), a vertical line drawn from the center of mass still falls inside the base.

Q. 5. Centre of gravity of Moon is slightly displaced towards Earth because

- (A) It is not a perfect uniform sphere
- (B) There is stronger gravitational force on the Moon's near side facing the earth
- (C) There is no atmosphere
- (D) Its orbit round the earth is elliptical

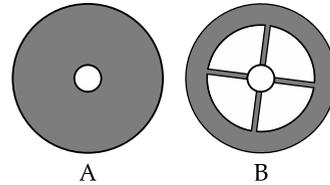
Ans. Option (B) is correct.

Explanation: The Moon's centre of mass is very close to its geometric centre (it is not exact because the Moon is not a perfect uniform sphere), but its centre of gravity is slightly displaced towards Earth due to uneven gravitational field. The gravitational force is stronger on the Moon's near side facing the earth.

II. Flywheel and sewing machine: There is a difference between inertia and moment of inertia of a body. Inertia depends on the mass of the body but the moment of inertia about an axis depends on the mass of the body and the distribution of its mass about the axis.

In the following figure, the masses of the two wheels are exactly equal but in the wheel (A) the

mass is uniformly distributed and in the wheel (B) most of the mass is situated at the rim. Both the wheels rotate about axis passing through the centre. It is noticed that while the two wheels are set in rotation and left, wheel (B) continues rotating for a longer time.



This means that the moment of inertia of wheel (B) is greater than the wheel (A). Also greater is the part of the mass of the body away from the axis of rotation, greater the moment of inertia of the body about the axis. Such a wheel is known as flywheel.

Consider a foot operated sewing machine. It has two wheels – one big and the other small. The wheels are connected by a rope. The bigger wheel acts as flywheel. The rope transfers the motion from this flywheel to the smaller wheel. Smaller wheel works as a pulley and also as a small flywheel.



We see even we stop supply of driving force to the bigger wheel it still continues to run for a short time because of its moment of inertia.

So, flywheel acts as an energy reservoir by storing and supplying mechanical energy when required. The kinetic energy stored in a flywheel is

$$E = \frac{1}{2} I \omega^2$$

where, I = moment of inertia and ω = angular velocity.

Q. 1. Which of the following statement is true?

- (A) Moment of inertia depends on the total mass and the distribution of mass from the axis of rotation
- (B) Inertia depends on the total mass and the distribution of mass from the axis of rotation
- (C) If the masses of two objects are equal then their moment of inertia are also equal
- (D) As mass of an object is distributed away from the axis of rotation, the moment of inertia decreases

Ans. Option (A) is correct.

Explanation: Inertia depends on the mass of the body but the moment of inertia about an axis depends on the mass of the body and the distribution of its mass about the axis.

$$\text{Moment of inertia} = \sum mr^2$$

Q. 2. How many flywheels are there in foot operated sewing machine ?

- (A) One (B) Two
- (C) Three (D) Zero

Ans. Option (B) is correct.

Explanation: In foot operated sewing machine there are two wheels – one big and the other small. The wheels are connected by a rope. The bigger wheel acts as flywheel. The rope transfers the motion from this flywheel to the smaller wheel. Smaller wheel works as a pulley and also as a small flywheel.

Q. 3. We see even we stop supply of driving force to the bigger wheel of foot operated sewing machine, it still continues to run for a short time. If the rim of this wheel is made thicker then

- (A) It will run for the same period when the driving force is stopped
- (B) It will run for shorter period when the driving force is stopped
- (C) It will run for longer period when the driving force is stopped
- (D) It will stop immediately when the driving force is stopped

Ans. Option (C) is correct.

Explanation: If the rim of the bigger wheel of foot operated sewing machine is made thicker then it will run for longer period when the driving force is stopped. This is due to the increase of its moment of inertia.

Q. 4. Energy stored in flywheel is

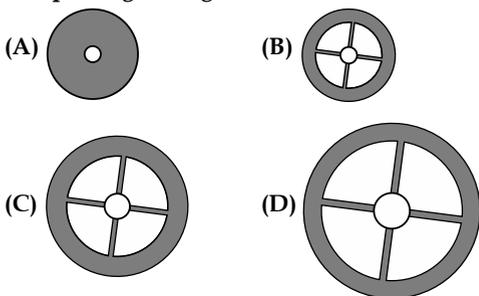
- (A) $\frac{1}{2} mv^2$
- (B) $\frac{1}{2} m\omega^2$
- (C) $\frac{1}{2} I\omega^2$
- (D) $\frac{1}{2} I\omega^2$

Ans. Option (D) is correct.

Explanation: Kinetic energy of an object moving in a straight line is $E = \frac{1}{2} mv^2$.

The kinetic energy of a spinning object is $E = \frac{1}{2} I\omega^2$.

Q. 5. Which one of the following wheel (having same mass) will have highest moment of inertia about axis passing through the centre?



Ans. Option (D) is correct.

Explanation: Mass of all the wheels are same. In wheel D, the mass is distributed furthest from the axis. Since moment of inertia = $\sum mr^2$, hence wheel D will have highest moment of inertia.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

A teacher gave a disc to the students and asked them to reduce its moment of inertia. There were many suggestions from the students.

Someone suggested, since moment of inertia depends on mass reduction of mass was the efficient method to reduce the moment of inertia.

Teacher wanted to know how the student wanted to reduce the mass. The student answered that by reducing the thickness of the disc or by giving it a shape of ring mass could be reduced.

Someone suggested, since moment of inertia of a disc depends on the square of the radius of the disc, reduction of radius is the most effective way to reduce its moment of inertia.

The teacher was not very happy. He was expecting some idea which does not require any modification of the object.

Q. 1. What is moment of inertia?

Ans. Moment of inertia is a physical quantity which resists the angular acceleration of a body.

Moment of inertia is the sum of the products of the mass of each particle in the body with the square of its distance from the axis of rotation.

Q. 2. What is the analogue of mass in rotational motion?

Ans. Moment of inertia is the analogue of mass in rotational motion.

Q. 3. Suggest a way to reduce the moment of inertia of the disc without reducing its mass or radius.

Ans. If the axis of rotation is considered along its diameter, then its moment of inertia is half of the moment of inertia about an axis perpendicular at the centre.

Q. 4. What is the radii of gyration when (i) the axis of rotation of the disc is along its diameter and (ii) the axis of rotation of the disc is perpendicular at the centre.

Ans. (i) when the axis of rotation of the disc is along its diameter, then

$$\text{Moment of inertia} = \frac{MR^2}{4}$$

$$\therefore k = \frac{R}{2}$$

(ii) when the axis of rotation of the disc is perpendicular at the centre, then

$$\text{Moment of inertia} = \frac{MR^2}{2}$$

$$\therefore k = \frac{R}{\sqrt{2}}$$

Q. 5. What will be the change in position of centre of mass if the disc is given a shape of a ring?

Ans. There will be no change in position.

UNIT-VI GRAVITATION

Study Time
Maximum Time: 2:45 hours
Maximum Questions: 85

CHAPTER

7

GRAVITATION



Syllabus

Kepler's laws of planetary motion, universal law of gravitation; Acceleration due to gravity and its variation with altitude and depth; Gravitational potential energy and gravitational potential, escape velocity, orbital velocity of a satellite.

Topic-1

Kepler's Laws, Universal Law of Gravitation, Acceleration Due to Gravity

TOPIC - 1

Kepler's Laws, Universal Law of Gravitation, Acceleration Due to Gravity

.... P. 146

TOPIC - 2

Gravitational Potential Energy & Satellites

.... P. 151



Revision Notes

➤ Kepler's Laws of Planetary Motion :

- (a) **Kepler's I Law (Law of Orbits)** : Each planet revolves around the Sun in an elliptical orbit. The Sun is situated at one foci of the ellipse.
- (b) **Kepler's II Law (Law of Areas)** : The position vector of the planet from the Sun sweeps out equal area in equal interval of time. That is the areal velocity of the planet around the Sun is constant.
- (c) **Kepler's III Law (Law of Periods)** : The square of the time period of any planet about the Sun is proportional to the cube of the semi-major axis of the elliptical orbit.

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Universal Law of Gravitation : It states that every body in universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them and its direction is along line joining the two masses.

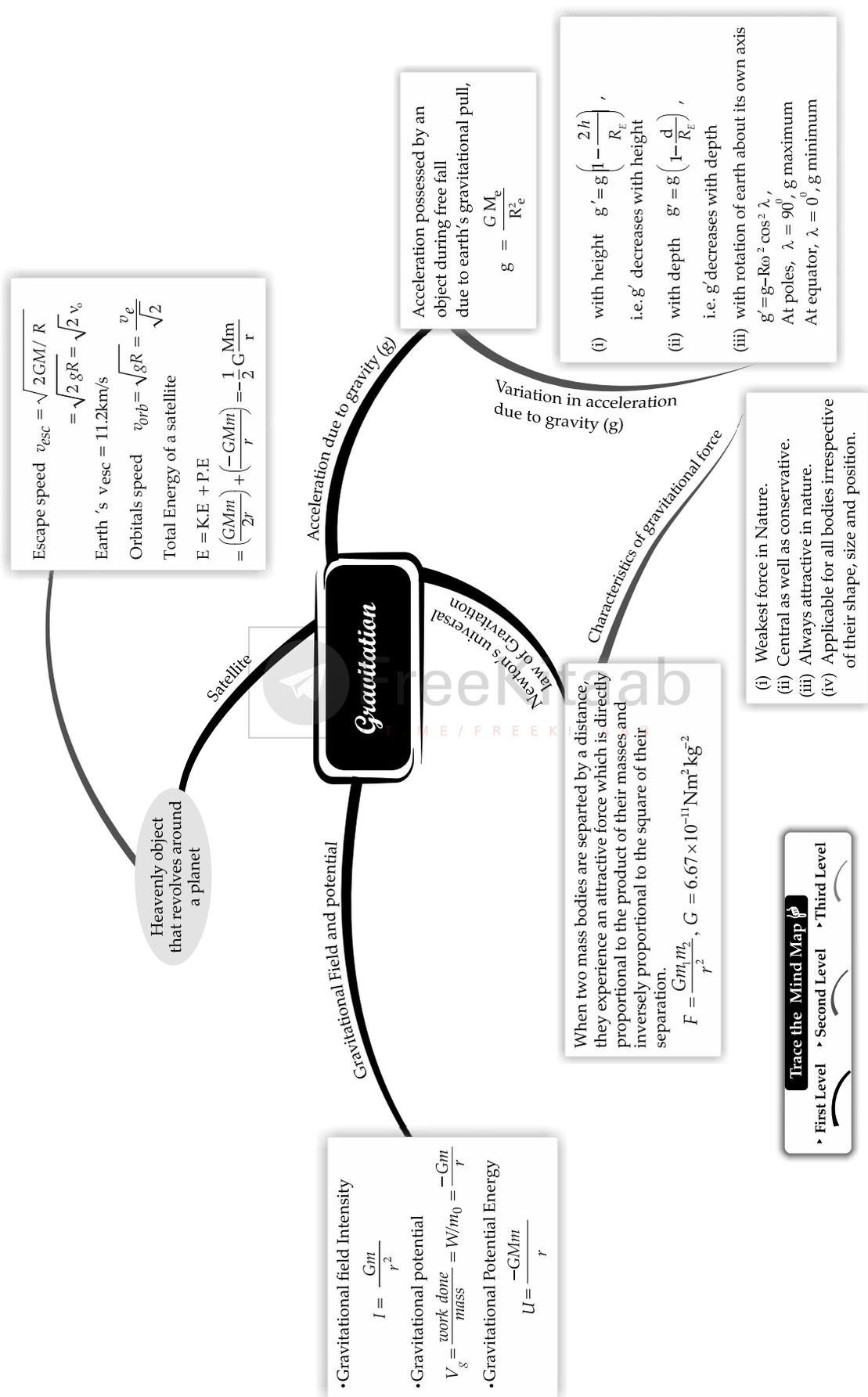
$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = \frac{G m_1 m_2}{r^2}$$

- **Gravitational constant (G)** : It is defined as the force of attraction acting between two bodies each of unit mass, whose centres are placed at unit distance apart. The value of G is constant throughout the universe. It is a scalar quantity. The dimensional formula of $G = [M^{-1}L^3T^{-2}]$. The value of $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.
The value of G being too small, we do not experience gravitational force in our daily life.
- **Gravity** : It is the force of attraction exerted by Earth towards its centre on a body lying on or near the surface of the Earth.

Scan to know
more about
this topic



Kepler's law



Heavenly object that revolves around a planet

Satellite

Escape speed $v_{esc} = \sqrt{2GM/R}$
 $= \sqrt{2gR} = \sqrt{2} v_e$
 Earth's $v_{esc} = 11.2 \text{ km/s}$

Orbitals speed $v_{orb} = \sqrt{gR} = \frac{v_e}{\sqrt{2}}$

Total Energy of a satellite
 $E = \text{K.E.} + \text{P.E.}$
 $= \left(\frac{GMm}{2r}\right) + \left(\frac{-GMm}{r}\right) = -\frac{1}{2} G \frac{Mm}{r}$

• Gravitational field Intensity
 $I = \frac{Gm}{r^2}$

• Gravitational potential
 $V_g = \frac{\text{work done}}{\text{mass}} = W/m_0 = \frac{-Gm}{r}$

• Gravitational Potential Energy
 $U = \frac{-GMm}{r}$

Newton's universal law of Gravitation

When two mass bodies are separated by a distance, they experience an attractive force which is directly proportional to the product of their masses and inversely proportional to the square of their separation.

$$F = \frac{Gm_1 m_2}{r^2}, G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

Acceleration due to gravity (g)

Acceleration possessed by an object during free fall due to earth's gravitational pull,

$$g = \frac{GM_e}{R_e^2}$$

Variation in acceleration due to gravity (g)

(i) with height $g' = g \left(1 - \frac{2h}{R_e}\right)$,
 i.e. g' decreases with height

(ii) with depth $g' = g \left(1 - \frac{d}{R_e}\right)$,
 i.e. g' decreases with depth

(iii) with rotation of earth about its own axis
 $g' = g - R \omega^2 \cos^2 \lambda$,
 At poles, $\lambda = 90^\circ$, g maximum
 At equator, $\lambda = 0^\circ$, g minimum

Characteristics of gravitational force

(i) Weakest force in Nature.
 (ii) Central as well as conservative.
 (iii) Always attractive in nature.
 (iv) Applicable for all bodies irrespective of their shape, size and position.

Trace the Mind Map

► First Level ► Second Level ► Third Level

Gravity is the measure of weight of the body.

The weight of the body = mass (m) \times acceleration due to gravity (g) = mg .

The unit of weight of the body will be the same as those of force.

Gravity is a vector quantity. It is always directed towards the centre of Earth. Gravity holds the atmosphere around the Earth.

- **Acceleration due to gravity (g)** : It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone.

Acceleration due to gravity is a vector quantity. It is directed towards the centre of Earth.

The unit of g is ms^{-2} and its dimensional formula is $[M^0L^1T^{-2}]$.

(a) The acceleration due to gravity does not depend upon (a) the mass of body, (b) shape or size of the body.

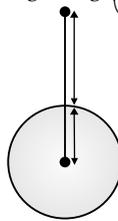
- **Variation of acceleration due to gravity.**

(a) **Effect of altitude :**

$$g' = g \left(1 - \frac{2h}{R} \right)$$

(b) **Effect of depth :**

$$g' = g \left(1 - \frac{d}{R} \right), d = \text{depth below Earth surface}$$



(c) **Rotation of Earth :**

$$g = g' + R\omega^2 \cos^2 \lambda$$

or

$$g' = g - R\omega^2 \cos^2 \lambda$$

where, ω is the angular velocity of rotation of Earth about its polar axis and λ is the latitude of a place.

(i) At the equator,

$$\lambda = 0^\circ,$$

so

$$g' = g - R\omega^2 \cos^2 0^\circ = g - R\omega^2$$

(ii) At the poles,

$$\lambda = 90^\circ,$$

so

$$g' = g - R\omega^2 \cos^2 90^\circ = g$$

Hence, the value of acceleration due to gravity increases from equator to pole due to rotation of Earth. It means the value of g increases with latitude.

(iii) When the Earth stops rotating about its own axis, there will be no change in the value of g on the poles, but there will be increase in the value of g by about 0.35% at the equator.

(d) **Shape of the Earth.** Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator. The polar radius of Earth is smaller than its equatorial radius by 21 km. As $g = GM/R^2$, so $g \propto 1/R^2$. It means the value of acceleration due to gravity increases as we go from equator to pole.



Key Words

- **Areal velocity** may be defined as the area swept by the radius vector in unit time.
- **Cavendish method** determines the value of G .
- **Geodesic** is the shortest distance between two points on Earth.
- **Aphelion** is the nearest point of planet from Sun.
- **Perihelion** is the farthest point of planet from Sun.



Key Formulae

- **Kepler's Law :**

$$T^2 = kr^3$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

Here, T = Time period and r = Semi major axis.

➤ **Newton's Gravitational Law :**

$$F = \frac{Gm_1m_2}{r^2}$$

➤ **Relation between g & G**

$$g = \frac{GM}{R^2}$$



Mnemonics

Concept: Kepler's laws of planetary motion:

Mnemonics: Take **e**ssential **f**oods **e**veryday **2/3** times.

Interpretation:

e - elliptical

f - focus

1st law: Planet moves in **e**lliptical orbit with sun

at one of the **foci**.

e equal - 2nd law: A planet covers the **e**qual area of space in **e**qual interval of time.

2/3 - 2/3

t - time period

3rd law: Semi-major axis of the orbit is proportional to **(time period)^{2/3}**



Very Short Answer Type Questions

(1 mark each)

Q. 1. Why is G called universal constant ? [R]

Ans. G is called a universal constant because its value is same everywhere. 1

Q. 2. Why does a rubber ball bounce higher on hills than in plains ? [U]

Ans. Acceleration due to gravity decreases with height so on hills g is less than plains. The ball bounces higher on hills because of this reason. 1



Answering Tip

➤ The value of acceleration due to gravity varies at a height ' h ', depth ' d ', on equators and on poles.

Q. 3. What is the variation in g at pole when Earth is at rest and rotating ? [U] [AI]

Ans. It remains unchanged. 1

Q. 4. Compare variation in g at pole and equator due to rotation of Earth ? [U]

Ans. Value of g is maximum at pole whereas minimum at equator. Effect of rotation of Earth on g is almost nil on poles and maximum at equator. 1

Q. 5. Mention one difference between g and G . [U]

Ans. The value of ' G ' remains the same throughout the universe while the value of ' g ' varies from place to place. 1



Commonly Made Error

➤ Student can not relate the different rate of bouncing of ball on hills and in plains to acceleration due to gravity.

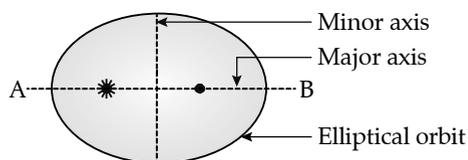


Short Answer Type Questions-I

(2 marks each)

Q. 1. What is Kepler's heliocentric theory ? [R]

Ans. According to this theory, all planets revolve around the Sun in elliptical orbits with Sun as one of the foci. 1



Q. 2. State Kepler's law of Planetary motion. Name the physical quantities which remain constant during the planetary motion, point without acceleration. [R] [NCT 2011]

Ans. Refer to Topic 1 'Revision Notes'.

(i) Mass

(ii) Angular Momentum

(iii) Total Energy 1 + 1



Commonly Made Error

➤ Student can not list physical quantities which remain constant during the planetary motion.



Answering Tip

➤ The clear understanding of Kepler's II law provide necessary backdrop for answering this part.

Q. 3. (i) According to Kepler's second law, the radius vector to a planet from the Sun sweeps out equal areas in equal interval of time. The

law is consequence of which conservation law ?

(ii) State Kepler's third law. [U] (NCT 2009)

Ans. (i) It is consequence of law of conservation of angular momentum. 1

(ii) Refer to Topic 1 'Revision Notes'. 1

Q. 4. Gravitational force is a weak force but still it is considered the most important force. Why ? [U]

Ans. Gravitational force plays an important role for initiating the birth of stars, for controlling the

entire structure of the universe and evolution of the universe. It helped to explain many natural phenomena. 2

Q. 5. Earth is continuously pulling Moon towards its centre. Why does not Moon fall on to Earth ? [U]

Ans. It is so because the gravitational attraction of Earth provides the necessary centripetal force to the Moon for its orbital motion around the Earth. Due to which the Moon is revolving around the Earth. 2

Short Answer Type Questions-II

(3 marks each)

Q. 1. What would happen if gravity suddenly disappear ? [U]

Ans. If gravity suddenly disappears,

- (i) All bodies will lose their weights.
- (ii) We shall be thrown away from the surface of Earth due to centrifugal force.
- (iii) The motion of planets around the Sun will cease because centripetal force shall not be provided.
- (iv) Motion of the satellite around Earth will also be not possible as no centripetal force will be provided. (1 mark each for any 3 points) 3



Answering Tip

- ▶ Student should familiarize themselves with the importance of the phenomenon of gravity.

Q. 2. We cannot move even a finger without disturbing all the stars. Explain. [U]

Ans. According to Newton's law of gravitation, everybody in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. When we move our finger, the distance of the objects with respect to finger changes, disturbing the entire universe including stars. 3



Commonly Made Error

- ▶ Student can not enlist the consequences of disappearance of gravity.

Long Answer Type Questions

(5 marks each)

Q. 1. Discuss the variation of g with height and depth. [U] [MSE Chandigarh 2009]

Ans. Refer to Topic 1 'Revision Notes'. 5

Q. 2. What are the main features of gravitational force ? [U]

Ans. Following are the main features of gravitational force :

- (i) It is always an attractive force. $\frac{1}{2}$
- (ii) It is independent of the medium between the particles. $\frac{1}{2}$
- (iii) It holds good over a wide range of distances (i.e., from interplanetary distances to interatomic distances). $\frac{1}{2}$
- (iv) It is an action-reaction pair, i.e., the force of attraction exerted by body A on body B is equal to the force of attraction exerted by body B on

body A. However, the acceleration of the two bodies will not be equal. 1

(v) The gravitational force between two particles is independent of presence or absence of other particles. $\frac{1}{2}$

(vi) The total gravitational force on one particle due to number of particles is the resultant of forces of attraction exerted on the given particle due to individual particles, i.e.,

$$F = F_1 + F_2 + F_3 + \dots \quad 1$$

It proves that the principle of superposition is valid.

(vii) It expresses, the force between point masses. $\frac{1}{2}$

(viii) It is a conservative force, i.e., the work done in moving a particle once around a closed path under the action of gravitational force is zero. $\frac{1}{2}$

Topic-2 Gravitational Potential Energy & Satellites



Revision Notes

- **Gravitational potential energy (U)** : The amount of work done in bringing a body from infinity to a point in gravitational field.

$$U = \frac{GMm}{r}$$

- **Escape Velocity** : The minimum velocity with which a body must be projected up in the space, so as to enable it to just overcome the gravitational pull.

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}. \quad \text{as } \left[g = \frac{GM}{R^2} \right]$$

Scan to know more about this topic



Gravitational potential Escape speed

- **Satellite** : A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.
 - Natural satellites** : All those satellites were made by nature. e.g., Jupiter–16 Moons. Saturn–18 Moons.
 - Artificial satellites** : All man-made satellites e.g., Aryabhata, etc.
- Gravitational forces are **central forces** as they act along the line joining the centres of the two bodies. The gravitational forces are **conservative forces**.



Key Words

- **Gravitational field** is the space around a material body in which its gravitational pull can be experienced.
- **Gravitational field intensity** of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.
- **Gravitational potential** at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.
- **Mass** of a body is the quantity of matter possessed by body.
- **Inertial mass of a body** is equal to the magnitude of external force required to produce unit acceleration in the body.
- **Gravitational mass** of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.
- **Centre of Gravity (C.G.)** of a body placed in the gravitational field is that point where the net gravitational force of the field acts.



Key Formulae

- **Gravitational Intensity** : $I = \frac{F}{m_0} = \frac{GM}{R^2} = g$

- **Gravitational Potential** : $V = \frac{W}{m_0}$

$$V = -\frac{GM}{r}$$

- **Gravitational Potential Energy** : $U = \frac{-GMm}{r}$

- **Satellite** :

(a) Orbital speed : $v = R\sqrt{\frac{g}{R+h}} = \sqrt{\frac{GM}{R+h}}$

(b) Time period of revolution : $T = \frac{2\pi}{R} \sqrt{\frac{(R+h)^3}{g}}$

(c) Height of satellite : $h = \left(\frac{T^2 R^2 g}{4\pi^2} \right) - R$

➤ **Escape speed**

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \quad \left[g = \frac{GM}{R^2} \right]$$



Very Short Answer Type Questions

(1 mark each)

Q. 1. What will be time period for a satellite very close to the surface of Earth ? U

Ans. Time period = $2\pi \sqrt{\frac{R_E}{g}}$
 $= 2\pi \sqrt{\frac{6400 \times 10^3}{9.8}} \text{ s } (\because R_E = 6400 \text{ km})$
 $= 85 \text{ minutes}$ 1

Q. 2. What is the gain in potential energy of an object of m raised from the surface of Earth to a height R ? (Acceleration due to gravity at the surface of Earth = g , radius of Earth = R). U

Ans. $\Delta U = U_2 - U_1$
 $= -\frac{GMm}{2R} - \left(-\frac{GMm}{R} \right)$
 $= \frac{GMm}{R} - \frac{GMm}{2R}$
 $= \frac{GMm}{2R} = \frac{1}{2} mgR$ 1

Q. 3. How does the average density of Earth depends on g ? U

Ans. $g = \frac{GM}{R^2}$

$$= \frac{G \times \frac{4}{3} \pi R^3 \rho}{R^2}$$

$\therefore g \propto \rho(\text{Density})$ 1

Q. 4. What is the relation between orbital and escape velocity ? R A I

Ans. $v_e = \sqrt{2} v_o$ 1

Q. 5. The escape velocity on Earth is 11.2 km s^{-1} . What is its value for a planet having double the radius and 8 times the mass of the Earth ? A

Ans. Escape velocity of earth

$$= \sqrt{\left(\frac{2GM}{R} \right)}$$

$$= 11.2 \text{ km/s}$$

Escape velocity of other planet

$$= \sqrt{\left(\frac{2G \times 8M}{2R} \right)}$$

$$= 2 \times \sqrt{\left(\frac{2GM}{R} \right)}$$

$$= 2 \times 11.2$$

$$= 22.4 \text{ km/s}$$
 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. Weight of a body should be greater in day or night. Why ? U

Ans. It should be greater in night. In the day time the body is pulled by Earth and Sun in two opposite directions. This will result into decrease in weight. 1

During night time the Earth and the Sun pull the body in the same direction so the weight will increase in night. 1



Answering Tip

➤ Student should familiarize themselves that weight of a body is due to the gravitational pull of both Sun and Earth.

Q. 2. What will happen if a satellite stops orbiting ? U

Ans. When a satellite stops orbiting, its kinetic energy becomes zero. This energy is converted into potential energy of the satellite. It then, starts falling towards the Earth while crossing the atmosphere with great speed and it may even burn. 2



Commonly Made Error

➤ Student can not reason out the difference in weight in day and during night.



Commonly Made Error

- ▶ Student can not comprehend the consequences of the situation 'if a satellite stops orbiting'.



Answering Tip

- ▶ Student should keep in mind that the energy remains conserved and if one form of energy is lost, it gets converted to other form.

Q. 3. Will 1 kg sugar be more at poles or at the equator ?

A A I

Ans. The value of g is larger at the poles than at the equator. If the sugar is weighed in a common balance then there will be no difference. If it is weighed by a spring balance, calibrated at the equator, then 1 kg of sugar will have a lesser amount at poles. **2**

Q. 4. Why are space rockets usually launched from west to east in the equatorial line ?

U

Ans. We know that Earth revolves from west to east about its polar axis. Therefore, all the particles

on the Earth have velocity from the west to east. This velocity is maximum in the equatorial line, as $v = R\omega$, where R is the radius of Earth and ω is the angular velocity of revolution of Earth about its polar axis. **1**

When a rocket is launched from west to east in equatorial plane, the maximum linear velocity is added to the launching velocity of the rocket, due to which launching becomes easier. **1**

Q. 5. Two satellites A and B go around a planet in circular orbits having radius $4R$ and R respectively. If the speed of the satellite A is $3v$, find the speed of the satellite B. **A**

Ans. As, $v_o = \sqrt{\frac{GM}{R}}$

so, For A $3v = \sqrt{\frac{GM}{4R}} \quad \dots(1) \frac{1}{2}$

and For B $v' = \sqrt{\frac{GM}{R}} \quad \dots(2) \frac{1}{2}$

Dividing (2) by (1) we get,

$\therefore \frac{v'}{3v} = 2 \quad \mathbf{1}$

or $v' = 6v$



Short Answer Type Questions-II

(3 marks each)

Q. 1. What is binding energy of a satellite ? **R**

Ans. The minimum energy required to free a satellite from the gravitational attraction is called binding energy. Binding energy is the negative value of total energy of satellite. Let a satellite of mass m be revolving around Earth of mass M and radius R . **1**

\therefore Total energy of satellite

$$= P.E. + K.E.$$

$$= -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{R} + \frac{mGM}{2R}$$

$$= -\frac{GMm}{2R} \quad \mathbf{1}$$

\therefore Binding energy of satellite

$$= -[\text{total energy of satellite}]$$

$$= \frac{GMm}{2R} \quad \mathbf{1}$$



Answering Tip

- ▶ Clear understanding and relation of the terms binding energy and total energy should be there.

Q. 2. What are the conditions under which a rocket fired from the Earth, launches an artificial satellite of Earth ? **U**

Ans. Following are the basic conditions :

- The rocket must take the satellite to a suitable height above surface of Earth. **1**
- From the desired height, the satellite must be projected with a suitable velocity, called the orbital velocity. **1**
- In the orbital path of satellite, the air resistance should be negligible so that its velocity does not decrease and it does not burn due to the heat produced. **1**

Q. 3. Derive an expression for the gravitational potential energy above the surface of Earth.

U [NCT 2008, 10; KVS 2008; MSE Chandigarh 2009]

Ans. Let the body of mass m be taken at height h above the surface of Earth. At any instant of time t it reaches at a distance x from the centre of Earth. The work done in raising through dx is.



Commonly Made Error

- ▶ Students fail to mention that binding energy is negative of the total energy of the satellite.

$$dW = \frac{GMm}{x^2} \cdot dx$$

$$= \Delta(P.E.) \quad 1$$

Hence, the work done in taking the body from surface of the Earth ($x = R$) to a height h ($x = R + h$) is

$$P.E. = W$$

$$= \int_R^{R+h} \frac{GMm}{x^2} dx$$

$$= GMm \int_R^{R+h} \frac{1}{x^2} dx$$

$$= -GMm \left[\frac{1}{x} \right]_R^{R+h}$$

$$= -GMm \left[\frac{1}{R+h} - \frac{1}{R} \right]$$

$$= -\frac{GMm}{R} \left[\frac{1}{1+\frac{h}{R}} - 1 \right]$$

$$= -\frac{GMm}{R} \left[\left(1 + \frac{h}{R}\right)^{-1} - 1 \right] \quad 1$$

Since, $R \gg h$

$$\therefore P.E. = -\frac{GMm}{R} \left[1 - \frac{h}{R} - 1 \right]$$

$$= \frac{GMmh}{R^2}$$

$$= \left(\frac{GM}{R^2} \right) mh \left[\text{as } g = \frac{GM}{R^2} \right]$$

$$= mgh$$

P.E. above the surface of Earth 1

$$= mgh$$



Commonly Made Error

► Students lack clear understanding about integration of integrand and in use of binomial theorem.



Answering Tip

► Student should have clear knowledge about mathematics which can help in solving numerical.



Long Answer Type Questions

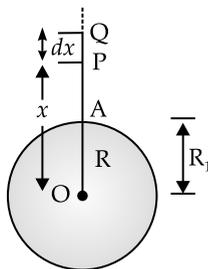
(5 marks)

- Q. 1. (i) Define escape velocity.
 (ii) Derive expression for the escape velocity of an object from the surface of planet.
 (iii) Does it depend on location from where it is projected? [NCT 2009; MSE Chandigarh 2008, 09]

[KVS 2013]

Ans. (i) Escape velocity on Earth (or any other planet) is defined as the minimum velocity with which the body has to be projected vertically upwards from the surface of Earth (or any other planet) so that it just crosses the gravitational field of Earth (or of that planet) and never returns on its own. 1

- (ii) Let Earth be a perfect sphere of mass M , radius R with centre at O . Let a body of mass m to be projected from a point A on the surface of Earth (planet). OA joined and extended further. Two points P and Q are taken at a distance x and $(x + dx)$ from the centre O of the Earth.



Gravitational force of attraction on the body at P is

$$F = GMm/x^2$$

This much force has to be applied on the body in the upward direction.

Work done in taking the body against gravitational attraction from P to Q is.

$$dW = F dx = \frac{GMm}{x^2} dx \quad 1$$

Total work done in taking the body against gravitational attraction from surface of Earth (i.e., $x = R$) to a region beyond the gravitational field of Earth (i.e., $x = \infty$) can be calculated by integrating the above expression with the limit, $x = R$ to $x = \infty$

Thus total work done is

$$W = \int_R^\infty \frac{GMm}{x^2} dx$$

$$= GMm \int_R^\infty x^{-2} dx$$

$$= GMm \left[\frac{x^{-2+1}}{-2+1} \right]_R^\infty$$

$$= -GMm \left[\frac{1}{x} \right]_R^\infty$$

$$= -GMm \left[\frac{1}{\infty} - \frac{1}{R} \right]$$

$$= \frac{GMm}{R} \quad 1$$

This work is done at the cost of kinetic energy given to the body at the surface of the Earth. If v_e is the escape velocity of the body projected from the surface of Earth, then

Kinetic energy of the body,

$$\begin{aligned} \frac{1}{2}mv_e^2 &= \frac{GMm}{R} \\ v_e^2 &= \frac{2GM}{R} \\ v_e &= \sqrt{\frac{2GM}{R}} \quad \dots(1) \end{aligned}$$

But, $g = \frac{GM}{R^2}$
 or, $GM = gR^2$
 Putting this value in (1),

$$v_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} \quad \dots(2) \quad 1$$

(iii) The value of escape velocity depends upon the mass and radius of the planet of the surface from which the body is to be projected. Clearly, the values of escape velocity of a body will be different for different planets. 1



Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. Escape velocity of an object of mass m is proportional to

- (A) m^2 (B) m
 (C) m^{-1} (D) m^0

Ans. Option (D) is correct.

Explanation: Escape velocity $= \sqrt{2gR}$,
 It is independent of mass.

Q. 2. An object of mass m is lifted at a height equal to the radius (R) of the earth. If the mass of the earth is M and acceleration due to gravity is g , then the change in gravitational potential energy of the object is

- (A) mgR (B) $2MgR$
 (C) $\frac{MgR}{2}$ (D) $\frac{mgR}{2}$

Ans. Option (D) is correct.

Explanation: Change in gravitational potential energy $= \Delta U = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R}$

$$\begin{aligned} \text{Putting } g &= \frac{GM}{R^2} \\ \Delta U &= \frac{mgR}{2} \end{aligned}$$

Q. 3. Two particles of equal mass m are moving along a circle of radius r under the action of their mutual gravitational attraction. The speed of each particle is

- (A) $\sqrt{\frac{Gm}{4r}}$ (B) $\sqrt{\frac{Gm}{2r}}$
 (C) $\sqrt{\frac{Gm}{r}}$ (D) \sqrt{Gmr}

Ans. Option (A) is correct.

Explanation: Gravitational attraction between the two particles $= F = \frac{Gm^2}{(2r)^2}$

$$\text{Required centripetal force} = \frac{mv^2}{r}$$

$$\therefore \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$$

$$\therefore v = \sqrt{\frac{Gm}{4r}}$$

Q. 4. If R is the radius of earth, then the acceleration due to gravity will be $g/4$ at a height _____ from the earth surface.

- (A) R (B) $2R$
 (C) $3R$ (D) $\frac{R}{2}$

Ans. Option (A) is correct.

Explanation: $\frac{g'}{g} = \frac{R^2}{(R+h)^2}$

or $\frac{1}{4} = \frac{R^2}{(R+h)^2}$

or $\frac{1}{2} = \frac{R}{(R+h)}$

$\therefore h = R$

Q. 5. The relation between gravitational potential (V) and gravitational intensity (E) is

- (A) $E = -\frac{d^2V}{dt^2}$ (B) $E = -\frac{dV}{dr}$
 (C) $E = \frac{d^2V}{dt^2}$ (D) $E = \frac{dV}{dr}$

Ans. Option (B) is correct.

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
 (B) Both A and R are true but R is NOT the correct explanation of A

- (C) A is true but R is false
(D) A is false and R is true

Q. 1. Assertion (A): At a depth equal to twice the height from the surface of earth the magnitudes of acceleration due to gravity are equal.

Reason (R): Acceleration due to gravity at height x and at depth y are $g\left(1 + \frac{2x}{R}\right)$ and $g(1 - y)$ respectively.

Ans. Option (C) is correct.

Explanation: Acceleration due to gravity at height x and at depth x are $g\left(1 - \frac{2x}{R}\right)$ and $g\left(1 - \frac{y}{R}\right)$ respectively.

So, when $y = 2x$ i.e., depth equal to twice the height from the surface, then acceleration due to gravity are equal.

So, the assertion is true but the reason is false.

Q. 2. Assertion (A): The magnitude of acceleration due to gravity does not depends upon mass of the body on which force acts.

Reason (R): Acceleration due to gravity is a constant quantity.

Ans. Option (C) is correct.

Explanation: Acceleration due to gravity = $g = \frac{GM}{R^2}$. So, it does not depends upon mass of the

body on which force acts. Hence, the assertion is true.

Acceleration due to gravity is not a constant quantity. It varies with latitude, altitude and depth. So, the reason is false.

Q. 3. Assertion (A): The acceleration due to gravity increases with height above the earth's surface.

Reason (R): Gravitational force increases with height above the earth's surface.

Ans. Acceleration due to gravity at an height h above the earth's surface is $g' = g \times \frac{R^2}{(R+h)^2}$

So, as height increases, acceleration due to gravity decreases. Hence, the assertion is false.

In everyday situations, the gravitational force on something does not change significantly as it rises above the Earth. Gravitational force at an altitude of 200km is around 95% of its magnitude at sea level.



NCERT Corner

Q. 1. Answer the following :

- You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
- An astronaut inside a small spaceship orbiting around the Earth cannot detect gravity. If the space station orbiting around the Earth has a large size can he hope to detect gravity?
- If you compare the gravitational force on the Earth due to the Sun to that due to the Moon, you would find that the Sun's pull is greater than Moon's pull. However, the tidal effect of the Moon's pull is greater than the tidal effect of the Sun. Why?

- Ans.** (a) No. Electrical forces depend upon the nature of the intervening medium while the gravitational forces don't depend upon the nature of the intervening medium. So, such shielding acts are not possible in case of gravitation, i.e., gravity screens are not possible.
- (b) Yes, astronaut can hope to detect gravity if the size of the spaceship is extremely large, then the magnitude of the gravity will become appreciable and hence, the gravitational effect of the spaceship may become measurable.
- (c) Earth-Moon distance is very small as compared to Earth-Sun distance. Tidal effect is inversely proportional to the cube of the distance. It is not governed by inverse square law like the gravitational force (which obeys inverse square

law). Hence, tidal effect of Moon is larger than that due to the Sun.

Q. 2. Choose the correct alternative

- Acceleration of gravity increases/decreases with increasing altitude.
- Acceleration due to gravity increases/decreases with increasing depth. (assume the Earth to be a sphere of uniform density).
- Acceleration due to gravity is independent of the mass of the Earth/mass of the body.

- (d) The formula $-GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the Earth.

Ans. (a) decreases

- decreases
- independent of mass of the body
- more

Q. 3. Choose the correct alternative :

- If the zero of potential energy is at infinity the total energy on orbiting satellite is negative of its kinetic/potential energy.
- The energy required to launch an orbiting satellite out of Earth's gravitational influence is more / less than the energy required to project a stationary object at the same height (as can the satellite) out of Earth influences.

[NCERT TB Q. No. 6]

- Ans. (a) Kinetic energy,
(b) Less.
- Q. 4. Does the escape speed of body from the Earth depend upon :**
- the mass of the body ?
 - the location from where it is projected ?
 - the direction of projection ?
 - the height of location from where the body is launched ?
- [NCERT TB Q. No. 7]

- Ans. (a) No, from the formula $v_e = \sqrt{\frac{2GM}{R}}$, it is clear that escape velocity does not depend on the mass of the body.
- (b) The escape velocity depends upon the value of gravitational potential at the point from where the body is projected. The gravitational potential energy of body $E = -\frac{GMm}{R}$ is slightly different at different points (the Earth is not a perfect sphere and hence R is different at different points). Because of this escape velocity depend slightly on the latitude of the place from where the body is projected.
- (c) The escape velocity of a body does not depend upon its direction of projection.
- (d) Since the gravitational potential energy at a point at the height h from the Earth surface is $\frac{GMm}{(R+h)}$, the escape velocity will be different for different values of h .

- Q. 5. A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant :**
- Linear speed,
 - Angular speed,
 - Angular momentum,
 - Kinetic energy,
 - Potential energy,
 - Total energy throughout it orbit ?

Neglect mass loss of the comet when it comes very close to the Sun ? [NCERT TB Q. No. 8]

- Ans. (a) The linear speed ($v = \omega R$) changes because the distance, (R) of the comet from the Sun changes due to it elliptical orbit around the Sun.
- (b) The angular speed of the comet also changes because it covers different angle in equal interval of time.
- (c) The angular momentum of the comet is same throughout due to the conservation of angular momentum in the absence of any torque.
- (d) Kinetic energy changes because linear speed is different at different points.
- (e) The potential energy at different points is different because the comet is not at the same distance from the Sun (the orbit is not circular).
- (f) The total energy of comet remain the same throughout the motion.



Commonly Made Error

- Confusion in identifying the physical quantities which remain constant in motion of the body in an elliptical orbit.



Answering Tip

- Student should study Kepler's II law in detail.

- Q. 6. Which of the following symptoms likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientation problem.** [NCERT TB Q. No. 9]

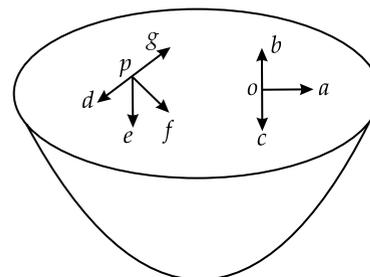
Ans. The astronaut in space will suffer from (a) swollen face, (b) headache and (c) orientation problem.

- (a) We know that legs carry the weight of the body in the normal position due to gravity pull. The astronaut in space is in weightlessness state. Hence, swollen feet may not effect his working. Due to weightlessness the astronaut may develop swollen face. As eyes, ears, nose are all embedded in face.
- (b) Headache is due to mental strain. It will persist whether a person is an astronaut in space or Earth.
- (c) Space also has orientation, we also have the frames of reference.

- Q. 7. In the following two exercises, choose the correct answer from the given ones. The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig).**

- (i) a, (ii) b, (iii) c, (iv) 0

[NCERT TB Q. No. 10]



- Ans. Inside a hollow spherical shell, potential will be same at all points. So, gravitational intensity, being negative of potential gradient will be zero. Due to zero gravitational intensity, the gravitational forces acting on any particle at any point inside a spherical shell will be symmetrically placed. So, if we remove the upper hemispherical shell, the net gravitational force acting on a particle at P will be downwards. Since gravitational intensity is gravitational force per unit mass, so, the direction of gravitational intensity will be along c. So, option (iii) is correct.

Q. 8. For the above problem, the direction of the gravitational intensity at any arbitrary point P is indicated by the arrow (i) *d*, (ii) *e*, (iii) *f*, (iv) *g*.

[NCERT TB Q. No. 11]

Ans. Using the above explanation, the direction of gravitational field intensity at P will be along *e*. So, option (ii) is correct.

Q. 9. A rocket is fired from the Earth towards the Sun. At what distance from the Earth's centre is the gravitational force on the rocket zero? Mass of the Sun = 2×10^{30} kg, mass of the Earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m.)

[NCERT TB Q. No. 12]

Ans. Here, $M_s = 2 \times 10^{30}$ kg;
 $M_e = 6 \times 10^{24}$ kg;
 $r = 1.5 \times 10^{11}$ m

Let x be the distance of a point from the Earth where gravitational forces on the rocket due to Sun and Earth become equal and opposite. Then distance of rocket from the Sun = $(r - x)$

If m is the mass of rocket then

$$\frac{GM_s m}{(r-x)^2} = \frac{GM_e m}{x^2}$$

or
$$\frac{(r-x)^2}{x^2} = \frac{M_s}{M_e}$$

$$\frac{r-x}{x} = \sqrt{\frac{M_s}{M_e}}$$

$$= \sqrt{\frac{2 \times 10^{30}}{6 \times 10^{24}}}$$

$$= \frac{10^3}{\sqrt{3}}$$

$$\frac{r}{x} - 1 = \frac{10^3}{\sqrt{3}}$$

$$\frac{r}{x} = 1 + \frac{10^3}{\sqrt{3}}$$

$$= \frac{\sqrt{3} + 10^3}{\sqrt{3}}$$

so,
$$x = \frac{r\sqrt{3}}{\sqrt{3} + 10^3}$$

$$= \frac{r\sqrt{3}}{\sqrt{3} + 10^3} \text{ m}$$

$$= \frac{1.5 \times 10^{11} \times 1.732}{1.732 + 1000} \text{ m}$$

$$= 2.55 \times 10^8 \text{ m}$$

Q. 10. How will you weight the Sun, i.e., estimate its mass? You will need to know the period of one of its planets and the radius of the planetary

orbit. The mean orbital radius of the Earth around the Sun is 1.5×10^8 km. Estimate the mass of the Sun. [NCERT TB Q. No. 13]

Ans. It is clear that Earth revolves around the Sun in an orbit of radius 1.5×10^{11} m and completes a revolution around the Sun in 365 days.

$$\therefore R = \text{radius of orbit of Earth} = 1.5 \times 10^{11} \text{ m}$$

$$T = \text{Time period of Earth around the Sun} = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s.}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

From the relation,

$$\frac{R^3}{T^2} = \frac{GM_s}{4\pi^2 R^2} \text{ where } s_s = \text{mass of Sun}$$

$$\text{or } M_s = \frac{4\pi^2 R^3}{T^2 G}$$

$$\text{or } M_s = \frac{4 \times 9.87 \times (1.5 \times 10^{11})^3}{(365 \times 24 \times 60 \times 60)^2 \times 6.67 \times 10^{-11}} \text{ kg}$$

$$\text{or } M_s = 2.0 \times 10^{30} \text{ kg.}$$

$$\therefore \text{Mass of Sun} = 2.0 \times 10^{30} \text{ kg.}$$

Q. 11. A saturn year is 29.5 times the Earth year. How far is the saturn from the Sun if the Earth is 1.5×10^8 km away from the Sun?

[NCERT TB Q. No. 14]

Ans. Here, $T_s = 29.5 T_e$
 $R_e = 1.5 \times 10^8$ km;
 $R_s = ?$

Using the relation,

$$\frac{T_s^2}{R_s^3} = \frac{T_e^2}{R_e^3}$$

$$\text{or } R_s = R_e \left(\frac{T_s}{T_e} \right)^{2/3}$$

$$= 1.5 \times 10^8 \left[\frac{29.5 T_e}{T_e} \right]^{2/3}$$

$$= 1.43 \times 10^9 \text{ km}$$

Q. 12. A body weighs 63 N on the surface of Earth. What is the gravitational force on it due to the Earth at a height equal to half the radius of Earth? **AI**

[NCERT TB Q. No. 15]

Ans. Let h = height above Earth's surface = $\frac{R}{2}$
where, R = radius of Earth

Now, the acceleration due to gravity at a height ' h ' above Earth's surface (g) is given by

$$g_h = g \left(1 + \frac{h}{R} \right)^{-2}$$

Given,
$$h = \frac{R}{2}$$

$$\therefore g_h = g \left(1 + \frac{R/2}{R} \right)^{-2}$$

or
$$\frac{g_h}{g} = \frac{1}{\left(1 + \frac{1}{2}\right)^2} = \frac{1}{\left(\frac{3}{2}\right)^2}$$

$$= \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

or
$$g_h = \frac{4}{9} \times g = \frac{4}{9}g \quad \dots(i)$$

Let m = mass of the body
If W and W_h be its weight at Earth's surface and at a height h above Earth's surface respectively, then $W = mg = 63$ N (given)

and
$$W_h = mg_h = m \times \frac{4}{9}g = \frac{4}{9}mg$$

or
$$W_s = \frac{4}{9} \times 63 = 28$$
 N

So,
$$W_h = 28$$
 N

Q. 13. Assuming the Earth to be a sphere of uniform mass density, how much would a body weight half way down to the centre of Earth, if it weighed 250 N on the surface ?

[NCERT TB Q. No. 16]

Ans. Suppose g, g_d be the acceleration due to gravity on Earth's surface and at a depth ' d ', from surface respectively.

Also, suppose W and W_d be the weight of a body on Earth's surface and at depth ' d ', respectively.

$\therefore W = mg = 250$ N ... (i)

and
$$W_d = mg_d \quad \dots(ii)$$

Now, we know that

$$g_d = g \left(1 - \frac{d}{R}\right) \quad \dots(iii)$$

Here, $d = \frac{R}{2}, R =$ radius of Earth... (iv)

\therefore From equations (iii) and (iv), we get

$$g_d = g \left(1 - \frac{R/2}{R}\right)$$

$$= g \left(1 - \frac{1}{2}\right) = g \times \left(\frac{1}{2}\right)$$

$$= \frac{g}{2} \quad \dots(v)$$

$\therefore W_d = mg_d$

$$= m \frac{g}{2} \quad [\text{by using equation (v)}]$$

$$= \frac{1}{2} mg = \frac{1}{2} W$$

$$= \frac{1}{2} \times 250 = 125$$
 N.

\therefore Weight of the body half way down to the centre of Earth = 125 N.

Q. 14. A rocket is fired vertically with a speed of 5 kms⁻¹ from the Earth's surface. How far from the Earth does the rocket go before returning to the Earth ?

Mass of the Earth = 6.0 × 10²⁴ kg, mean radius of Earth = 6.4 × 10⁶ m, G = 6.67 × 10⁻¹¹ Nm² kg⁻².

[A1] [NCERT TB Q. No. 17]

Ans. Suppose v be the initial speed of the rocket which reaches a height h above the surface of Earth where its velocity vanishes, i.e., becomes zero.

When m be the mass of the rocket, then its total energy at Earth's surface is

$$K.E. + P.E. = \frac{1}{2}mv^2 - \frac{GMm}{R} \quad \dots(i)$$

where, M = Mass of Earth
 R = Radius of Earth
 G = Universal gravitational constant

At highest point,

$$K.E. = 0 \quad (\because \text{Velocity} = 0)$$

and
$$P.E. = -\frac{GMm}{R+h} \quad \dots(ii)$$

\therefore Total energy of the rocket at a height h is given by

$$= K.E. + P.E. = 0 + P.E.$$

$$= -\frac{GMm}{R+h} \quad \dots(iii)$$

From the law of conservation of energy.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{1}{2}v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= \frac{GM}{R} \left(1 - \frac{R}{R+h}\right) \left(\because \frac{GM}{R^2} = g\right)$$

or
$$\frac{1}{2}v^2 = gR \left(\frac{R+h-R}{R+h}\right)$$

$$= \frac{gR}{R+h}h$$

or
$$2gRh = v^2(R+h)$$

or
$$Rv^2 = 2ghR - v^2h = h(2gR - v^2)$$

$\therefore h = \frac{Rv^2}{2gR - v^2} \quad \dots(iv)$

Given, $v = 5$ kms⁻¹
 $= 5000$ ms⁻¹ (given)

and $R = 6.4 \times 10^6$ m.

Putting these values in eqn. (iv), we have

$$\therefore h = \frac{6.4 \times 10^6 \times (5 \times 10^3)^2}{2 \times 9.8 \times 6.4 \times 10^6 - (5 \times 10^3)^2}$$

$$= 1.6 \times 10^6$$
 m

\therefore Distance from centre of Earth is given by

$$= R + h = 6.4 \times 10^6 + 1.6 \times 10^6$$

$$= 8000$$
 km

Q. 15. Escape velocity of projectile on a planet's surface is 11.2 km s⁻¹. If a body is projected at double the speed, find its speed at an infinite distance from the planet. [NCERT TB Q. No. 18]

Ans. From principle of conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_e^2$$

or
$$v = \sqrt{v_i^2 - v_e^2}$$

$$= \sqrt{(2v_e)^2 - v_e^2}$$

$$= \sqrt{3} v_e$$

$$= 1.732 \times 11.2 \text{ km/s}$$

[∵ $\sqrt{3} = 1.732$]

$$= 31.68 \text{ km s}^{-1}$$

Q. 16. A satellite orbits the Earth at a height of 400 km above the surface. How much energy must be expended to launch the satellite out of the Earth's gravitational influence ?

Mass of the satellite = 200 kg; mass of the Earth = 6.0×10^{24} kg, radius of Earth = 6.4×10^6 m; $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{-kg}^{-2}$.

[NCERT TB Q. No. 19]

Ans. Given : $M = 6 \times 10^{24}$ kg
 $m = 200$ kg
 $R = 6.4 \times 10^6$ m
 $x = 0.4 \times 10^6$ m

Using, total energy of satellite in orbit,

$$E = -\frac{GMm}{2(R+x)}$$

$$E = -\frac{(6.67 \times 10^{-11})(6 \times 10^{24})200}{2(6.4 \times 10^6 + 0.4 \times 10^6)} \text{ J}$$

we get

$$= -\frac{6.67 \times 6 \times 2 \times 10^{15}}{2 \times 6.8 \times 10^6} \text{ J}$$

$$= -5.89 \times 10^9 \text{ J}$$

Energy required to send the satellite

$$= 5.9 \times 10^9 \text{ J}$$

Q. 17. Two stars each of one solar mass ($= 2 \times 10^{30}$ kg) are approaching each other for a head on collision. When they are at distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).

[NCERT TB Q. No. 20]

Ans. Mass of each star, $M = 2 \times 10^{30}$ kg
 Initial distance between two stars, $r = 10^9$ km
 $= 10^{12}$ m

Initial potential energy of the system = $\left(-\frac{GMm}{r}\right)$

Total kinetic energy of the system

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2$$

Where v is the speed of stars with which they collide. When the stars are about to collide the distance between their centres, $r' = 2R$, where R is their radius.

∴ Final potential energy of two stars = $-\frac{GMm}{2R}$

Since gain in K.E = loss in P.E

$$\therefore Mv^2 = -\frac{GMM}{r} - \left(-\frac{GMM}{2R}\right) = \frac{-GMM}{r} + \frac{GMM}{2R}$$

$$2 \times 10^{30}v^2 = \frac{-6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{10^{12}} + \frac{6.67 \times 10^{-11} \times (2 \times 10^{30})^2}{2 \times 10^7}$$

$$= -2.668 \times 10^{38} + 1.334 \times 10^{43}$$

$$= 1.334 \times 10^{43} \text{ J}$$

$$\therefore v = \sqrt{\frac{1.334 \times 10^{43}}{2 \times 10^{30}}} = \sqrt{0.66 \times 10^{13}} \text{ m/s}$$

$$= \sqrt{6.67 \times 10^{12}} = 2.583 \times 10^6 \text{ ms}^{-1}$$



Commonly Made Error

- ▶ Students fail to apply law of conservation of energy properly.



Answering Tip

- ▶ Student should solve the numerical problem step by step to obtain desired results.

Q. 18. Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid point of the line joining the centres of the spheres. Is an object placed at the point is in equilibrium ? If so, is the equilibrium stable or unstable ?

[NCERT TB Q. No. 21]

Ans. Here $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$
 $M = 100$ kg
 $R = 0.1$ m

Distance between the two spheres $d = 1.0$ m

Suppose that the distance of either sphere from the mid point of the line joining their centre is r . Then

$$r = \frac{d}{2} = 0.5 \text{ m.}$$

The gravitational field at the mid point due to two spheres will be equal and opposite.

Hence, the resultant gravitational field at the mid point = 0

The gravitational potential at the mid point.

$$\begin{aligned}
 &= \left(-\frac{GM}{r}\right) \times 2 \\
 &= \frac{-6.67 \times 10^{-11} \times 100 \times 2}{0.5} \text{ Jkg} \\
 &= -2.668 \times 10^{-8} \text{ Jkg}^{-1}
 \end{aligned}$$

Any object placed at point x will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effecting force in that direction.



Commonly Made Error

- ▶ Students can not obtain correct values for gravitational field and potential.



Answering Tip

- ▶ Students should properly understand the terms gravitational field and gravitational potential.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

- Q. 1.** The Earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the Earth, the acceleration due to gravity
- (A) will be directed towards the centre but not the same everywhere.
 (B) will have the same value everywhere but not directed towards the centre.
 (C) will be same everywhere in magnitude directed towards the centre.
 (D) cannot be zero at any point.

Ans. Option (D) is correct.

- Q. 2.** As observed from Earth, the Sun appears to move in an approximate circular orbit. For the motion of another planet like Mercury as observed from Earth, this would
- (A) be similarly true.
 (B) not be true because the force between Earth and Mercury is not inverse square law.
 (C) not be true because the major gravitational force on Mercury is due to Sun.
 (D) not to be true because Mercury is influenced by forces other than gravitational forces.

Ans. Option (C) is correct.

- Q. 3.** Different points in Earth are at slightly different distances from the Sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the c.m. (centre of mass) causing translation and a net torque at the c.m. causing rotation around an axis through the c.m. For the Earth-Sun system (approximating the Earth as a uniform density sphere)
- (A) the torque is zero.
 (B) the torque causes the Earth to spin.
 (C) the rigid body result is not applicable since the Earth is not even approximately a rigid body.
 (D) the torque causes the Earth to move around the Sun.

Ans. Option (A) is correct.

- Q. 4.** Satellites orbiting the Earth have finite life and sometimes debris of satellites fall to the Earth. This is because,
- (A) the solar cells and batteries in satellites run out.
 (B) the laws of gravitational predict a trajectory spiralling inwards.
 (C) of viscous forces causing the speed of satellite and hence height to gradually decrease.
 (D) of collisions with other satellites.

Ans. Option (C) is correct.

- Q. 5.** Both Earth and Moon are subject to the gravitational force of the Sun. As observed from the Sun, the orbit of the Moon
- (A) will be elliptical
 (B) will not be strictly elliptical because the total gravitational force on it is not central.
 (C) is not elliptical but will necessarily be a closed curve.
 (D) deviates considerably from being elliptical due to influence of planets other than Earth.

Ans. Option (B) is correct.

- Q. 6.** In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They
- (A) will not move around the Sun since they have very small masses compared to the Sun.
 (B) will move in an irregular way because of their small masses and will drift away into outer space.
 (C) will move around the Sun in closed orbits but not obey Kepler's laws.
 (D) will move in orbits like planets and obey Kepler's laws.

Ans. Option (D) is correct.

- Q. 7. Choose the wrong option.** A1
- (A) Inertial mass is a measure of difficulty of accelerating body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.
- (B) That the gravitational mass and inertial mass are equal is an experimental result.
- (C) That the acceleration due to gravity on Earth is the same for all bodies is due to the equality of gravitational mass and inertial mass.
- (D) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot.

Ans. Option (D) is correct.



Commonly Made Error

- Students can not comprehend the terms 'gravitational mass' and 'inertial mass' properly leading to wrong conclusions.

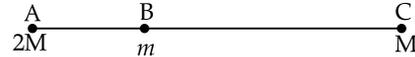


Answering Tip

- Student should have a clear understanding of the terms gravitational mass and inertial mass.

- Q. 8. Particles of masses $2M$, m and M are respectively at points A, B and C with $AB=1/2 (BC)$. m is much-much smaller than M and at time $t = 0$. They are all at rest.**

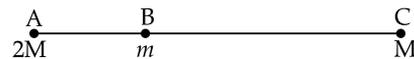
At subsequent times before any collision takes place:



- (A) m will remain at rest.
 (B) m will move towards M.
 (C) m will move towards $2M$.
 (D) m will have oscillatory motion.

Ans. Option (C) is correct.

Explanation:



Gravitational force, $F = \frac{GMm}{r^2}$

Let $AB = r$

∴ Force on B due to A,

$$\therefore F_{BA} = \frac{G(2Mm)}{r^2} = 2F$$

Now, Force on B due to C, -

$$\therefore F_{BC} = \frac{GMm}{(BC)^2} = \frac{GMm}{4r^2} \quad [\because BC = 2AB]$$

$$F_{BC} = \frac{F}{4} \text{ or } F_{BA} > F_{BC}$$

Hence, m will move towards A i.e., $2M$.



Very Short Answer Type Questions

- Q. 9. Molecules in air in the atmosphere are attracted by gravitational force of the Earth. Explain why all of them do not fall into the Earth just like an apple falling from a tree.**

[NCERT Exemplar Q. No. 17]

- Ans.** Air molecules in the atmosphere experience the vertically downward force due to gravity just like an apple falling from a tree. Due to thermal motion, air molecules move randomly, their velocity is not in the vertical direction. The downward force gravity causes the density of air in the atmosphere close to Earth higher than the density as we go up. But in apple's case, only vertical motion dominates because of heavier molecules than air molecules.

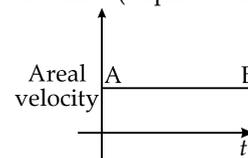
- Q. 10. Give one example each central force and non-central force.** [NCERT Exemplar Q. No. 18]

- Ans. Examples of Central force :** gravitational force of a point mass, electrostatic force due to a point charge. **(Any one)**

Examples of Non-central force: spin-dependent nuclear forces, magnetic force between two current carrying loops. **(Any one)**

- Q. 11. Draw areal velocity versus time graph for mars.**
 [NCERT Exemplar Q. No. 19]

- Ans.** The graph between areal velocity and time is a straight line parallel to time axis because areal velocity of a planet revolving around the Sun is constant with time. (Kepler's second law).



- Q. 12. How is the gravitational force between two point masses affected when they are dipped in water keeping the separation between them the same?**

[NCERT Exemplar Q. No. 21]

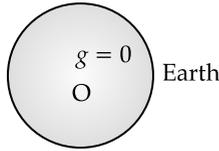
- Ans.** By Universal law of gravitation,

$$F = \frac{Gm_1m_2}{r^2}$$

The force acting between two point masses m_1 and m_2 , is independent of the nature of medium between them. Therefore, Gravitational force acting between two masses will remain unaffected when they are dipped in water.

- Q. 13. Is it possible for a body to have inertia but no weight?** [NCERT Exemplar Q. No. 22]

Ans. Yes, because weight (mg) of a body can be zero as it depend upon acceleration due to gravity but every body will always have inertia (i.e., mass)



For example: At O, $g = 0$, at the centre of the Earth, the weight of a body is zero but it has inertia (i.e., mass).

Q. 14. We can shield a charge from electric fields by putting it inside a hollow conductor. Can we shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

[NCERT Exemplar Q. No. 23]

Ans. Refer Q1(a) of NCERT corner.

Q. 15. An astronaut inside a small spaceship orbiting around the Earth cannot detect gravity. If the space station orbiting around the Earth has a large size, can he hope to detect gravity?

[NCERT Exemplar Q. No. 24]

Ans. Refer question 1(b) of NCERT Corner.

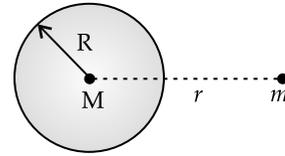
Q. 16. The gravitational force between a hollow spherical shell (of radius R and uniform density) and a point mass is F . Show the nature of F vs. r graph where r is the distance of the point from the centre of the hollow spherical shell of uniform density.

[NCERT Exemplar Q. No. 25]

Ans. Gravitational force is F .

Shell's density is constant and it is ρ

As density of shell is uniform it can be treated as point mass.



$$\text{Mass of shell, } M = \rho \times \frac{4}{3} \pi R^3$$

Gravitational force between M and m,

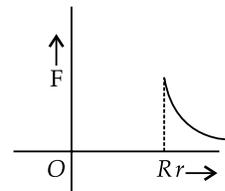
$$F = \frac{GMm}{r^2}$$

But force inside the shell is zero.

$$F = 0 \text{ for } r < R$$

$$F = \frac{GMm}{r^2} \text{ for } r > R$$

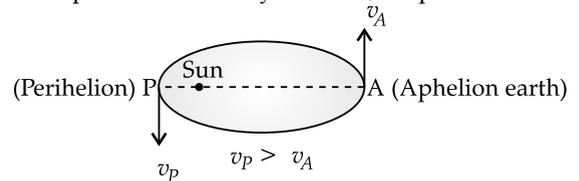
Variation of F versus r .



Q. 17. Out of aphelion and perihelion, where is the speed of the Earth more and why? [A1]

[NCERT Exemplar Q. No. 26]

Ans. Areal velocity of Earth around the Sun is constant from Kepler's II law, so the speed of the Earth is more at the perihelion than at the aphelion. The Earth has to cover greater linear distance to keep the areal velocity constant, at aphelion.

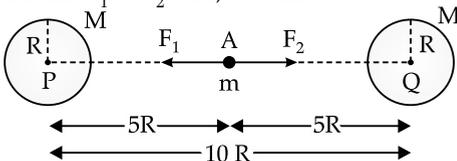


Short Answer Type Questions

Q. 18. Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid point of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reason for your answer.

[NCERT Exemplar Q. No. 29]

Ans. Given: $m_1 = m_2 = M$, $r = 10R$



Let mass m is placed at mid-point A (line joining the centres of P & Q sphere)

$$\text{Now, } |F_1| = \frac{GMm}{(5R)^2}$$

$$|F_1| = |F_2| = \frac{GMm}{25R^2}$$

F_1 & F_2 are equal and opposite forces are acting on m at A.

Net force $F_1 = -F_2$ or $F_1 + F_2 = 0$

So, mass m is in equilibrium.

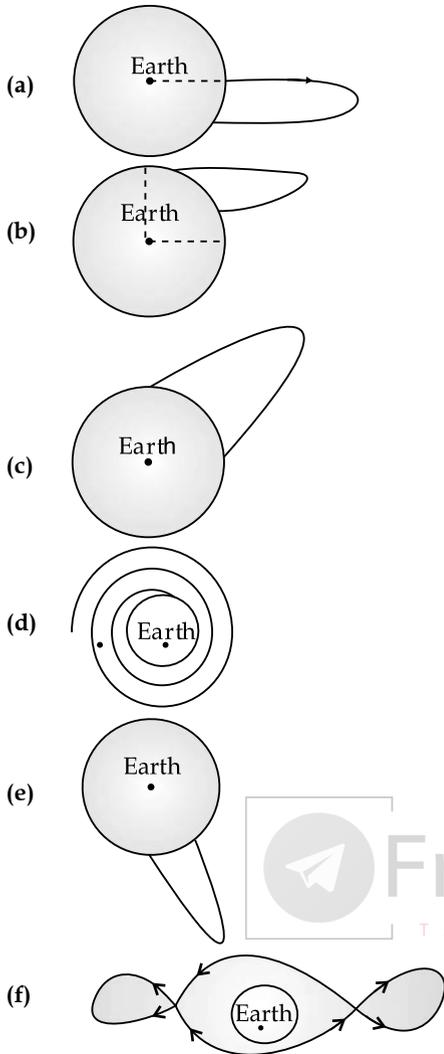
If m is slightly displaced x from A towards P then

$$F_1 = \frac{GMm}{(5R-x)^2} \text{ \& } F_2 = \frac{GMm}{(5R+x)^2} \text{ or } F_1 > F_2$$

That means resultant force acting on A is towards P. Hence, equilibrium is unstable equilibrium.

Q. 19. Shown are several curves (Fig. (a), (b), (c), (d), (e) and (f)). Explain with reason which ones amongst them can be possible trajectories traced by a projectile (neglect air friction).

[NCERT Exemplar Q. No. 31]

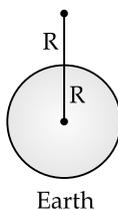


Ans. The trajectory of a particle under gravitational force of the Earth will be a conic section (for motion outside the Earth) with the centre of the Earth as a focus. Only (c) meets this requirement because in this centre of Earth is the focus of trajectory.

Q. 20. An object of mass m is raised from the surface of the Earth to a height of equal to the radius of the Earth, that is, taken from a distance R to $2R$ from the centre of the Earth. What is the gain in its potential energy?

[NCERT Exemplar Q. No. 32]

Ans. Given: An object is raised distance $R \rightarrow 2R$
Potential energy of body on the surface of Earth = $\frac{-GMm}{R}$



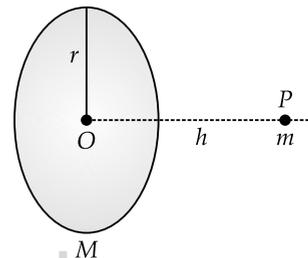
P.E. of object at height equal to radius of Earth = $\frac{-GMm}{2R}$

$$\begin{aligned} \text{Gain in Potential Energy} &= P.E_f - P.E_i \\ &= \frac{-GMm}{2R} - \left(\frac{-GMm}{R} \right) \\ &= \frac{GMm}{R} \left[-\frac{1}{2} + 1 \right] \\ &= \frac{GMm}{2R} \end{aligned}$$

As, $GM = gR^2$

$$\text{Gain in P.E.} = \frac{gR^2 m}{2R} = \frac{1}{2} mgR.$$

Q. 21. A mass m is placed at P a distance h along the normal through the centre O of a thin circular ring of mass M and radius r .



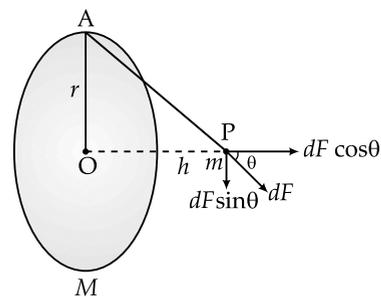
If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h=r$?

[NCERT Exemplar Q. No. 33]

Ans. Gravitational force F at P

$$F_h = \frac{GMm \cos \theta}{AP^2} = \frac{GMmh}{(r^2 + h^2)^{3/2}}$$

$$\therefore \cos \theta = \frac{h}{(r^2 + h^2)^{1/2}}$$



$$\frac{F_r}{F_{2r}} = \frac{\frac{GMm \cdot r}{(r^2 + r^2)^{3/2}}}{\frac{GMm \cdot 2r}{[r^2 + (2r)^2]^{3/2}}}$$

or,

$$\frac{F_r}{F_{2r}} = \frac{(r^2 + 4r^2)^{3/2}}{2(2r^2)^{3/2}}$$

$$\text{or, } \frac{F_r}{F_{2r}} = \frac{(5r^2)^{3/2}}{2\sqrt{2}r^3}$$

$$\text{or, } \frac{F_r}{F_{2r}} = \frac{5\sqrt{5}r^3}{4\sqrt{2}r^3}$$

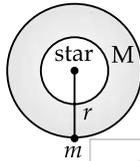
$$\text{or, } \frac{F_r}{F_{2r}} = \frac{5}{4}\sqrt{\frac{5}{2}}$$

$$\text{or, } \frac{F_{2r}}{F_r} = \frac{4}{5}\sqrt{\frac{2}{5}}$$

$$\therefore F_{2r} = \frac{4}{5}\sqrt{\frac{2}{5}}F_r$$

Long Answer Type Questions

- Q. 22. A star like the Sun has several bodies moving around it at different distances. Consider that all of them are moving in circular orbits. Let r be the distance of the body from the centre of the star and let its linear velocity be v , angular velocity ω , kinetic energy K , gravitational potential energy U , total energy E and angular momentum L . As the radius r of the orbit increases, determine which of the above quantities increase and which ones decrease. [NCERT Exemplar Q. No. 34]



Ans. Consider a body of mass m is rotating around the star in circular path of radius r .

(i) Orbital velocity –

$$v = \sqrt{\frac{GM}{r}} \text{ or } v \propto \frac{1}{\sqrt{r}}$$

Orbital velocity decreases as r increases.

(ii) Angular velocity = $\frac{2\pi}{T}$

By Kepler's III law

$$T^2 \propto r^3 \text{ or } T^2 = Kr^3$$

$$\omega = \frac{2\pi}{Kr^{3/2}} \text{ or } \omega \propto \frac{1}{\sqrt{r^3}}$$

Hence, angular velocity decreases as r increases.

(iii) Kinetic Energy $K = \frac{1}{2}m\frac{GM}{r}$ or $K \propto \frac{1}{r}$

Hence, K , decreases on increasing the radius.

(iv) Gravitational Potential Energy,

$$U = \frac{-GMm}{r}$$

$$\text{or } U \propto \frac{-1}{r}$$

So, on increasing radius of circular orbit the U increases. (due to negative sign)

(v) Total Energy,

$$E = K + U = \frac{GMm}{2r} + \left(-\frac{Gmm}{r}\right)$$

$$E = -\frac{GMm}{2r}$$

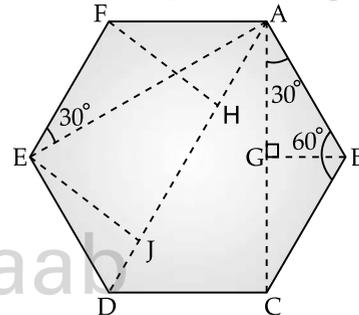
So, on increasing the radius, E will also be increases.

(vi) Angular momentum $L = mvr = mr\sqrt{\frac{GM}{r}}$

$L = m\sqrt{GM}r$ or $L \propto \sqrt{r}$. So, L increases as r increases.

- Q. 23. Six point masses of mass m each are at the vertices of a regular hexagon of side L . Calculate the force on any of the masses.

[NCERT Exemplar Q. No. 35]



Ans. Let us consider the following diagram in which size point masses are placed at six vertices, A, B, C, D, E, F.

$$AC = AG + GC = 2AG$$

$$\text{or, } AC = 2L \cos 30^\circ$$

$$\therefore AC = \sqrt{3}L$$

$$\text{Similarly, } AE = \sqrt{3}L$$

$$AD = AH + HJ + JD$$

$$\text{or, } AD = L \sin 30^\circ + L + L \sin 30^\circ$$

$$\therefore AD = 2L$$

Force on mass at A due to mass at B,

$$F_1 = \frac{Gmm}{L^2} \text{ along A to B} \\ = \frac{Gm^2}{L^2}$$

Force on mass at A due to mass at C,

$$F_2 = \frac{Gm \cdot m}{(\sqrt{3}L)^2} \\ = \frac{Gm^2}{3L^2} \text{ along A to C}$$

$$[\because AC = \sqrt{3}L]$$

Force on mass at A due to mass at D,

$$F_3 = \frac{Gmm}{(2L)^2}$$

$$= \frac{Gm^2}{4L^2} \text{ along A to D}$$

[∵ AD = 2L]

Force on mass at A due to mass at E,

$$F_4 = \frac{Gmm}{(\sqrt{3}L)^2}$$

$$= \frac{Gm^2}{3L^2} \text{ along A to E}$$

Force on mass at A due to mass at F,

$$F_5 = \frac{Gmm}{L^2}$$

$$= \frac{Gm^2}{L^2} \text{ along A to F}$$

Resultant force due to F_1 and F_5 ,

$$F_{15} = \sqrt{F_1^2 + F_5^2 + 2F_1F_5 \cos 120^\circ}$$

$$= \frac{Gm^2}{L^2} \text{ along A to D [Angle between } F_1 \text{ and } F_5 = 120^\circ]$$

Resultant force due to F_2 and F_4 ,

$$F_{24} = \sqrt{F_2^2 + F_4^2 + 2F_2F_4 \cos 60^\circ}$$

$$= \frac{\sqrt{3}Gm^2}{3L^2} = \frac{Gm^2}{\sqrt{3}L^2} \text{ along A to D}$$

$$\therefore \text{ Net force along A to D} = F_{15} + F_{24} + F_3$$

$$= \frac{Gm^2}{L^2} + \frac{Gm^2}{\sqrt{3}L^2} + \frac{Gm^2}{4L^2}$$

$$= \frac{Gm^2}{L^2} \left(1 + \frac{1}{\sqrt{3}} + \frac{1}{4} \right)$$



Commonly Made Error

- Students can not calculate the distance between masses kept at the vertices of a regular hexagon.



Answering Tip

- Student should familiarize themselves with various mathematical shapes and their properties.



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the given text and answer any four the following questions on the basis of the same.

- I. A black hole is a place in space where gravity pulls so much that even light can not get out. The gravity is so strong because matter has been squeezed into a tiny space.

From Newton's universal law of gravitation,

$$F = \frac{GMm}{R^2}$$

The photons have no mass. From Einstein's mass-energy equivalence relation $E = mc^2$

According to quantum theory of radiation,

$$E = hv$$

$$\therefore m = \frac{h}{c\lambda}$$

Putting in the expression of Newton's law of gravitation,

$$\text{Force acting on the light particles} = G \frac{Mh}{c\lambda R^2}$$

Because no light can get out, people can't see black holes. They are invisible. Space telescopes with special tools can help find black holes.

A beam of light just grazing the edge of the sun also bend a little. This may be observed during total solar eclipse. The position of a star will appear closer to the sun.

Black holes can be big or small. Scientists think the smallest black holes are as small as just one atom.

These black holes are very tiny but have the extremely high density. Another kind of black hole is called "stellar." Its mass can be up to 20 times more than the mass of the sun. There may be many, many stellar mass black holes in milky Way. The largest black holes are called "super massive." These black holes have masses those are more than 1 million suns together. Scientists have found proof that every large galaxy contains a super massive black hole at its centre. The super massive black hole at the centre of the Milky Way galaxy is called Sagittarius A. It has a mass equal to about 4 million suns and would fit inside a very large ball that could hold a few million Earths. A black hole cannot be seen because strong gravity pulls all of the light into the middle of the black hole. But scientists can see how the strong gravity affects the stars and gas around the black hole. If a black hole of the same mass as the sun replaces the sun, Earth and the other planets will orbit the black hole as they orbit the sun now.

Q. 1. The gravity of black hole is so strong because

- (A) Its density is too low
- (B) Its density is too high
- (C) Its volume is too small
- (D) Both (A) and (C)

Ans. Option (B) is correct.

Explanation: The gravity of black hole is so strong because matter has been squeezed into a tiny space i.e., density is too high.

Q. 2. The super massive black hole at the center of the Milky Way galaxy is

- (A) Sagittarius A (B) Sagittarius B2
(C) Cygnus X-1 (D) Cygnus X-3

Ans. Option (A) is correct.

Explanation: The largest black holes are called "super massive." These black holes have masses more than 1 million Sun's together. Scientists have found proof that every large galaxy contains a super massive black hole at its centre. The super massive black hole at the centre of the Milky Way galaxy is called Sagittarius A. It has a mass equal to about 4 million suns.

Q. 3. What will happen to the earth and other planets if a black hole of same mass as the sun replaces the sun?

- (A) Earth and other planets will fall into the black hole
(B) Earth and other planets will orbit the black hole as they orbit the Sun now.
(C) The orbits of earth and other planets will be much shorter.
(D) Earth and other planet will stop orbiting.

Ans. Option (B) is correct.

Explanation: If a black hole the same mass as the sun replace the sun, Earth and the other planets will orbit the black hole as they orbit the sun now.

Q. 4. The attractive force acting on a light particle by a black hole is

- (A) $G \frac{Mh}{c\lambda R^2}$ (B) $G \frac{Mch}{hR^2}$
(C) $G \frac{Mc}{h\lambda R^2}$ (D) $G \frac{M\lambda}{chR^2}$

Ans. Option (A) is correct.

Explanation: From Newton's universal law of gravitation, $F = \frac{GMm}{R^2}$

The photons have no mass. From Einstein's mass-energy equivalence relation

$$E = mc^2.$$

According to quantum theory of radiation,

$$E = hv$$

$$\therefore m = \frac{h}{c\lambda}$$

Putting in the expression of Newton's law of gravitation,

$$\text{Force acting on the light particles} = \frac{GMh}{c\lambda R^2}$$

Q. 5. A beam of light just grazing the edge of the sun also bend a little. It is observable during

- (A) Total solar eclipse (B) Partial solar eclipse
(C) Full lunar eclipse (D) Partial lunar eclipse

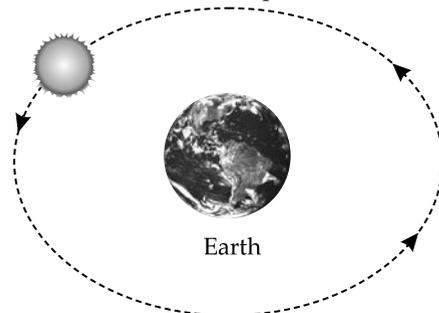
Ans. Option (A) is correct.

Explanation: How to observe a beam of light just grazing the edge of the sun bends? This can be observed during a total solar eclipse. Carefully measuring the location of a star, it appears closer to the sun. In 1919, teams of astronomers observed the solar eclipse from Brazil and from an island in the Atlantic. They intercepted the path of the total solar eclipse, and carefully photographed the positions of nearby stars. Several months later, they announced that Einstein was right! The rays really bend !

B Subjective Questions

I. Read the given text and answer any four of the following questions on the basis of the same.

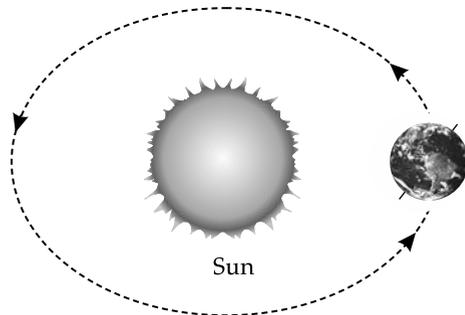
In astronomy, the geocentric theory of the universe is the idea that the Earth is the center of the universe and other objects go around it. Belief in this system was common in ancient Greece. It was embraced by both Aristotle and Ptolemy and most Greek philosophers assumed that the Sun, Moon, stars and visible planets circle the Earth.



Two common observations were believed to support the idea that the Earth is in the center of the Universe. The first is that the stars (including the Sun and planets) appear to revolve around the Earth day as seen by the sun rising in the east and setting in the west every day. The second is the common sense perception that the Earth is solid and stable; it is not moving but is at rest.

From the late 16th century onward it was gradually replaced by the heliocentric model of Copernicus, Galileo and Kepler.

In astronomy, the heliocentric theory is the idea that the Sun is at the center of the Solar System. This theory explained many of the observations of astronomers. Some of its revolutionary ideas were that the Earth rotates on its axis daily and revolves around the Sun once a year.



In 1515, a Polish priest Nicolaus Copernicus proposed that the Earth was a planet like Venus or Saturn and that all planets circled the Sun. Afraid of criticism, he did not publish his theory until 1543. He was afraid of Church's disapproval of his theory.

While Copernicus rightly observed that the planets revolve around the Sun, it was Kepler who correctly defined their orbits. At the age of 27, Kepler became the assistant of a wealthy astronomer, Tycho Brahe, who asked him to define the orbit of Mars. Brahe had collected a lifetime of astronomical observations, which, on his death, passed into Kepler's hands. Using these observations, Kepler found that the orbits of the planets followed three laws.

Q. 1. What is geocentric theory?

Ans. In geocentric theory the Earth is considered to be at the center of the universe and other celestial

objects go around it. It was assumed that the Sun, Moon, stars and other planets encircle the Earth.

Q. 2. Which observations supported geocentric theory?

Ans. Two common observations supported the geocentric theory:

- (i) The Sun, other stars and planets appear to revolve around the Earth
- (ii) The common sense perception that the Earth is solid and stable; hence it is not moving; it is at rest.

Q. 3. What is heliocentric theory ?

Ans. The heliocentric theory is the idea that the Sun is at the center of the Solar System. The Earth and other planets rotate on their own axes daily and revolve around the Sun once a year.

Q. 4. Who first proposed the heliocentric theory and why he kept it in abeyance for a long time?

Ans. In 1515, a Polish priest Nicolaus Copernicus proposed that the Earth is a planet like Venus, Saturn, Jupiter etc. All of the planets encircle the Sun.

Being afraid of disapproval of his theory by the churches, he kept his theory in abeyance until 1543.

Q. 5. Who correctly defined the orbits of different planets? How he got the necessary data for this discovery?

Ans. Johannes Kepler correctly defined the orbits of the planets. He was the assistant of an astronomer, Tycho Brahe. Brahe, in his lifetime, had collected a huge amount of data from astronomical observations. After his death, Kepler got access to this data and discovered the laws of planetary motion.

□□

UNIT-VII

PROPERTIES OF BULK MATTER

Study Time
Maximum Time: 2:30 hours
Maximum Questions: 77

CHAPTER

8

MECHANICAL PROPERTIES OF SOLIDS



Syllabus

Elasticity, Stress-strain relationship, Hooke's law, Young's modulus, Bulk modulus, Shear modulus, Modulus of rigidity, Poisson's ratio, elastic energy.

Topic-1 Elastic Behaviour of Solids

TOPIC - 1

Elastic Behaviour of Solids

.... P. 169

TOPIC - 2

Modulus of Elasticity

.... P. 173



Revision Notes

- **Stress** is defined as the restoring force acting per unit area of a deformed body, i.e.,

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$$

The S.I. unit of stress is N/m^2 and its dimensional formula = $[\text{ML}^{-1}\text{T}^{-2}]$. Stress is a tensor quantity. Normal Stress have following three types :

- Longitudinal stress** : If a body changes its length under a deforming force and the stress is normal to the surface of the body then the stress is called longitudinal stress. The longitudinal stress can be a tensile stress or compression stress. The longitudinal stress produced because of increase in length of body under deforming force is known as **tensile stress**. The longitudinal stress produced due to decrease in length of body under a deforming force is known as **compression stress**.
- Volumetric stress** : If a body changes its volume under a normal deforming force acting on every surface of the body, the stress set up in the body is volumetric stress.
- Tangential stress** : It is also called shearing stress. When a deforming force applied tangentially to the surface of the body changes the shape of the body without changing its volume, the stress set up is known as tangential stress. The shape of the body changes or the body gets twisted due to tangential stress.

- **Strain** is defined as the ratio of change in configuration of the body because of a deforming force on it, to the original configuration of the body it means

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

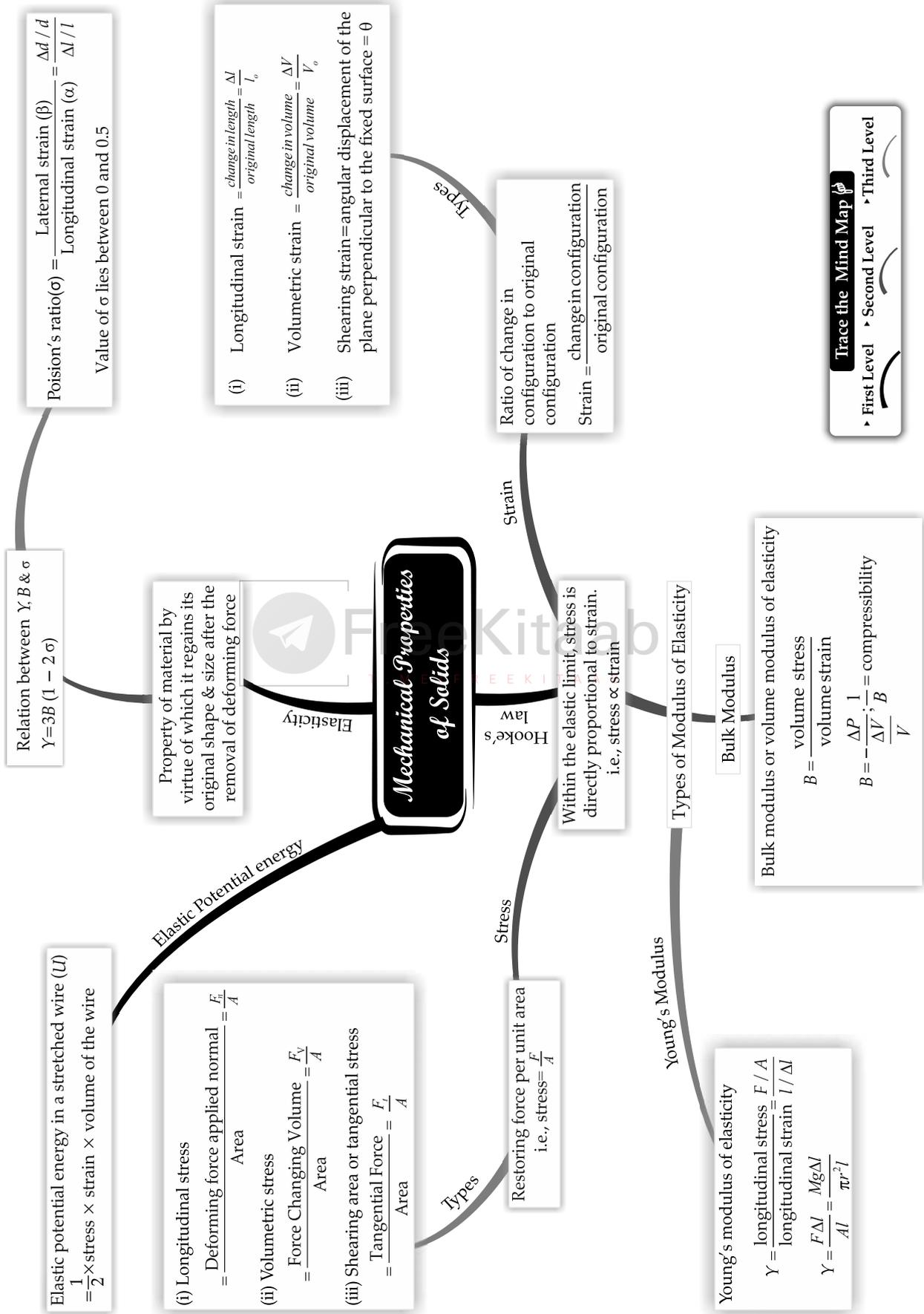
Strain can be of following three types, depending upon the direction of force applied :

- Longitudinal strain** = $\frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l}$
- Volumetric strain** = $\frac{\text{change of volume}}{\text{original volume}} = \frac{\Delta V}{V}$

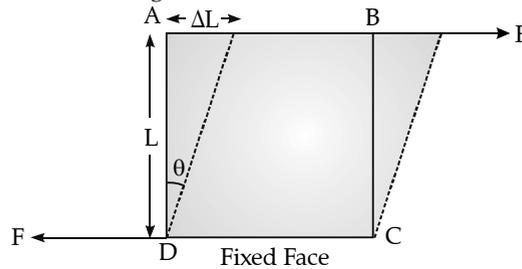
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Stress, strain Curve



(c) **Shearing strain** is produced when the deforming force is applied parallel to the surface of a body and body changes its shape without changing its volume. Shearing strain is defined as the angle through which a vertical line perpendicular to the fixed surface gets rotated under the effect of a tangential deforming force.



Shearing strain is also defined as the ratio of displacement of a surface (ΔL) under the tangential deforming force to the perpendicular distance (L) of the displaced surface from the fixed surface, i.e.,

Shearing strain,
$$\theta = \frac{\Delta L}{L}$$

Strain has no units and dimensions.

(a) If a beam is bent, both compression strain as well as extension strain are produced.

- Hooke's law states that within elastic limit, stress is directly proportional to strain, i.e., **Stress \propto Strain**.

Key Words

- **Deforming force** is that force which when applied changes the configuration of the body.
- **Elasticity** is the property of the body by virtue of which the body regains its original configuration (length, volume or shape) when the deforming forces are removed.
- **Perfectly elastic body** is that body which perfectly regains its original form on removing the external deforming force from it, e.g., quartz.
- **Plastic body** is that body which does not regain its original form at all on the removal of deforming force, however small the deforming force may be, e.g., putty and paraffin wax.
- **Elastic limit** is the upper limit of deforming force up to which if deforming force is removed, the body regains its original form completely and beyond which if the force is increased, the body loses its property of elasticity and it gets permanently deformed. Elastic limit is the property of a body whereas elasticity is the property of material of a body.

Key Formulae

- Normal stress (S) = F/A
- Breaking force = Breaking stress \times area of cross-section
- Longitudinal strain = $\frac{\Delta l}{l}$
- Volumetric strain = $\frac{\Delta V}{V}$
- Shearing strain, $\theta = \frac{\Delta L}{L}$

Very Short Answer Type Questions

(1 mark each)

Q. 1. What is ultimate or tensile strength ? R AI

Ans. The stress at which the specimen breaks or ruptures ultimately is called ultimate or tensile strength. 1

Q. 2. Define compressibility. R

Ans. Compressibility is the reciprocal of the bulk modulus, i.e., compressibility = $\frac{1}{K}$. 1

Q. 3. Which is more elastic : Water or air ? Give reason.

[U] [KVS 2013]

Ans. Elasticity is reciprocal of compressibility. Air is more compressive than water. Hence water is more elastic than air. 1



Commonly Made Error

- ▶ Student can not compare the elasticity of air and water.



Answering Tip

- ▶ Student should understand that the bulk modulus of elasticity is reciprocal of compressibility.

Q. 4. How can the structure of a crystal be studied? [U]

Ans. With the help of X-rays. 1

Q. 5. Name two classes of solids. [A]

Ans. Solids are of two types :
(i) Crystalline solids
(ii) Glassy or amorphous solids. 1



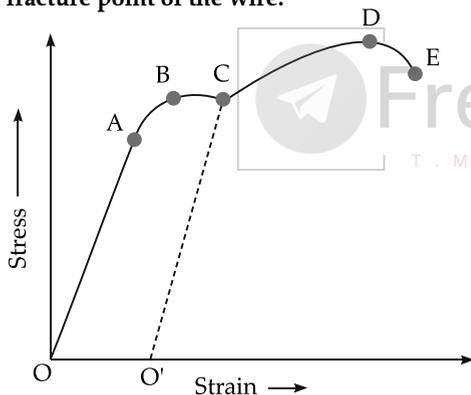
Short Answer Type Questions-I

(2 marks each)

Q. 1. What are glassy solids. [R]

Ans. The solids in which atoms and molecules are not arranged in definite and regular manner are called glassy or amorphous solids, e.g., glass, rubber, sulphur etc. 2

Q. 2. The stress-strain graph for a metal wire is given in figure. Upto the point B, the wire returns to its original state O along the curve BAO, when it is gradually unloaded. Point E corresponds to the fracture point of the wire.



- (a) Upto which point of curve, is Hooke's law obeyed? This point is also called 'Proportion limit'.
- (b) Which point on the curve corresponds to elastic limit and yield point of the wire ?
- (c) Indicate the elastic and plastic regions of the stress-strain curve.
- (d) What change happens when the wire is loaded upto a stress corresponding to point C on curve, and then unloaded gradually? [U] [NCT 2008]

Ans. (a) Upto point A, Hooke's law is obeyed because the graph is straight line from O to A. ½
(b) Point B. From the graph it is clear that the wire returns to its original position after being unloaded upto point B only, hence B is elastic limit. ½
(c) Elastic region—O to B
Plastic region—B to E ½
(d) Strain is directly proportional to load upto point A and after A, strain increases by greater amount as compared to first case (i.e., O to A) for a given increase in load. Beyond the elastic limit B, the curve does not retrace backwards as the wire is

unloaded but returns along dotted line CO'. Point O' corresponds to strain at zero load, which shows there is a permanent strain in the wire. ½



Commonly Made Error

- ▶ Student can not identify the various points on the stress versus strain curve.



Answering Tip

- ▶ Student should understand Hooke's law properly using stress - strain graph.

Q. 3. A silica glass rod has a diameter of 1 cm and is 10 cm long. Estimate the largest mass that can be hung from it without breaking it. [A]
(Breaking strength of glass is $50 \times 10^6 \text{ Nm}^{-2}$.)

Ans. Using $\text{Stress} = \frac{F}{A}$, ½

$$\begin{aligned} \text{we get } F &= \text{stress} \times A \\ &= (50 \times 10^6) \times \pi \times \left(\frac{10^{-2}}{2}\right)^2 \text{ N} \end{aligned} \quad ½$$

$$\begin{aligned} \text{Largest mass} &= \frac{F}{g} = \frac{(50 \times 10^6) \left(\frac{\pi \times 10^{-4}}{4}\right)}{10} \text{ kg} \\ &= 392.7 \text{ kg} \end{aligned} \quad 1$$



Commonly Made Error

- ▶ Student can not calculate the mass that can be hung from the glass without breaking it.



Answering Tip

- ▶ Student should understand that when the mass is hung then the force acting on the mass is 'mg' (i.e. the force of gravity).

Topic-2 Modulus of Elasticity



Revision Notes

- **Modulus of elasticity or coefficient of elasticity (E)** of a body is defined as the ratio of stress to the corresponding strain produced, within the elastic limit, i.e.,

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Modulus of elasticity is of three types :

- (a) **Young's modulus of elasticity (Y)** is defined as the ratio of longitudinal stress to the longitudinal strain, within the elastic limit, i.e.,

$$\begin{aligned}
 Y &= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} \\
 &= \frac{F/A}{\Delta l/l} = \frac{F}{A} \times \frac{l}{\Delta l}
 \end{aligned}$$

Y is the property of solid material only. Y increases on mixing the impurity in the solid and decreases on increasing the temperature of the solid body.



- (b) **Bulk modulus of elasticity (K)** is first defined by Maxwell. It is defined as the ratio of volume stress to the volumetric strain, within the elastic limit, i.e.,



$$\begin{aligned}
 K &= - \frac{\text{Volume stress}}{\text{Volumetric strain}} \\
 K &= \frac{P}{\frac{\Delta V}{V}} = - \frac{PV}{\Delta V}
 \end{aligned}$$

K is the property for solids, liquids and gases.

- **Modulus of Rigidity (η)** is defined as the ratio of tangential stress to the shearing strain, within the elastic limit, i.e.,

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

η is the characteristic of solid material only as the liquids and gases do not have fixed shape. η for liquid is zero.

- **Poisson's ratio (σ)**. It is defined as the ratio of lateral strain to the longitudinal strain, i.e.,

$$\begin{aligned}
 \sigma &= \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \\
 &= \frac{\Delta D/D}{\Delta l/l} = - \frac{\Delta D.l}{D.\Delta l}
 \end{aligned}$$

Numerically value of σ lies between -1 and + $\frac{1}{2}$ but practical value of σ lies between 0 and

+ $\frac{1}{2}$.



Key Words

- **Compressibility** is defined as the reciprocal of bulk modulus of elasticity.

$$\begin{aligned}
 \text{Compressibility (c)} &= \frac{1}{K} \\
 &= - \frac{\Delta V}{PV}
 \end{aligned}$$

- **Elastic fatigue** is the loss in strength of a material caused due to repeated alternating strains to which the material is subjected.
- Yield strength of a material is defined as the maximum stress it can sustain without crossing the elastic limit.

Key Formulae

- Young's modulus $Y = \frac{Fl}{A\Delta l}$
- Bulk modulus $K = -\frac{FV}{A\Delta V} = -P \frac{V}{\Delta V}$
- Modulus of Rigidity $\eta = \frac{FL}{A\Delta L}$
- Poisson's Ratio $\sigma = \frac{-\Delta D.l}{D.\Delta l}$
- **Relation Among Various Elastic Constants :**
 - (i) Relation between Y, K and σ , $Y = 3K(1 - 2\sigma)$
 - (ii) Relation between Y, η and σ , $Y = 2\eta(1 + \sigma)$
 - (iii) Relation between K, η and σ , $\sigma = \frac{3K - 2\eta}{2\eta + 6K}$
 - (iv) Relation between Y, K and η , $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$
- Elastic potential energy in stretched wire, $U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume of wire}$
- Elastic potential energy per unit volume of wire,

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} Y \times (\text{Strain})^2$$
- Work done in a stretching wire $W = \frac{1}{2} \times \text{Load} \times \text{Extension}.$



Mnemonics

Concept: Value of Poisson's ratio.

Mnemonics: Priyanka Roy started from **zero** mark and ran for **half** an hour.

Interpretation:

P - Poisson's

R - Ratio

zero - 0

half - $\frac{1}{2}$

Value of Poisson's ratio lies between 0 and $\frac{1}{2}$.



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is Hooke's law and Modulus of Elasticity?

[R] (DDE)

Ans. According to Hooke's law, within the elastic limit, stress is proportional to the strain.

$$\text{i.e., } \frac{\text{stress}}{\text{strain}} = \text{constant} = E$$

The constant E is called Modulus of Elasticity. 1

Q. 2. What is Young's Modulus?

[R]

Ans. It is defined as the ratio of longitudinal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

1

Q. 3. What are the factors affecting Elasticity?

[U] (DDE)

Ans. Factors are

(1) **Hammering and rolling:** This result is in an increase in the elasticity of material.

(2) **Annealing:** Annealing results in decrease in the elasticity of material

(3) **Temperature:** Elasticity decreases with rise in temperature but the elasticity of invar steel (alloy) does not change with change in temperature.

(4) **Impurities:** This type of effect depends upon the nature of impurities. 1

Q. 4. Why do spring balances show wrong readings after they have been used for a long time?

[U] (DDE)

Ans. This happens due to elastic fatigue in spring. 1

Q. 5. Which is more elastic, steel or rubber ? Explain

[U] [A] (DDE)

Ans. $Y_s = \frac{F L}{A \Delta L_s}$ and $Y_R = \frac{F L}{A \Delta L_R}$

For same force applied to wires made of steel and rubber of same length and same area of cross section,

$$\Delta L_s < \Delta L_R$$

1

$$\frac{Y_s}{Y_R} = \frac{\Delta L_R}{\Delta L_s} > 1$$

$$\therefore Y_s > Y_R$$

So, steel is more elastic. 1



Commonly Made Error

► Student can not compare the elasticity of steel and rubber.



Answering Tip

► Student should compare the elasticity of steel and rubber by comparing the extension produced in steel and rubber wires of same length, same area of cross section and considering same force applied on them.

Short Answer Type Questions

(2 marks each)

Q. 1. Two rods of different material having coefficient of linear expansion α_1 and α_2 and Young's modulus Y_1 and Y_2 respectively are fixed between two rigid walls. The rods are heated to same high temperature. If $\alpha_1 : \alpha_2 :: 2 : 3$ and the thermal stress in two rods is same, then find the ratio Y_1/Y_2 . [A]

Ans. Stress = $Y \times$ strain

$$\therefore Y_1 \alpha_1 t_1 = Y_2 \alpha_2 t_1$$

or $Y_1 \alpha_1 = Y_2 \alpha_2$ 1

or $\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1}$

$\therefore \frac{Y_1}{Y_2} = \frac{3}{2}$ 1



Commonly Made Error

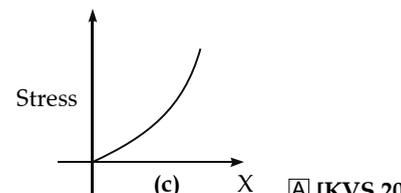
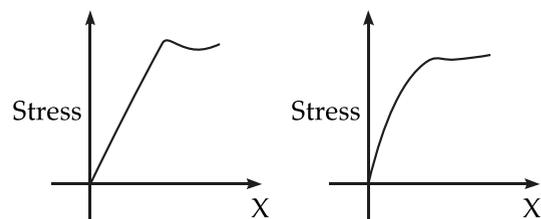
► Student can not link strain to the coefficient of linear expansion.



Answering Tip

► Strain = coefficient of linear expansion \times change in temperature.

Q. 2. Following are the graphs of elastic materials. Which one corresponds to that of brittle material ?



[A] [KVS 2008]

Ans. (b) is more brittle. A material is said to be brittle if there is small stress-strain variation beyond elastic point and fracture point of brittle material lies close to elastic point. 2



Objective Type Questions

(1 mark each)

A Multiple Choice Questions

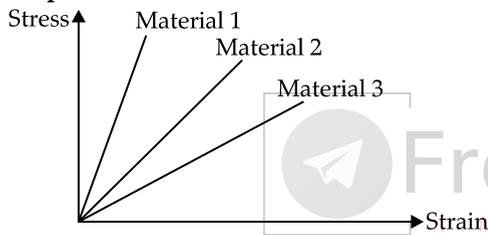
Q. 1. Rigidity modulus and Young's modulus of are respectively η and Y . A copper wire of length L and area of cross-section A is so pulled that its length becomes $5L$ and area of cross-section becomes $\frac{A}{5}$. So,

- (A) Its Y increases, η decreases.
- (B) Its η increases, Y decreases.
- (C) Its Y and η both increases.
- (D) Its Y and η both remains unchanged

Ans. Option (D) is correct.

Explanation: Modulus of elasticity does not change with dimension. It depends on material only.

Q. 2. Stress-strain graphs of 3 materials are shown upto their elastic limits.



Which of the following statements is true?

- (A) All the materials have same elasticity.
- (B) Material 1 has the highest and material 3 has the lowest elasticity.
- (C) Material 3 has the highest and material 1 has the lowest elasticity.
- (D) From stress-strain graph the comparison of elasticity of different materials is not possible.

Ans. Option (C) is correct.

Explanation: $Y = \frac{\text{stress}}{\text{strain}} = \frac{1}{\text{slope}}$

So, Material 3 has the highest and material 1 has the lowest elasticity.

Q. 3. Which of the following cannot be the value of Poisson's ratio?

- (A) 0.1 (B) 0.3
- (C) 0.5 (D) 0.8

Ans. Option (D) is correct.

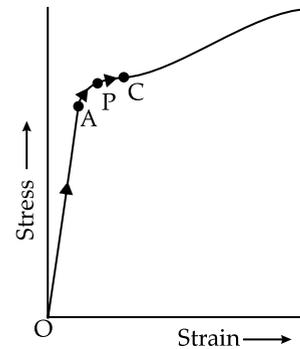
Explanation: Value of Poisson's ratio lies between 0 and 0.5.

Q. 4. Work done in stretching an wire per unit volume is

- (A) Stress \times strain (B) $\frac{\text{Stress}}{\text{strain}}$
- (C) $\frac{1}{2}$ stress \times strain (D) 2 stress \times strain

Ans. Option (C) is correct.

Q. 5. In the following stress strain graph the yield point is



- (A) A (B) B
- (C) C (D) None of these

Ans. Option (b) is correct.

Explanation: The point B is known as yield point since, this is the elastic limit.

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is true

Q. 1. Assertion (A): Strain is the ratio of change in dimension of a body to its original dimension.

Reason (R): Dimension of strain is $[ML^{-1}T^{-2}]$.

Ans. Option (C) is correct.

Explanation: Strain = $\frac{\text{Change in dimension}}{\text{Original dimension}}$,

Hence, the assertion is true.

It is a dimensionless quantity. So, the reason is false.

Q. 2. Assertion (A): Young's modulus of a perfectly plastic body is zero.

Reason (R): Perfectly plastic body cannot regain its original shape after application of pressure.

Ans. Option (B) is correct.

Explanation: Restoring force of a perfectly plastic body is zero since, it cannot regain its original shape after application of pressure.

Young's modulus = $\frac{\text{Restoring force per unit area}}{\text{Elongation per unit length}}$

$$= \frac{0}{\text{Elongation per unit length}} = 0$$

So, the assertion and reason both are true but the reason does not explain the assertion.

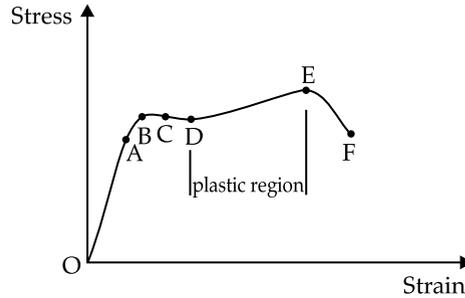
Q. 3. Assertion (A): Ductile material is used for making thin wires.

Reason (R): In stress-strain graph of a ductile material, the plastic region is too short.

Ans. Option (C) is correct.

Explanation: Ductile material is malleable. Hence it is used for making thin wires. So, the assertion is true.

In stress-strain graph of a ductile material, the plastic region is extensive.



NCERT Corner

Q. 1. A steel wire of length 4.7 m and cross-section $3.0 \times 10^{-5} \text{ m}^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper ?

Ans. Using the relation,

$$Y = \frac{Fl}{A\Delta l}$$

$$Y_s = \frac{Fl}{A\Delta l} = \frac{F \times 4.7}{(3 \times 10^{-5})\Delta l}$$

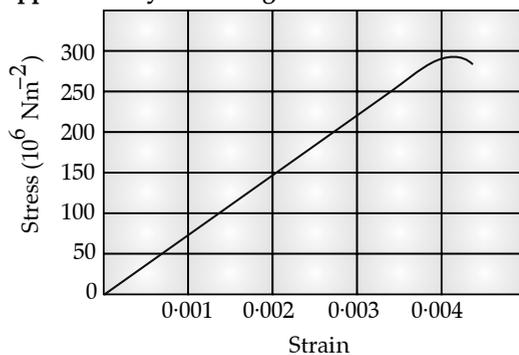
and

$$Y_c = \frac{Fl}{A\Delta l} = \frac{F \times 3.5}{(4 \times 10^{-5})\Delta l}$$

then

$$\frac{Y_s}{Y_c} = \frac{F \times 4.7}{(3 \times 10^{-5})\Delta l} \times \frac{(4 \times 10^{-5})\Delta l}{F \times 3.5} = 1.79$$

Q. 2. Figure shows the strain-stress curve for a given material. What are (a) Young's modulus, and (b) approximate yield strength for this material ?



Ans. From the graph for a stress of $150 \times 10^6 \text{ Nm}^{-2}$, the strain is 0.002.

(a) Modulus of the material (Y) is given by

$$Y = \frac{150 \times 10^6}{0.002} = \frac{150 \times 10^6}{2 \times 10^{-3}} = 75 \times 10^9 \text{ Nm}^{-2} = 7.5 \times 10^{10} \text{ Nm}^{-2}$$

(b) Yield strength of a material is defined as the maximum stress it can sustain without crossing the elastic limit.

\therefore From graph, the approximate yield strength of the given material

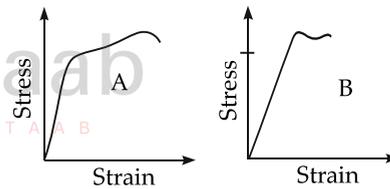
$$= 300 \times 10^6 \text{ Nm}^{-2} = 3 \times 10^8 \text{ Nm}^{-2}$$

Q. 3. The stress-strain graphs for materials A and B are shown in figure. The graphs are drawn on the same scale.

(a) Which of the materials has the greater Young's modulus ?

(b) Which of the two is stronger material ?

[NCT 2009]



Ans. (a) From graph it is clear that for a given strain, stress for A is more than that of B. Hence, Young's modulus (= stress/strain) is greater for A than that of B.

(b) A is stronger than B. Strength of a material is measured by the amount of stress required to cause fracture, corresponding to the point of fracture.

Q. 4. Read the following two statements below carefully and state, with reasons, if it is true or false:

(a) The Young's modulus of rubber is greater than that of steel.

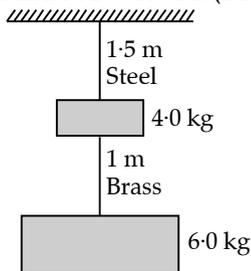
(b) The stretching of a coil is determined by its shear modulus.

Ans. (a) Incorrect. This is because if steel and rubber wires of same length and area of cross-section are subjected to same deforming force, then the extension produced in steel is less than the extension produced in rubber, so $Y_s > Y_r$. In other words, for producing same strain in steel and rubber, more stress is required in case of steel.

(b) **Correct.** The reason is that when a coil spring is stretched, there is neither a change in the length of the coil (i.e., length of the wire forming the coil spring) nor a change in its volume. Since, the

change takes place in the shape of the coil spring, its stretching is determined by its shear modulus.

- Q. 5.** Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of steel and brass wires. Young's modulus of steel and copper are respectively 2.0×10^{11} Pa and 0.91×10^{11} Pa. ($1 \text{ Pa} = 1 \text{ Nm}^{-2}$).



Ans. For steel wire,

$$\begin{aligned} \text{Total force, } F_1 &= 4 + 6 = 10 \text{ kgf} \\ &= 10 \times 9.8 \text{ N} \\ l_1 &= 1.5 \text{ m, } \Delta l_1 = ? \\ 2r_1 &= \text{Diameter of wire} = 0.25 \text{ cm.} \\ r_1 &= \frac{0.25 \times 10^{-2}}{2} \text{ m} \\ &= 0.125 \times 10^{-2} \text{ m} \end{aligned}$$

\therefore Area $A_1 = \pi r_1^2$,
Let Y_1 be the Young's modulus of steel wire,

$$\text{Then } Y_1 = \frac{F_1 / A_1}{\Delta l_1 / l_1}$$

$$\begin{aligned} \text{or } \Delta l_1 &= \frac{F_1 l_1}{A_1 Y_1} \\ &= \frac{(10 \times 9.8)(1.5) \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} \\ &= 1.5 \times 10^{-4} \text{ m.} \end{aligned}$$

In case of brass wire

$$\begin{aligned} F_2 &= 6.0 \text{ kgf} = 6 \times 9.8 \text{ N} \\ Y_2 &= 0.91 \times 10^{11} \text{ pa} \\ 2r_2 &= 0.25 \text{ cm} \\ \therefore r_2 &= \frac{0.25}{2} \text{ cm} \\ &= 0.125 \times 10^{-2} \text{ m} \\ \therefore l_2 &= 1.0 \text{ m, } \Delta l_2 = ? \\ \therefore \Delta l_2 &= \frac{F_2 l_2}{A_2 Y_2} \\ &= \frac{(6 \times 9.8) \times 1.0 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 0.91 \times 10^{11}} \\ &= 1.3 \times 10^{-4} \text{ m.} \end{aligned}$$



Commonly Made Error

- ▶ Student can not find the force acting on steel and brass wires correctly.



Answering Tip

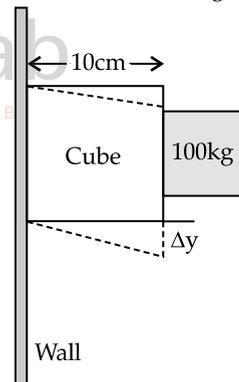
- ▶ A free body diagram can help in giving a better visualization and a clear picture.

- Q. 6.** The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face ?

Ans. Given : Side of cube, $L = 10 \text{ cm} = 0.1 \text{ m}$

$$\begin{aligned} \therefore A &= \text{area of the face (1)} \\ &= L \times L = L^2 \\ &= (0.1)^2 \\ &= 0.01 \text{ m}^2. \end{aligned}$$

Mass attached to Face (1),
 $M = 100 \text{ kg}$



When F be the force on face (1) due to this mass. Then

$$F = Mg = 100 \times 9.8 \text{ N.}$$

$$\begin{aligned} \therefore \text{Shear stress on the face is given} \\ &= \frac{F}{A} = \frac{100 \times 9.8}{0.01} \text{ Nm}^{-2} \\ &= 9.8 \times 10^4 \text{ Nm}^{-2} \end{aligned}$$

Shear modulus of aluminium,

$$\eta = 25 \text{ GPa} = 25 \times 10^9 \text{ Nm}^{-2}$$

Using the relation,

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} \quad \dots(1)$$

Suppose $\Delta y =$ Vertical displacement of the face = ?

$$\begin{aligned} \therefore \text{Shearing strain,} \\ \frac{\Delta y}{L} &= \frac{\text{Shearing stress}}{\eta} \end{aligned}$$

$$\begin{aligned} \text{or } \Delta y &= \frac{\text{Shearing stress}}{\eta} \times L \\ &= \frac{9.8 \times 10^4 \times 0.1}{25 \times 10^9} \\ \text{or } \Delta y &= 0.0392 \times 10^{-5} \text{ m} \\ \text{or } &= 3.92 \times 10^{-7} \text{ m} \\ &\approx 4 \times 10^{-7} \text{ m} \end{aligned}$$

Q. 7. Four identical hollow cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. The Young's modulus of steel is 2.0×10^{11} Pa.

Ans. Cross-sectional area of a column

$$\begin{aligned} &= \pi (r_2^2 - r_1^2) \\ &= \pi (0.6^2 - 0.3^2) \\ &= 0.27\pi \text{ m}^2 \end{aligned}$$

Force on one column,

$$F = \frac{50000 \times 9.8}{4}$$

Compressive strain

$$\begin{aligned} Y &= \frac{F/A}{\Delta l/l} \\ &= \frac{\Delta l}{l} = \frac{F}{AY} \\ &= \frac{50000 \times 9.8}{4(0.27\pi)(2 \times 10^{11})} \\ &= 7.21 \times 10^{-7} \end{aligned}$$



Commonly Made Error

- ▶ The mass of the whole structure consisting of four cylindrical columns is considered in calculating the force on one column.



Answering Tip

- ▶ Student should first enlist the quantities given in the numerical problem and then solve it step by step to avoid mistakes.

Q. 8. A piece of copper having a rectangular cross-section of $15.2 \text{ mm} \times 19.1 \text{ mm}$ is pulled in tension with 44,500N force, producing only elastic deformation. Calculate the resulting strain ? (Y for copper = $1.1 \times 10^{11} \text{ Nm}^{-2}$.)

Ans. Given :

$$\begin{aligned} Y &= 1.1 \times 10^{11} \text{ Nm}^{-2} \\ A &= \text{Area of cross-section} \\ &= 15.2 \text{ mm} \times 19.1 \text{ mm} \\ &= 15.2 \times 10^{-3} \text{ m} \times 19.1 \times 10^{-3} \text{ m}^2 \end{aligned}$$

Force, $F = 44500 \text{ N}$.

$$\therefore Y = \frac{\text{Stress}}{\text{Strain}}$$

or $\text{Strain} = \frac{\text{Stress}}{Y} = \frac{F}{AY}$

or $\text{Strain} = \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 1.1 \times 10^{11}}$
 $= 1.39 \times 10^{-3}$.

Q. 9. A steel cable with a radius of 1.5 cm supports a chair lift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2} , what is the maximum load the cable can support ?

Ans. Given : Radius of steel cable,

$$r = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}.$$

$$\text{Maximum Stress} = 10^8 \text{ Nm}^{-2}.$$

\therefore Area of cross-section of cable

$$A = \pi r^2 = \pi (1.5 \times 10^{-2})^2 \text{ m}^2$$

Maximum load the cable can stand

$$= \text{Maximum force}$$

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross-section}}$$

or Maximum force = Maximum stress

$$\times \text{Area of cross-section}$$

or $F_{\text{max}} = 10^8 \times \pi \times (1.5 \times 10^{-2})^2$

or $F_{\text{max}} = 3.142 \times 2.25 \times 10^8 \times 10^{-4} \text{ N}$.

or Maximum load the cable can withstand

$$= 7.07 \times 10^4 \text{ N}.$$

Q. 10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension. Given Young modulus of copper and iron are respectively $110 \times 10^9 \text{ Nm}^{-2}$ and $190 \times 10^9 \text{ Nm}^{-2}$.

Ans. Suppose Y_1 and Y_2 be the Young's modulus of copper and iron wires respectively

$$Y_1 = 110 \times 10^9 \text{ Nm}^{-2}$$

$$Y_2 = 190 \times 10^9 \text{ Nm}^{-2}$$

Also suppose A_1 and A_2 be the areas of cross-section of copper and iron wires respectively. If d_1 and d_2 be their respective diameters.

Then $A_1 = \pi d_1^2/4$

and $A_2 = \pi d_2^2/4$.

$$\frac{A_1}{A_2} = \frac{d_1^2}{d_2^2} = \left(\frac{d_1}{d_2}\right)^2$$

\therefore

$$L = 2 \text{ m}.$$

Suppose Δl be the extension produced in each wire.

Suppose F = Tension produced in each wire

\therefore Using relation,

$$Y = \frac{\text{Stress}}{\text{Strain}},$$

we get, Strain for copper wire = $\frac{F/A_1}{Y_1}$

and Strain for iron wire = $\frac{F/A_2}{Y_2}$

Since, the bar is supported symmetrically,
∴ The two strains are equal

$$\therefore \frac{F}{A_1 Y_1} = \frac{F}{A_2 Y_2}$$

or $A_1 Y_1 = A_2 Y_2$

$$\frac{A_1}{A_2} = \frac{Y_2}{Y_1}$$

$$\frac{\pi d_1^2/4}{\pi d_2^2/4} = \frac{Y_2}{Y_1}$$

$$\begin{aligned} \frac{d_1}{d_2} &= \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} \\ &= \sqrt{\frac{19}{11}} = \sqrt{1.73} \end{aligned}$$

It means, $\frac{d_1}{d_2} = 1.31$

or $d_1 : d_2 = 1.31 : 1$.

Q. 11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm². Calculate the elongation of wire when the mass is at the lowest point of its path. $Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$

Ans. Given: $m = 14.5 \text{ kg}, l = r = 1 \text{ m}$
 $\omega = 2 \text{ rps} = 2 \times 2\pi \text{ rad/s}$
 $A = 0.065 \times 10^{-4} \text{ m}^2$

Tension in wire at lowest position on vertical circle = $F = mg + m\omega^2 r$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times \frac{22}{7} \times \frac{22}{7} \times 1$$

$$= 142.1 + 2291.6$$

$$= 2433.7 \text{ N}$$

$$Y = \frac{Fl}{A\Delta l}$$

or $\Delta l = \frac{Yl}{AY}$

$$= \frac{2433.7 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= 1.87 \times 10^{-3} \text{ m}$$

$$\Delta l = 1.87 \text{ mm}$$



Commonly Made Error

► Student can not compute the tension in the lowest point of the string.



Answering Tip

► Student should study motion of mass in a vertical circle.

Q. 12. Compute the bulk modulus of water from the following data : Initial volume = 100 litre, pressure increase = 100 atmosphere, final volume = 100.5 litre (1 atmosphere = $1.013 \times 10^5 \text{ pa}$).

Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Ans. Bulk Modulus,

$$\begin{aligned} B &= \frac{-P}{\frac{\Delta V}{V}} \\ &= P \frac{V}{\Delta V} \\ &= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}} \end{aligned}$$

or $B_w = 2.026 \times 10^9 \text{ Pa}$

Bulk Modulus of air, $B_A = 1.0 \times 10^5 \text{ Pa}$

$$\begin{aligned} \therefore \frac{\text{Bulk Modulus of water, } B_w}{\text{Bulk Modulus of air, } B_A} &= \frac{2.026 \times 10^9}{1.0 \times 10^5} \\ &= 2.026 \times 10^4 \end{aligned}$$

This ratio is too large because gases are more compressible than those of liquids. Interatomic forces are more strong in liquids than gases.

Q. 13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3 \text{ kg m}^{-3}$? Compressibility of water is $45.8 \times 10^{-11} \text{ Pa}^{-1}$ and 1 atm = $1.03 \times 10^5 \text{ Pa}$.

Ans. Compressibility = $\frac{1}{K} = (45.8 \times 10^{-11}) \text{ Pa}^{-1}$

Pressure, $P = 80 \text{ atm}$
 $= 80 \times 1.013 \times 10^5 \text{ Pa}$

Let change in volume,

$$\Delta V = \frac{M}{\rho} - \frac{M}{\rho_d} = M \left(\frac{1}{\rho} - \frac{1}{\rho_d} \right)$$

∴ Volumetric strain

$$= \frac{\Delta V}{V}$$

$$= M \left[\frac{1}{\rho} - \frac{1}{\rho_d} \right] \frac{\rho}{M} = 1 - \frac{\rho}{\rho_d}$$

where ρ is density of water at surface and ρ_d is the density of water at depth.

$$\frac{\Delta V}{V} = \frac{1}{K}$$

$$1 - \left(\frac{1.03 \times 10^3}{\rho_d} \right) = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11}$$

$$\rho_d = 1.034 \times 10^3 \text{ kgm}^{-3}.$$

Q. 14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm. K for glass slab = $37 \times 10^9 \text{ Nm}^{-2}$ [AI]

Ans. Given :

$$P = 10 \text{ atm}$$

$$= 10 \times 1.013 \times 10^5 \text{ Pa}$$

$$K \text{ for glass slab} = 37 \times 10^9 \text{ Nm}^{-2}.$$

Fractional change in volume of glass slab

$$= \frac{\Delta V}{V} = ?$$

From the relation

$$K = \frac{P}{\frac{\Delta V}{V}}$$

$$\frac{\Delta V}{V} = \frac{P}{K} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9}$$

$$= \frac{1.013}{37 \times 10^3} = 0.0274 \times 10^{-3}$$

$$= 2.74 \times 10^{-5}$$

$$\approx 0.003\%.$$

Q. 15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of $7 \times 10^6 \text{ Pa}$. K for copper = $140 \times 10^9 \text{ Pa}$.

Ans. Given :

$$L = 10 \text{ cm} = 0.1 \text{ m}$$

$$K = \text{bulk modulus of Cu}$$

$$= 140 \times 10^9 \text{ Pa}$$

$$P = 7 \times 10^6 \text{ Pa}$$

$\Delta V =$ Volume contraction of solid copper cube = ?

$$\therefore V = L^3 = (0.1)^3 = 0.001 \text{ m}^3.$$

Using formula, $K = - \frac{P}{\left(\frac{\Delta V}{V} \right)}$

we get $\Delta V = - \frac{PV}{K} = \frac{7 \times 10^6 \times 0.001}{140 \times 10^9} \text{ m}^3$

$$= - \frac{1}{20} \times 10^{-6} \text{ m}^3$$

$$= -0.05 \times 10^{-6} \text{ m}^3$$

$$= -5 \times 10^{-2} \text{ cm}^3$$

Here, negative sign shows volume contraction.

Q. 16. How much should the pressure on a litre of water be changed to compress it by 0.10%. Bulk modulus of water = $2.2 \times 10^9 \text{ Nm}^{-2}$ [AI]

Ans. Here

$$V = 1 \text{ litre}$$

$$\Delta V = -0.10\% \text{ of } V$$

$$= - \frac{0.10}{100} \times 1 = - \frac{1}{1000} \text{ litre}$$

Suppose $\Delta P =$ change in pressure required for compression of 1 litre of water.

$$K = \text{bulk modulus of water} = 2.2 \times 10^9 \text{ Nm}^{-2}$$

From the relation

$$K = - \frac{P}{\left(\frac{\Delta V}{V} \right)}$$

we get

$$\Delta P = -K \cdot \frac{\Delta V}{V}$$

$$\Delta P = 2.2 \times 10^9 \times \left(\frac{1}{1000} \right)$$

$$\Delta P = \frac{2.2 \times 10^9}{1000}$$

$$= 2.2 \times 10^6 \text{ Nm}^{-2}.$$



Commonly Made Error

- ▶ Students fail to comprehend the term 'compress it by 0.10%'.



Answering Tip

- ▶ Student should have practice in solving numerical problems related to percentage.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

- Q. 1. Modulus of rigidity of ideal liquids is**
- (A) infinity.
 - (B) zero
 - (C) unity.
 - (D) some finite small non-constant value.

Ans. Option (B) is correct.



Commonly Made Error

- ▶ Student can not estimate the value of modulus of rigidity for an ideal liquid.



Answering Tip

- ▶ A liquid cannot sustain tangential force. So modulus of rigidity is zero.

Q. 2. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will

- (A) be double.
- (B) be half.
- (C) be four times.
- (D) remain same.

Ans. Option (D) is correct.



Commonly Made Error

- ▶ Student can not judge the effect of reducing the length of the wire to the breaking force.



Answering Tip

- ▶ Breaking force is independent of the length of the wire. It rather depends directly on the area of cross section.

Q. 3. The temperature of a wire is doubled. The Young's modulus of elasticity

- (A) will also double.
- (B) will become four times.
- (C) will remain same.
- (D) will decrease.

Ans. Option (D) is correct.

Q. 4. A spring is stretched by applying a load to its free end. The strain produced in the spring is

- (A) volumetric.
- (B) shear.
- (C) longitudinal and shear.
- (D) longitudinal.

Ans. Option (C) is correct.

Q. 5. A rigid bar of mass M is supported symmetrically by three wires each of length l . Those at each end are of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to

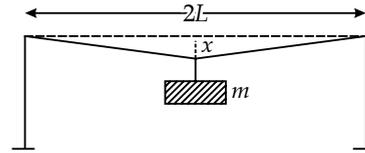
- (A) $Y_{\text{copper}}/Y_{\text{iron}}$
- (B) $\sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}}$
- (C) $\frac{Y_{\text{iron}}^2}{Y_{\text{copper}}^2}$
- (D) $\frac{Y_{\text{iron}}}{Y_{\text{copper}}}$

Ans. Option (B) is correct.

Explanation: Try yourself. Refer Q.10. of NCERT Corner.

Q. 6. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic

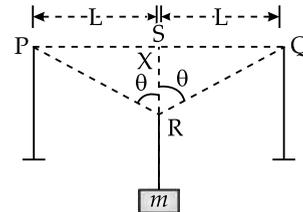
limit, horizontally between two pillars. A mass m is suspended from the mid point of the wire. Strain in the wire is



- (A) $\frac{x^2}{2L^2}$
- (B) $\frac{x}{L}$
- (C) $\frac{x^2}{L}$
- (D) $\frac{x^2}{2L}$

Ans. Option (A) is correct.

Explanation:



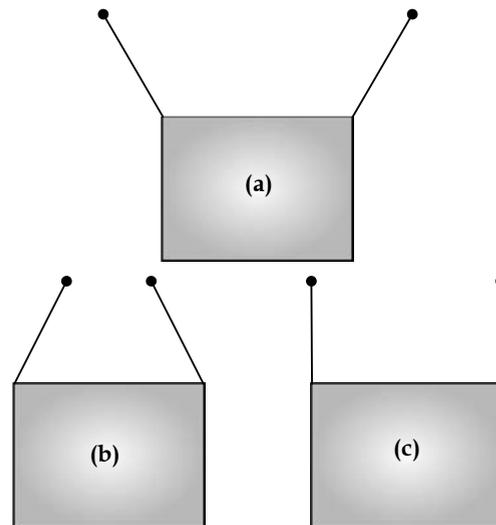
$$\text{Increase in length, } \Delta L = (PR + RQ) - PQ = 2PR - PQ$$

$$\Delta L = 2(L^2 + x^2)^{1/2} - 2L = 2L \left(1 + \frac{x^2}{L^2}\right)^{1/2} - 2L$$

$$= 2L \left[1 + \frac{1}{2} \frac{x^2}{L^2}\right] - 2L \text{ (by binomial theorem)}$$

$$\text{strain} = \frac{\Delta L}{2L} = \frac{x^2}{2L^2}$$

Q. 7. A rectangular frame is to be suspended symmetrically by two strings of equal length on two supports. It can be done in one of the following three ways:



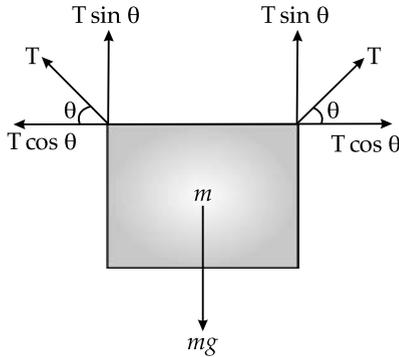
The tension in the strings will be

- (A) The same in all cases.
- (B) least in (A).

- (C) least in (B).
(D) least in (C).

Ans. Option (C) is correct.

Explanation:



Let M be the mass and θ be the angle which the tension T in the string makes with the horizontal.

$$2T \sin \theta = mg$$

$$T = \frac{mg}{2 \sin \theta} \text{ or } T \propto \frac{1}{\sin \theta}$$

T is least if $\sin \theta$ has maximum value i.e.,
 $\sin \theta = 1 = \sin 90^\circ$

Q. 8. Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods.

- (A) Both the rods will elongate but there shall be no perceptible change in shape.
(B) The steel rod will elongate and change shape but the rubber rod will only elongate.
(C) The steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse.
(D) The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre.

Ans. Option (D) is correct.

Very Short Answer Type Questions

Q. 9. The Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress?

[NCERT Exemplar Q. No. 14]

Ans. Young's modulus,

$$Y = \frac{\text{stress}}{\text{Longitudinal strain}}$$

$Y \propto \text{stress}$ (for same longitudinal strain)

$$\therefore \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} = \frac{\text{stress}_{\text{steel}}}{\text{stress}_{\text{rubber}}} \quad \dots(1)$$

(Given: $Y_{\text{steel}} > Y_{\text{rubber}}$),

$$\therefore \frac{Y_{\text{steel}}}{Y_{\text{rubber}}} > 1$$

From eqn (1) $\frac{\text{stress}_{\text{steel}}}{\text{stress}_{\text{rubber}}} > 1$ or $\text{stress}_{\text{steel}} > \text{stress}_{\text{rubber}}$

Q. 10. Is stress a vector quantity?

[NCERT Exemplar Q. No. 15]

Ans. No.

$$\text{Stress} = \frac{\text{Magnitude of restoring force by solid}}{\text{area of cross-section}}$$

Therefore, stress is tensor quantity



Answering Tip

- Student should have clear knowledge about scalar, vector and tensor quantities.

Q. 11 Identical springs of steel and copper are equally stretched. On which, more work will have to be done? [NCERT Exemplar Q. No. 16]

Ans. Work done in stretching a wire, $W = \frac{1}{2} F \times \Delta l$.

[F = applied force, Δl = extension in wire]

Springs are equally stretched, therefore, for same force (F).

$$W \propto \Delta l \quad \dots(i)$$

Y (Young's modulus)

$$Y = \frac{F}{A} \times \frac{l}{\Delta l} \text{ or } \Delta l = \frac{F}{A} \times \frac{l}{Y}$$

Both springs are identical,

$$\Delta l \propto \frac{1}{Y} \quad \dots(ii)$$

From (i) & (ii),

$$W \propto \frac{1}{Y} \text{ or } \frac{W_{\text{steel}}}{W_{\text{copper}}} = \frac{Y_{\text{copper}}}{Y_{\text{steel}}} < 1 \ (\because Y_{\text{steel}} > Y_{\text{copper}})$$

$\therefore W_{\text{steel}} < W_{\text{copper}}$. Hence, more work will be done for stretching copper spring.

Q. 12. What is the Young's modulus for a perfect rigid body? [NCERT Exemplar Q. No. 17]



Commonly Made Error

- Student lack understanding about tensor quantities.

Ans. Young's Modulus, $Y = \frac{F}{A} \times \frac{l}{\Delta l}$

A rigid body cannot be deformed by applying any deforming force, $\therefore \Delta l = 0$.

or $Y = \frac{F}{A} \times \frac{l}{0} = \infty$

\therefore For perfect rigid body, Young's modulus is infinity

Q. 13. What is the Bulk modulus for a perfect rigid body? [NCERT Exemplar Q. No. 18]

Ans. Bulk modulus, $K = \frac{-P}{\frac{\Delta V}{V}} = \frac{P}{\frac{\Delta V}{V}}$

As perfect rigid body does not change its shape even after infinite force. Hence, $\Delta V = 0$

or $K = \frac{PV}{0} = \infty$

Therefore, Bulk modulus for a perfect rigid body is infinity.

Short Answer Type Questions

Q. 14. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f , its length increases by l . Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

[NCERT Exemplar Q. No. 19]

Ans. Young's Modulus $= \frac{f}{A} \times \frac{L}{l}$, where l is the increase in length of wire l

For I wire, $Y = \frac{f}{\pi r^2} \times \frac{L}{l}$ (i)

For II wire, the increase in length be l' .

Then $Y = \frac{2f}{4\pi r^2} \times \frac{2L}{l'}$
 $Y = \frac{f}{\pi r^2} \times \frac{L}{l'}$ (ii)

From eqⁿ(s) (i) and (ii)

$$\frac{f}{\pi r^2} \times \frac{L}{l} = \frac{f}{\pi r^2} \times \frac{L}{l'}$$

$\therefore l = l'$

Q. 15. A steel rod ($Y = 2.0 \times 10^{11} \text{ Nm}^{-2}$, and $\alpha = 10^{-5} \text{ }^\circ\text{C}^{-1}$) of length 1 m and area of cross-section 1 cm^2 is heated from 0°C to 200°C without being allowed to extend or bend. What is the tension produced in the rod? [AI]

[NCERT Exemplar Q. No. 20]

Ans. Here, the equation of thermal expansion for linear expansion will be applied because of increase in temperature of the rod, length, increase.

$$\Delta T = 200^\circ\text{C} - 0^\circ\text{C} = 200^\circ\text{C}, \alpha = 10^{-5} \text{ }^\circ\text{C}^{-1}, l = 1 \text{ m}$$

$$\text{Area of cross-section, } A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$$

$$\therefore \frac{\Delta l}{l} = \alpha \Delta T = 10^{-5} \times 200 = 2 \times 10^{-3}$$

$$\begin{aligned} \text{Tension produced in steel rod} &= YA\alpha\Delta T \\ &= 2.0 \times 10^{11} \times 1 \times 10^{-4} \times 2 \times 10^{-3} \\ &= 4 \times 10^4 \text{ N.} \end{aligned}$$

Q. 16. To what depth must a rubber ball be taken in deep sea so that its volume is decreased by

0.1%. (The bulk modulus of rubber is $9.8 \times 10^8 \text{ Nm}^{-2}$, and the density of sea water is 10^3 kgm^{-3} .) [NCERT Exemplar Q. No. 21]

Ans. Bulk Modulus, $B = 9.8 \times 10^8 \text{ N/m}^2$
 Density of sea water (ρ) = 10^3 Kg/m^3
 Volume decrease (Percentage),

$$\left(\frac{\Delta V}{V} \times 100 \right) = 0.1\%$$

$$\begin{aligned} \frac{\Delta V}{V} &= \frac{0.1}{100} \\ &= \frac{1}{1000} = 1 \times 10^{-3} \end{aligned}$$

Let rubber ball be taken up to depth h
 \therefore change in pressure, $P = h\rho g$
 Bulk Modulus,

$$B = \left| \frac{P}{\frac{\Delta V}{V}} \right| = \frac{h\rho g}{\frac{\Delta V}{V}}$$

$$\begin{aligned} \text{or, } h &= \frac{B \times (\frac{\Delta V}{V})}{\rho g} \\ &= \frac{9.8 \times 10^8 \times 1 \times 10^{-3}}{10^3 \times 9.8} \\ h &= 100 \text{ m} \end{aligned}$$



Commonly Made Error

► Student do not consider pressure, $P = h\rho g$



Answering Tip

► Student should familiarize themselves with pressure related terminology.

Q. 17. A truck is pulling a car out of a ditch by means of a steel cable that is 9.1 m long and has a radius of 5 mm. When the car just begins to move, the tension in the cable is 800 N. How much has the cable stretched? (Young's modulus for steel is $2 \times 10^{11} \text{ Nm}^{-2}$.) [NCERT Exemplar Q. No. 22]

Ans. Given:

Steel cable's length, $l = 9.1 \text{ m}$

Radius, $r = 5\text{mm} = 5 \times 10^{-3}\text{ m}$.
Tension in cable, $F = 800\text{ N}$
Young's modulus = $2 \times 10^{11}\text{ N/m}^2$
Young's modulus ,

$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

or

$$\Delta l = \frac{F}{\pi r^2} \times \frac{l}{Y}$$

$$= \frac{800 \times 9.1}{3.14 \times (5 \times 10^{-3})^2 \times 2 \times 10^{11}}$$

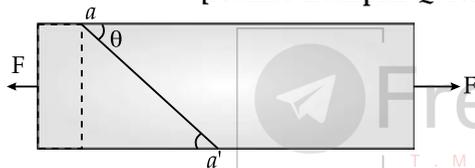
$$= 4.64 \times 10^{-4}\text{ m}$$

👁 Long Answer Type Questions

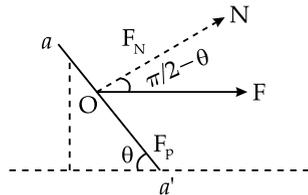
Q. 19. Consider a long steel bar under a tensile stress due to forces F acting at the edges along the length of the bar. Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?

- (a) For what angle is the tensile stress maximum?
(b) For what angle is the shearing stress maximum?

[NCERT Exemplar Q. No. 24]



Ans. Let the cross sectional area of the bar be A .



Consider the equilibrium of the plane aa' . A force F must be acting on this plane making an angle $\frac{\pi}{2} - \theta$ with the normal ON . Resolving F into components, along the plane (F_p) and normal to the plane (F_n)

$$F_p = F \cos \theta$$

$$F_n = F \sin \theta$$

Let the area of the aa' be A' . Then

$$\frac{A}{A'} = \sin \theta$$

$$\therefore A' = \frac{A}{\sin \theta} \quad \dots(1)$$

Tensile stress = $\frac{\text{Normal force}}{\text{Area}}$

$$= \frac{F \sin \theta}{A'} = \frac{F \sin \theta}{A/\sin \theta} = \frac{F \sin^2 \theta}{A} \quad (\text{from (1)})$$

Q. 18. Two identical solid balls, one of ivory and the other of wet-clay, are dropped from the same height on the floor. Which one will rise to a greater height after striking the floor and why?

[NCERT Exemplar Q. No. 23]

Ans. Since, the ivory ball is more elastic than the wet-clay ball, therefore, ivory ball will tend to retain its shape instantaneously after the collision. Hence, there will be a large energy and momentum transfer to the ivory ball as compared to the wet clay ball. Thus, the ivory ball will rise higher after striking the floor.

$$\text{shearing stress} = \frac{\text{parallel force}}{\text{Area}} = \frac{F \cos \theta}{A/\sin \theta}$$

$$= \frac{F}{A} \cos \theta \cdot \sin \theta = \frac{F}{2A} \cdot 2 \cos \theta \sin \theta$$

$$= \frac{F}{2A} \cdot \sin 2\theta$$

(a) Maximum Tensile stress, $\sin^2 \theta = 1$

or $\sin \theta = 1$

$$\theta = \frac{\pi}{2}$$

(b) Maximum shearing stress, $\sin 2\theta = 1$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$



Commonly Made Error

- ▶ Student do not consider the area of the surface correctly.



Answering Tip

- ▶ Student should carefully consider the dimensions of the surface on which stress is to be evaluated.

Q. 20. (a) A steel wire of mass μ per unit length with a circular cross section has a radius of 0.1 cm. The wire is of length 10 m when measured lying horizontal, and hangs from a hook on the wall. A mass of 25 kg is hung from the free end of the wire. Assuming the wire to be uniform and lateral strains \ll longitudinal strains, find the extension in the length of the wire. The density of steel is 7860 kg m^{-3} (Young's modulus $Y = 2 \times 10^{11}\text{ Nm}^{-2}$). [NCERT Exemplar Q. No. 25]

(b) If the yield strength of steel is $2.5 \times 10^8 \text{ Nm}^{-2}$, what is the maximum weight that can be hung at the lower end of the wire?

Ans. (a) Different forces are acting on the wire which is hanging, because different part of wire will be acted upon by different forces. Therefore, Consider a small element dx at a distance x from the load ($x = 0$). Let $T(x)$ and $T(x + dx)$ are tensions on the two cross sections a distance dx apart. Then

$$T(x + dx) - T(x) = dm g$$

$$= \mu g dx \text{ (where } \mu \text{ is the mass/length)}$$

$$[\because dm = \mu dx]$$

$$\text{or, } dT = \mu g dx \quad [\because dT = T(x + dx) - T(x)]$$

$$\text{or } T(x) = \mu g x + C \quad \text{[Integrating]}$$

$$\text{At } x=0, T(0) = Mg \text{ or } C = Mg$$

$$\therefore T(x) = \mu g x + Mg$$

Let the length dx at x increase by dr . then

$$\text{Young's Modulus, } Y = \frac{\text{stress}}{\text{strain}}$$

$$\frac{T(x)}{A} = Y$$

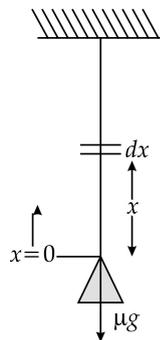
$$\frac{dT(x)}{dx} = \frac{1}{YA} T(x)$$

$$\text{or } \frac{dr}{dx} = \frac{1}{YA} T(x)$$

$$\text{or } dr = \frac{1}{YA} \int_0^L (\mu g x + Mg) dx$$

$$\text{or } r = \frac{1}{YA} \left[\frac{\mu g x^2}{2} + Mg x \right]_0^L$$

$$\text{or } r = \frac{1}{YA} \left[\frac{mgL^2}{2} + MgL \right] \quad [\because \mu L = m]$$



(m is the mass of the wire)

$$A = \pi \times (10^{-3})^2 \text{ m}^2,$$

$$Y = 200 \times 10^{11} \text{ Nm}^{-2}$$

$$m = \pi \times (10^{-3})^2 \times 10 \times 7860 \text{ kg}$$

$$\therefore r = \frac{1}{2 \times 10^{11} \times \pi \times 10^{-6}}$$

$$\left[\frac{\pi \times 786 \times 10^{-4} \times 10 \times 10^2}{2} + 25 \times 10 \times 10 \right]$$

$$= [196.5 \times 10^{-6} + 3.98 \times 10^{-3}] \approx 4 \times 10^{-3} \text{ m}$$

(b) Clearly, the maximum tension would be at $x=L$
 $T = \mu g L + Mg = (m + M)g \quad [\because m = \mu L]$

$$\text{The yield force} = \text{yield strength} \times \text{area}$$

$$= 250 \times 10^6 \times \pi \times (10^{-3})^2 = 250 \times \pi \text{ N}$$

At yield point, $T = \text{Yield Force}$

$$(M + m)g = 250 \times \pi$$

$$\therefore m = \pi \times (10^{-3})^2 \times 10 \times 7860 \ll M,$$

$$\therefore Mg \approx 250 \times \pi$$

$$\text{Hence, } M = \frac{250 \times \pi}{10} = 25 \times \pi \approx 75 \text{ kg.}$$

Q. 21. A steel rod of length $2l$, cross sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform.) [NCERT Exemplar Q. No. 26]

Ans. Consider an element at r of width dr . Let $T(r)$ and $T(r + dr)$ be the tensions at the two edges, respectively.

$$\text{Net centrifugal force in element} = \omega^2 r dm$$

$$[\omega = \text{angular velocity of rod}]$$

$$\text{or, Net centrifugal force} = \omega^2 r \mu dr$$

$$[\because \mu = \text{mass/length}]$$

$$-T(r + dr) + T(r) = \mu \omega^2 r dr$$

$$-dT = \mu \omega^2 r dr$$

\therefore Tension and centrifugal force are opposite.

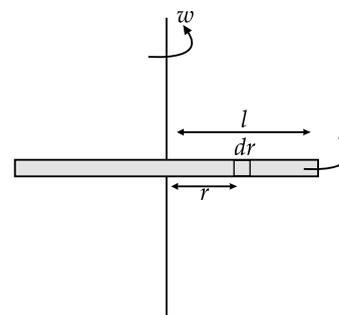
$$\therefore - \int_{T=0}^T dT = \int_{r=l}^{r=r} \mu \omega^2 r dr$$

$$T = \mu \omega^2 \frac{r^2}{2} + c$$

$$T = \frac{\mu \omega^2}{2} (l^2 - r^2) + c$$

$$\text{At } r=l, T=0, \therefore c=0$$

$$\therefore T(r) = \frac{\mu \omega^2}{2} (l^2 - r^2)$$



Let the increase in length of the element dr be $d\delta$ therefore, Young's Modulus,

$$\begin{aligned} Y &= \frac{\text{stress}}{\text{strain}} \\ &= \frac{T(r)}{d\delta} \\ &= \frac{A}{dr} \end{aligned}$$

$$Y = \frac{(\mu\omega^2/2)(l^2 - r^2) / A}{\frac{d(\delta)}{dr}}$$

$$\therefore \frac{d\delta}{dr} = \frac{1}{YA} \frac{\mu\omega^2}{2} (l^2 - r^2)$$

$$\therefore d\delta = \frac{1}{YA} \frac{\mu\omega^2}{2} (l^2 - r^2) dr$$

Change in length, $\delta = \frac{1}{YA} \frac{\mu\omega^2}{2} \int_0^l (l^2 - r^2) dr$

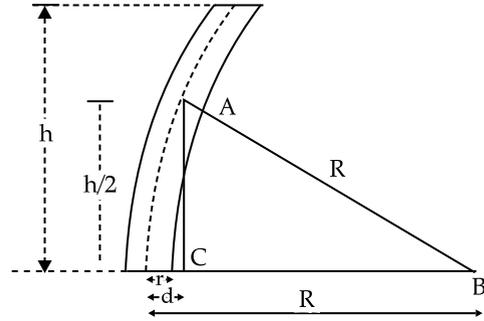
$$= \frac{1}{YA} \frac{\mu\omega^2}{2} \left[l^3 - \frac{l^3}{3} \right] = \frac{1}{3YA} \mu\omega^2 l^3$$

The total change in length is $2\delta = \frac{2}{3YA} \mu\omega^2 l^3$

Q. 22. In nature, the failure of structural members usually results from large torque because of twisting or bending rather than due to tensile or compressive strains. This process of structural breakdown is called buckling and in cases of tall cylindrical structures like trees, the torque is caused by its own weight bending the structure. Thus the vertical through the centre of gravity does not fall within the base. The elastic torque caused because of this bending about the central axis of the tree is given by $\frac{Y\pi r^4}{4R}$. Y is the Young's modulus, r is the radius of the trunk and R is the radius of curvature of the bent surface along the height of the tree containing the centre of gravity (the neutral surface). Estimate the critical height of tree for a given radius of the trunk.

[NCERT Exemplar Q. No. 28]

Ans. Let us consider the following diagram.



From question,

Torque on trunk of tree = $\frac{Y\pi r^4}{4R}$

[r = radius of tree
 R = radius of curvature of bent surface]

When the tree is about to buckle,

$$\text{Torque} = Wd = \frac{Y\pi r^4}{4R}$$

If $R \gg h$, then the centre of gravity is at a height $h/2$ from the ground.

From $\triangle ABC$

$$R^2 = (R - d)^2 + \left(\frac{1}{2}h\right)^2$$

$$R^2 = R^2 - 2Rd + \frac{1}{4}h^2$$

If $d \ll R$, $\therefore d = \frac{h^2}{8R}$

If W_0 is the weight/volume

$$\frac{Y\pi r^4}{4R} = W_0 (\pi r^2 h) \frac{h^2}{8R}$$

[\therefore Torque is due to weight]

$$\text{or } h = \left(\frac{2Y}{W_0}\right)^{1/3} r^{2/3}$$

Thus, critical height, $h = \left(\frac{2Y}{W_0}\right)^{1/3} r^{2/3}$



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any 4 of the following question on the basis of the same.

I. Elasticity vs. plasticity:

Objects get deformed when pushed, pulled, and twisted. Elasticity is the measure of the amount that the object can return to its original shape after these external forces and pressure are removed. The opposite of elasticity is plasticity. When something is stretched, and it stays stretched, the

material is said to be plastic. Such deformation is said to be plastic deformation.

In elastic deformation, atoms of the material are displaced temporarily from their original lattice site. They return back to their original position after the removal of external force. In plastic deformation, atoms of the solid are displaced permanently from their original lattice site. They don't return back to the original position even after the removal of external load. So, elastic deformation is temporary, whereas plastic deformation is permanent.

Amount of elastic deformation is very small. But the amount of plastic deformation is quite large. External force required for elastic deformation of solid is quite small. Force required for plastic deformation is much higher. Total energy absorbed by the material during elastic and plastic deformation region is called module of toughness.

Energy absorbed by the material during elastic deformation is called module of resilience.

Most materials have an amount of force or pressure for which they deform elastically. If more force or pressure is applied, then they undergo plastic deformation. Materials those have a fair amount of plastic deformation before breaking are said to be ductile. Materials those can't stretch or bend much without breaking are said to be brittle. Copper, aluminium etc. are ductile materials. For this reason those are used for making wires. Glass and ceramics are often brittle; they will not bend; they will break.

Q. 1. Which of the following statement is false?

- (A) A body is said to be plastic when it deforms due to application of force and return to its original shape when the deforming force is removed.
- (B) External force required for elastic deformation of solid is quite small.
- (C) In plastic deformation, atoms of the solid are displaced permanently from their original lattice site.
- (D) Most materials have an amount of force or pressure for which they deform elastically. If more force or pressure is applied, then they undergo plastic deformation.

Ans. Option (A) is correct.

Explanation: A body is said to be elastic when it deforms due to application of force and returns to its original shape when the deforming force is removed. The opposite of elasticity is plasticity.

When something is stretched, and it stays stretched, the material is said to be plastic.

Q. 2. Hooks law is applicable for

- (A) Plastic materials (B) Elastic materials
- (C) Both (A) and (B) (D) Brittle materials

Ans. Option (B) is correct.

Explanation: Hooks law is applicable only for elastic materials which states that, for relatively small deformation of an object, the displacement or size of the deformation is directly proportional to the deforming force. Under these conditions, the object returns to its original shape and size upon removal of the load.

Q. 3. Aluminium is a materials.

- (A) Brittle (B) Plastic
- (C) Ductile (D) Both (A) and (C)

Ans. Option (C) is correct.

Explanation: Aluminium is a ductile material. It can undergo substantial plastic deformation prior to fracture.

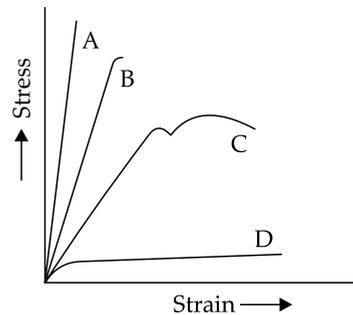
Q. 4. Ceramic is a material

- (A) Brittle (B) Plastic
- (C) Ductile (D) Both (A) and (C)

Ans. Option (A) is correct.

Explanation: Ceramic when subjected to little stress, it fractures with little elastic deformation and without significant plastic deformation. Hence, it is a brittle material.

Q. 5. Which of the following 4 stress-strain graphs represent a ductile material and a brittle material?



- (A) A is for a brittle material, B is for a ductile material
- (B) A is for a brittle material, D is for a ductile material
- (C) A is for a brittle material, C is for a ductile material
- (D) C is for a brittle material, A is for a ductile material

Ans. Option (C) is correct.

Explanation: A typical stress-strain curve for a brittle material is linear. Hence, graph A is for brittle material. Graph C is for a ductile material.

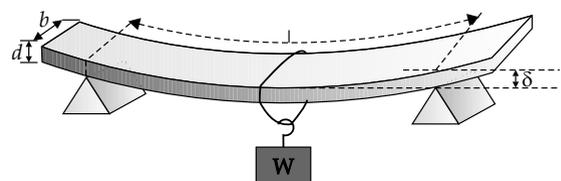
II. Sagging of a bridge

A bridge is designed such that it can withstand the load of the flowing traffic, the force of winds and its own weight.

Let us consider the case of a beam loaded at the centre and supported near its ends as shown in Figure.

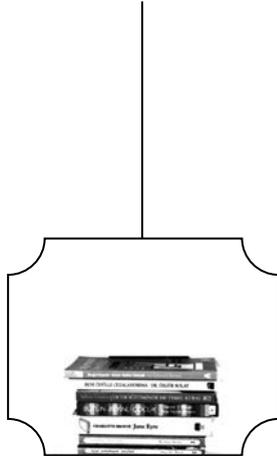
A beam of length l , breadth b , and depth d when loaded at the centre by a load W sags by an amount given by

$$\delta = \frac{Wl^3}{4bd^3Y}$$



One day after keeping an additional book in the rack the wire broke. Principal was surprised. He carefully observed the wire. There was no rusting. Principal was confused. He called the physics teacher and requested him to explain the phenomenon.

Principal had arts background. So, physics teacher explained him the basics of elasticity, Hook's law and breaking stress.

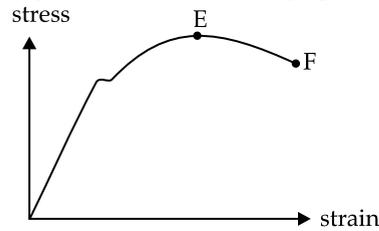


Q. 1. What is breaking stress?

Ans. Breaking stress is the maximum stress that the body can withstand before it finally breaks. It is defined as the ratio of breaking force per unit area.

Q. 2. Draw a simple stress-strain graph and indicate the region in which the wire was lying just before breaking?

Ans. Just before breaking, the wire was lying in the EF region of the stress-strain graph.



Q. 3. Weight of all the books placed on the hanging rack was 60N. Radius of the wire was 2.0 mm. Find the approximate breaking stress of the steel wire.

Ans. Breaking stress = Force / Area of cross-section

$$\text{or, Breaking stress} = \frac{60}{\pi(0.002)^2}$$

$$\therefore \text{ Breaking stress} = 4.77 \text{ N/m}^2$$

Q. 4. On which parameters does breaking stress depend?

Ans. Breaking stress depends on the material only.

Q. 5. Breaking stress of a steel wire of length L and radius r is F. What will be the change in breaking stress if the radius is doubled and length is halved?

Ans. Breaking stress will remain same since it does not depend on the dimension of the wire. It depends on the material only.



CHAPTER 9

Study Time
Maximum Time: 3 : 30 hours
Maximum Questions: 90

MECHANICAL PROPERTIES OF FLUIDS

Syllabus

Pressure due to a fluid column, Pascal's law and its applications (hydraulic lift and hydraulic brakes). effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications.

Surface energy and surface tension, angle of contact, excess pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

Topic-1 Fluids at Rest

TOPIC - 1 Fluids at Rest P. 191
TOPIC - 2 Surface Energy & Surface Tension P. 196
TOPIC - 3 Viscosity & Bernoulli's Theorem P. 201

Revision Notes

➤ **Pressure :**

(i) Pressure is defined as the thrust acting per unit area of the surface in contact with liquid, i.e.,

$$P = \frac{\text{Thrust}(F)}{\text{Area}(A)} = \frac{F}{A} = h\rho g$$

(ii) Liquid pressure is independent of shape of the liquid surface as well as area of the liquid surface, but depends upon height of liquid column.

(iii) Total pressure at a depth h below liquid surface is $P = h\rho g + P_0$, where P_0 is the atmospheric pressure.

(iv) S.I. unit of pressure is Nm^{-2} or pascal (denoted by Pa) and its dimensional formula is $[\text{ML}^{-1}\text{T}^{-2}]$.

(v) Pressure is a scalar quantity because a liquid at rest exerts equal pressure in all directions at all points in the same horizontal plane.

➤ **Pascal's Law :** It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same. Pascal's law also states that the increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of liquid provided the gravity effect is neglected.

➤ **Atmospheric pressure :**

(i) It is defined as the pressure exerted by atmosphere.

(ii) At S.T.P, the value of atmospheric pressure is $1.01 \times 10^5 \text{ Nm}^{-2}$ or $1.01 \times 10^6 \text{ dyne/cm}^2$.

➤ **Archimedes' principle :** It states that when a body is immersed partly or wholly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.

Observed weight of body = True weight – Weight of liquid displaced.

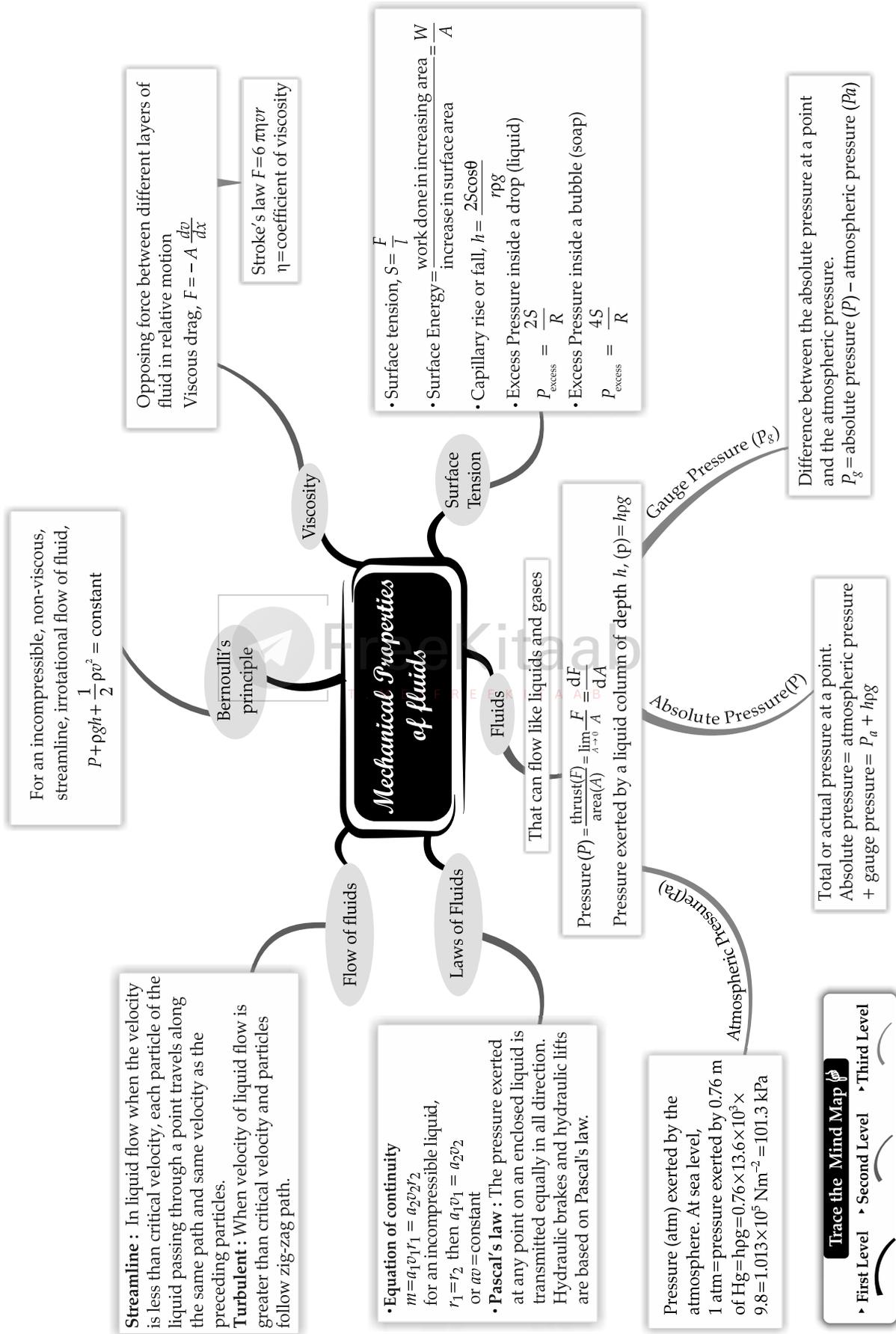
If w is the observed weight of body of density ρ when immersed in a liquid of density σ , then

$$\begin{aligned}
 w &= Mg - mg \\
 &= Ah\rho g - Ah\sigma g
 \end{aligned}$$

Scan to know
more about
this topic



Application of
Pascal's Law



Trace the Mind Map

- First Level
- Second Level
- Third Level

$$= Ahg(\rho - \sigma)$$

$$= Ahg\rho\left(1 - \frac{\sigma}{\rho}\right) = W\left(1 - \frac{\sigma}{\rho}\right)$$

\therefore True weight,
$$W = \frac{\text{apparent weight}}{(1 - \sigma/\rho)}$$

- **Laws of floatation :** It states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to or greater than the weight of the body.
 - (a) When true weight of the body $W > w$ (weight of the liquid displaced), the body will sink to the bottom of the liquid. It will be so when the density of solid body (ρ) is greater than the density of liquid (σ), i.e., $\rho > \sigma$.
 - (b) When $W < w$, the body will rise above the surface of liquid to such an extent that the weight of the liquid displaced by immersed part of the body (i.e., upward thrust) becomes greater than the weight of the body. The body then will float. In this case the density of solid body is less than the density of liquid, i.e., $\rho < \sigma$.
 - (c) When $W = w$, the body is at rest anywhere in the liquid. The body will float with its whole volume just immersed in the liquid. In this case the density of body is equal to density of liquid, i.e., $\rho = \sigma$.
- There will be equilibrium of floating body when
- (i) Weight of liquid displaced by the immersed part of body is equal to the weight of the body.
 - (ii) The centre of gravity of the body and the centre of buoyancy lie along the same vertical line.
 - (iii) If the centre of gravity of the body lies vertically below the meta centre then body is in stable equilibrium. The body will be in unstable equilibrium if centre of gravity lies vertically above the meta centre.

Key Words

- **Fluid** is the name given to a substance which begins to flow when external force is applied on it. It includes liquid and gas.
- **Thrust :** The total normal force exerted by liquid at rest on a given surface in contact with it is known as thrust of liquid on that surface. It is due to collision of molecules of liquid while moving at random, with the walls of the container and rebounding from them.
- **Buoyancy** is the upward force acting on the body immersed in a fluid.
- **Metacentre** is a point where the vertical line passing through the centre of buoyancy intersects the central line.

Key Formulae

- Pressure = $\frac{F}{A} = h\rho g$ (due to h height of liquid)
 - h = height, ρ = Density of liquid, g = Acceleration due to gravity.
- Gauge pressure = Total pressure – Atmospheric pressure
- For Hydraulic lift; $\frac{F_1}{A_1} = \frac{F_2}{A_2}$
 - F_1, F_2 = Forces on pistons of area of cross - sections A_1, A_2
- Density = $\frac{\text{Mass}}{\text{Volume}}$, Relative density = $\frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$
- **Archimedes' Principle :**
 - Loss of weight of body in liquid = Weight of liquid displaced = Volume \times Density of liquid $\times g$
- **Law of floatation :**
 - A body will float if, weight of body = Weight of liquid displaced.

Very Short Answer Type Questions

(1 mark each)

- Q. 1. Define Pascal's law.** R
- Ans.** It states that if the effect of gravity is neglected, pressure in a fluid at rest is the same at all points. 1



Commonly Made Error

- Students forget to mention 'fluid at rest' and 'effect of gravity is neglected' terms in the statement for pascal's law.



Answering Tip

► Students should memorize important keywords for answering theoretical questions.

Q. 2. What is column pressure ? [R]

Ans. Pressure exerted by a liquid due to its height is called column pressure. 1

Q. 3. Define centre of buoyancy. [R]

Ans. Centre of buoyancy is a point where buoyant force acts on the body. 1

Q. 4. The dams of water reservoir are made thick near the bottom. Why ? [U]

Ans. Pressure exerted by a liquid column of height h , $P = h\rho g$. As h is large, so P is quite large. Due to it the bottom are made thick. 1

Q. 1. What is atmospheric pressure ? Give its different units. [R]

Ans. It is pressure exerted by the atmosphere of our Earth.

Units used in medicine, physiology etc. ½

1 mm of mercury = 1 torr

1 torr = 133 Pa ½

Units of meteorology etc. ½

1 bar = 10^5 Pa

1 millibar = 10^2 Pa ½

Q. 2. What are absolute pressure and gauge pressure ? [R]

Ans. Pressure at a point is given by the relation

$$P = P_a + h\rho g \quad 1$$

Where, P_a is the atmospheric pressure and $h\rho g$ is the column pressure. Here, P is the absolute pressure and $(P - P_a)$ is the gauge pressure normally measured. 1



Commonly Made Error

► Student can not explicate absolute and gauge pressure.



Answering Tip

► Student should familiarize themselves with pressure related terminology - absolute and gauge pressure.

Q. 3. The density of the atmosphere at sea level is 1.29 kg m^{-3} . Assume that it does not change with altitude. Then how high would the atmosphere extend? [A]

Ans. Here, $1.01 \times 10^5 = \rho gh$ 1

$$\text{or } h = \frac{1.01 \times 10^5}{\rho g} = \frac{1.01 \times 10^5}{1.29 \times 9.8}$$

$$= 7989 \text{ m.} \quad 1$$

Q. 5. Give S.I. unit of pressure and equate it to pascal. [U]

Ans. Atmosphere (atm) is a common unit of pressure.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa.} \quad 1$$

Q. 6. Why is a slight blow on a cork of bottle filled with a liquid sufficient to break the bottle ? [U]

Ans. The blow on the cork of bottle exerts a pressure on the liquid. This pressure gets transmitted to the entire bottle through the liquid. This is sufficient to break the bottle. 1

Q. 7. What is force on a man due to atmospheric pressure ? Why one does not feel it ? [U]

Ans. It is about $2 \times 10^5 \text{ N}$. One does not feel it because his blood pressure is slightly greater than the atmospheric pressure. 1

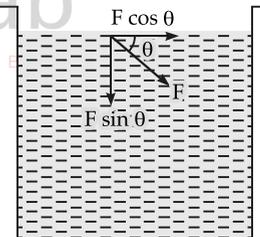


Short Answer Type Questions-I

(2 marks each)

Q. 4. Show that a liquid in a state of equilibrium of rest exerts a force perpendicular to the surface only. [U] [KVS 2014]

Ans. Let a given quantity of liquid exert force F in a direction θ with the horizontal. Resolving, we get $F \cos \theta$ along horizontal surface of liquid and $F \sin \theta$ along the vertical. ½



As there is no flow of liquid so $F \cos \theta$ should be zero. ½

Since, $F \neq 0$, $\cos \theta = 0$

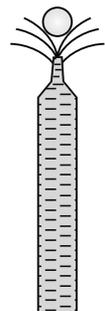
$$\text{or } \theta = 90^\circ$$

i.e., a liquid at rest exerts normal force to the walls of the container. 1

Q. 5. If a small ping-pong ball is placed in a vertical jet of water or air, it will rise to a certain height above the nozzle and stay at that level. Explain. [U]

Ans. Due to the high velocity of the jet of water, the pressure between the ball and jet decreases. The greater (atmospheric) pressure on the other side of the ball pushes it against the jet and the ball remains suspended. The high velocity of water takes the ball upwards along with it and makes it to spin. 1

A ping-pong ball supported on a jet of water. 1



Short Answer Type Questions-II

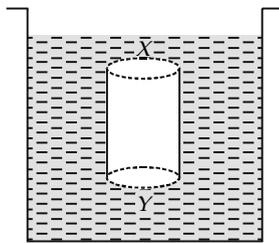
(3 marks each)

Q. 1. What is column pressure ? Derive a relation for the same. □

Ans. Pressure exerted by a liquid due to its height is called column pressure. ½

Consider two points X and Y to be lying on the top and bottom circular faces of an imaginary cylinder of liquid. Let area of the circular faces be a each and height of the cylinder be h . If pressure exerted at point X is P_x and at Y is P_y , then

$$P_x = \frac{F_x}{a} \quad \frac{1}{2}$$



$F_x = P_x a$ acting downward. Weight of this cylinder, $W = mg = V\rho g = ah\rho g$ is also acting downward so total downward force = $F_x + W = P_x a + ah\rho g$ ½

The lower face of the cylinder experiences upward force given by

$$F_y = P_y a.$$

In equilibrium, $F_y = F_x + W$

i.e., $P_y a = P_x a + ah\rho g$

or $(P_y - P_x) = h\rho g$

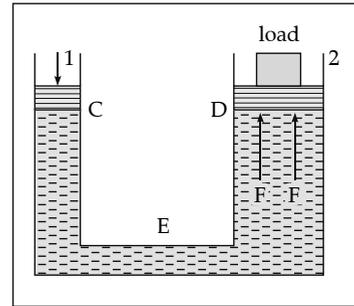
or $P = h\rho g$ 1

Q. 2. State Pascal's law of transmission of fluid pressure. Explain how is Pascal's law applied in a hydraulic lift. (with suitable diagram).

□ [NCT 2009, 10; MSE Chandigarh 2009]

Ans. Pascal law : If gravity effect is neglected the pressure at every point of liquid in equilibrium at rest is same. 1

Hydraulic lift. It is used to lift the heavy loads. Its working is based on Pascal's law. A simple hydraulic lift is shown in Figure. Here, C and D are two cylinders of different areas of cross section. They are connected to each other with a pipe E. Each cylinder is provided with airtight frictionless piston. Let a , and A be the areas of cross-sections of the pistons in C and D respectively, where $a < A$. The cylinders are filled with an incompressible liquid.



Let a downward force f be applied on the piston of C. Then the pressure exerted on the liquid, $P = f/a$.

According to Pascal's law, this pressure is transmitted equally to piston of cylinder D.

∴ Upward force acting on the piston of cylinder D will be

$$F = PA = \frac{f}{a} A = f \frac{A}{a}. \quad 1$$

Q. 3. A razor blade can be made to float on water. What forces act on this blade ? Is Archimedes' principle applicable ? □

Ans. When a razor blade is made to float on water, three forces act on the blade :

- (i) Weight of the blade acting vertically downwards. 1
- (ii) Reaction on blade exerted by the liquid surface acting vertically upwards. 1
- (iii) Force of the surface tension on circumference of the blade acting tangentially to the liquid surface. 1

In this case, as no portion of razor blade is immersed in water, hence Archimedes principle is not applicable.



Commonly Made Error

► Students lack understanding of the Archimedes' principle.



Answering Tip

► The understanding that the Archimedes' principle is valid for objects fully or partially immersed in water should be there.

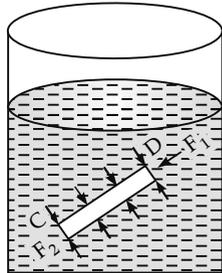
Long Answer Type Question

(5 marks)

Q.1. State and prove Pascal's law and give its experimental proof also. □

Ans. It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium at rest is same. ½

Proof : Consider two points C and D inside the liquid in a container which is in equilibrium of rest. Imagine a right circular cylinder with axis CD of uniform cross-sectional area A such that points C and D lie on flat faces of the cylinder in figure. 1



The liquid inside the cylinder is in equilibrium under the action of forces exerted by the liquid outside the cylinder. These forces are acting every where \perp to the surface of the cylinder. Thus force on the flat faces of the cylinder at C and D will \perp to the forces on the curved surface of the cylinder. Since the liquid is in equilibrium, therefore, the sum of forces acting on the curved surface of the cylinder must be zero. If P_1 and P_2 are the pressure at points C and D and F_1 and F_2 are the forces acting on the flat faces of the cylinder due to liquid, then

$$F_1 = P_1 A$$

and $F_2 = P_2 A$ 1

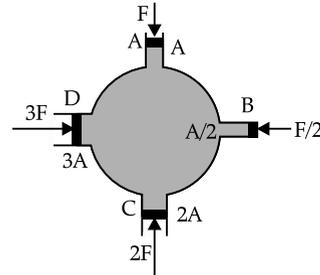
Since the liquid is in equilibrium, therefore

$$F_1 = F_2$$

$$P_1 A = P_2 A$$

$$P_1 = P_2$$

It means the pressure at C and D are the same. ½



Experimental Proof. Consider a spherical vessel having four cylindrical tubes A, B, C and D each fitted with air tight frictionless piston of area of cross-section $a, a/2, 2a$ and $3a$ respectively.

Fill the vessel with an incompressible liquid so that no air gap is left inside the vessel and piston fitted in the various cylindrical tubes. Push the piston A with force F . The pressure developed on the liquid $= F/a = P$ (say). ½

It is seen that all other pistons will be pushed outwards. To keep the pistons at their respective original positions, the force $F/2, 2F$ and $3F$ respectively required to be applied on pistons of tubes B, C and D respectively to hold them. Now pressure developed on liquid in tubes, B, C and D are $\frac{F/2}{a/2}, 2F/2a, 3F/3a$ i.e., equal to F/a . This

indicates that the pressure applied is transmitted equally to all parts of the liquid. This proves Pascal's law. 1

Topic-2 Surface Energy & Surface Tension

Revision Notes

- **Surface Tension :** It is the property of the liquid by virtue of which the free surface of the liquid at rest tends to have the minimum surface area and as such it behaves as if covered with a stretched membrane.
 - (a) Quantitatively, surface tension of a liquid is measured as the force acting per unit length of a line imagined to be drawn tangentially any where on the free surface of the liquid at rest. It acts at right angles to this line on both the sides and along the tangent to the liquid surface, i.e., $S = F/l$.
 - (b) Surface tension of a liquid is also defined as the amount of work done in increasing the free surface of liquid at rest by unity at constant temperature, i.e., $S = W/A$.
or $W = S \times A = \text{Surface tension} \times \text{Area of liquid surface formed.}$
 - (c) Surface tension is a molecular phenomenon and it arises due to electromagnetic forces. The explanation of surface tension was first given by Laplace.

- (d) S.I. unit of surface tension is Nm^{-1} or Jm^{-2} and C.G.S. unit is dyne-cm^{-1} or erg-cm^{-2} .
 - (e) Dimensional formula of surface tension = $[ML^0T^{-2}]$
 - (f) Surface tension is a scalar quantity as it has no specific direction for a given liquid.
 - (g) Surface tension does not depend upon the area of the free surface of liquid at rest.
- **Surface Energy** : It is defined as the amount of work done against the force of surface tension in forming the liquid surface of a given area at a constant temperature, i.e.,
- $$\text{Surface energy} = \text{Work done} = \text{Surface tension} \times \text{Surface area of liquid.}$$
- The S.I. unit of surface energy is Joule and C.G.S. unit is erg.
- (a) If small drops combine together to form a big drop, the surface area decreases, so surface energy decreases. Therefore the energy is released. If this energy is taken by drop, the temperature of drop increases.
 - (b) If a big drop is splitted into number of smaller drops, the surface area of drops increases. Hence, surface energy increases. So energy is spent. If this energy is pronated by drop, the temperature of drop decreases e.g., spray.

Key Words

- **Surface film** is the top most layer of liquid at rest with thickness equal to the molecular range.
- **Angle of contact** between a liquid and solid in contact is defined as the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid.
- **Capillary tube** is a tube with a fine and uniform bore throughout its length.
- **Capillarity** is the phenomenon of rise or fall of liquid in a capillary tube.

Key Formulae

- Surface tension, $S = F/l$
- Surface energy, $E = \text{Work done}$
- Work done, $W = S \times \text{Increase in area}$
- Excess of pressure inside the liquid drop is

$$P = P_i - P_o = \frac{2S}{r};$$

$$P_i = \text{Pressure inside bubble}$$
- Excess of pressure inside the soap bubble is

$$P = P_i - P_o = \frac{4S}{r};$$

$$P_o = \text{Pressure outside bubble}$$
- Total pressure in the air bubble at a depth h below the surface of liquid of density ρ is

$$P = P_o + h\rho g + \frac{2S}{r}$$
- Ascent/Decent Formula : $h = \frac{2S \cos \theta}{r\rho g}$

where,
 r = radius of capillary tube
 ρ = density
 S = Surface tension
 θ = angle of contact

Mnemonics

Concept: Excess pressure inside liquid drop, air bubble and soap bubble.

Mnemonics: Emily purchased two Swiss rolls for Lata didi and Amit bhaiya and four swiss rolls for Shanti bahin.

Interpretation:

E - Excess
P - pressure
T - Two
s - Surface tension

r - Radius
L - Liquid
d - drop

A - Air
b - bubble

Excess pressure in liquid drop and in air bubble are $2S/R$.

f - four
s - Surface tension
r - Radius
S - soap
b - bubble

Excess pressure in soap bubble is $4S/R$.



Very Short Answer Type Questions

(1 mark each)

Q. 1. When wax is rubbed on cloth, the cloth becomes water proof. Why ? U

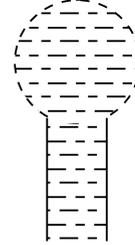
Ans. The capillaries formed in threads disappear when wax is rubbed on cloth. 1

Q. 2. What is the effect of temperature on surface tension ? U

Ans. Surface tension of liquids decreases with increase in temperature. 1

Q. 3. What will happen if the length of the capillary tube smaller than the height to which the liquid rises ? U

Ans. The liquid will not over flow in the capillary. It will be full upto the top of the capillary and at open end it will bulge out. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. Why does the free surface of a liquid behave like an elastic stretched membrane ? U

Ans. The liquid molecules on the surface experience a downward force. So, they have greater potential energy. 1

In order to have minimum energy, the free surface tends to contract to minimum area and hence behaves like an elastic stretched membrane. 1

Q. 2. Why do the hair of a shaving brush cling together when taking out of water ? U AI

Ans. When the brush is taken out of water, thin water film is formed at the tips of the hair. It contracts due to surface tension and so, the hair cling together. 2

Q. 3. Why the tip of the nib of a pen is split ? U

Ans. The tip of the nib of a pen is split in order to provide a capillary which helps the ink to rise to the end of the nib and enables it to write continuously. 2

Q. 4. Why does an iron needle float on clean water but sink when some detergent is added to this water ? U

Ans. Due to surface tension, the free surface of liquid at rest behaves like a stretched membrane. When an iron needle floats on the surface of clean water, its weight is supported by the stretched membrane. When some detergent is added to this water, its surface tension decreases. 1

As a result of it, the stretched membrane on the surface of water is weakened and is not able to support the weight of needle. Hence, needle sinks in such water. 1

Q. 5. Explain why some oil spreads uniformly on water when others float as drops. U

Ans. The surface tension of some oil is less than surface tension of water. When such oil are dropped on the surface of water, they are pulled in all directions on water surface and spread uniformly on water. 1

In the case of other oils, whose surface tension is equal to or greater than that of water, they float as drops on water. 1



Short Answer Type Questions-II

(3 marks each)

Q. 1. Two capillaries of same length and radii in the ratio 1 : 2 are connected in series. A liquid flows through them in a streamlined condition. If the pressure across the two extreme ends of the combination is 1 m of water, find the pressure difference across first capillary. A

Ans. Here, $\frac{\pi P_1 r^4}{8\eta l} = \frac{\pi P_2 (2r)^4}{8\eta l}$ 1

i.e., $P_1 = 16P_2$ 1

Given that $P_1 + P_2 = 1 \text{ m}$ 1

$$P_1 + \frac{P_1}{16} = 1$$

or $P_1 = \frac{16}{17} = 0.94 \text{ m}$ 1



Commonly Made Error

► Students can not evaluate the pressure difference across the capillary in combination.



Answering Tip

► Students should familiarize with capillary action in series combination which is similar to resistance in series.

Q. 2. Explain why small drops of mercury are spherical and large drops become flat ? U

Ans. In case of a small drop of mercury force of gravity is small and force of surface tension plays a vital role. Therefore, the free surface of drop tends to have minimum surface area. For given volume the sphere has minimum area. Hence the small drops are of spherical shape. 2

👁 Long Answer Type Questions

(5 marks each)

Q. 1. Find the excess pressure :

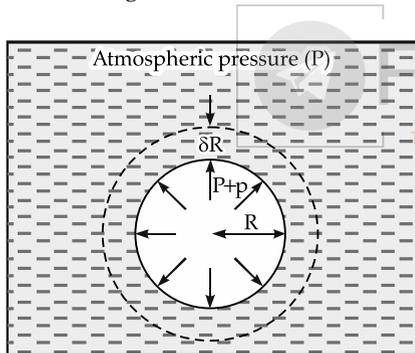
- (i) inside a liquid drop,
- (ii) inside soap bubble,
- (iii) inside an air bubble in liquid. U

[MSE Chandigarh 2009]

Ans. (i) Inside a liquid drop. Consider a liquid drop of radius R . The molecules lying on the surface of liquid drop, due to surface tension will experience resultant force acting inward perpendicular to the surface.

Let S = Surface tension of liquid drop
 P = excess pressure inside the drop

Due to excess of pressure, let there be increase in the radius of the drop by a small quantity δR as shown in figure.



1/2

Then work done by the excess pressure.

$$W = \text{Force} \times \text{Displacement}$$

$$= (\text{Excess pressure} \times \text{Area} \times \text{Increase in radius})$$

$$= p \times 4\pi R^2 \times \delta R \quad \dots(i) \quad \frac{1}{2}$$

Increase in surface area of the drop

$$= \text{Final surface area} - \text{Initial surface area}$$

$$= 4\pi(R + \delta R)^2 - 4\pi R^2$$

$$= 4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$$

$$= 8\pi R \delta R \quad \frac{1}{2}$$

[Neglecting, $(\delta R)^2$ being very very small]

\therefore Increase in surface energy

$$= \text{increase in surface area} \times \text{surface tension}$$

$$= 8\pi R(\delta R) \times S \quad \dots(ii) \quad \frac{1}{2}$$

As the increase in surface energy is at the cost of work done by the excess pressure, therefore from (i) and (ii) $p \times 4\pi R^2 \times \delta R = 8\pi R \delta R \times S$ 1/2

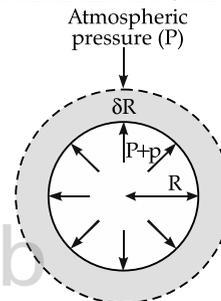
$$p = \frac{2S}{R}$$

In the case of large mercury drop, the gravitational pull becomes more effective than the surface tension and exerts downward pull on the drop so that its centre of gravity may lie at lowest possible position. Hence, the large drop of mercury becomes elliptical or flat in shape. 1

(ii) Inside a soap bubble. Consider a soap bubble of radius R the molecules lying on the surface of liquid bubble will experience a resultant force acting on water perpendicular to the surface due to the surface tension.

Let S = surface tension of the soap solution,
 p = excess of pressure inside the bubble.

Due to it, let there be increase in the radius of the bubble by a small amount δR in fig.



1/2

Then work done,

$$W = \text{Force} \times \text{Distance}$$

$$= (\text{Excess pressure} \times \text{Area}) \times \text{Increase in radius}$$

$$= p \times 4\pi R^2 \times \delta R \quad \dots(iii) \quad \frac{1}{2}$$

The soap bubble has two free surface, one outside the bubble and one inside the bubble when soap solution and air are in contact.

\therefore The effective increase in surface area of the bubble

$$= 2[\text{Final S.A.} - \text{Initial S.A.}]$$

$$= 2[4\pi(R + \delta R)^2 - 4\pi R^2]$$

$$= 2 \times 4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$$

$$= 8\pi \times 2R(\delta R)$$

[Neglecting $(\delta R)^2$, being very small]

$$= 16\pi R \delta R \quad \frac{1}{2}$$

\therefore Increase in surface energy

$$= \text{Increase in surface area} \times \text{Surface tension}$$

$$= 16\pi R \delta R \times S \quad \dots(iv)$$

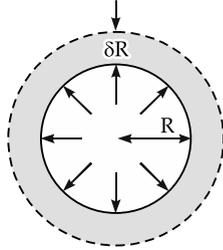
\therefore Increase in surface energy is as the cost of work done by the excess pressure therefore from (iii) and (iv),

$$p \times 4\pi R^2 \times (\delta R) = 16\pi R(\delta R) \times S$$

or
$$p = \frac{4S}{R} \quad \frac{1}{2}$$

(iii) **Inside an air bubble in liquid.** Consider an air bubble of radius R . Just inside a liquid of surface tension S .

Atmospheric pressure + Pressure of liquid = P



The air bubble will have only one free surface as shown in figure. It can be shown that the pressure inside the air bubble is given by

$$P = \frac{2S}{R} \quad \frac{1}{2}$$



Commonly Made Error

- Students do not consider the two free surface of a soap bubble for calculating surface tension.



Answering Tip

- Student should keep in mind that for a droplet, there is only one surface but for a soap bubble there are two surfaces which provides surface tension.

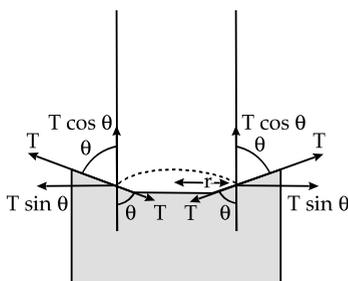
Q. 2. Derive the formula for rise of liquid in a capillary tube (Ascent formula).

Ans. A uniform capillary tube (radius r) is vertically immersed in liquid (density ρ , surface tension T). Liquid rises in the tube upto height h .

Let us consider that the liquid meniscus is concave upward. The meniscus which is in contact with the glass is circular and its circumference is $2\pi r$.

Due to surface tension, the liquid surface exerts a force on the surface of capillary tube which is equal to T per unit length of the circumference $2\pi r$. According to Newton's 3rd law, the capillary tube wall also exerts a reactionary force T which is directed outward.

If θ is the angle of contact, then the two components of this outward directed force T are $T \sin \theta$ and $T \cos \theta$.



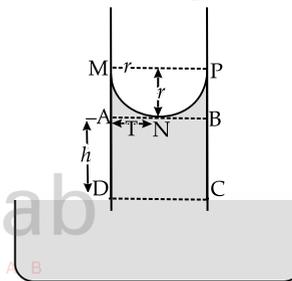
Considering the entire circumference $2\pi r$, for each horizontal component $T \sin \theta$ there is equal and opposite component which cancels out each other. Vertical components being in same direction, the total force in upward direction becomes $2\pi r T \cos \theta$. Liquid rises up inside the capillary tube due to this upward force till it is balanced by the weight of the liquid column.

Let us consider the liquid raises upto height h .

To find the weight of the liquid column raised, for simplicity of calculation, let us consider the liquid meniscus MNP be a hemisphere of radius r .

Volume of the liquid column = Volume of the cylinder $ABCD$ of height h and radius r + Volume $MANBP$

or, Volume of the liquid column = Volume of the cylinder $ABCD$ of height h and radius r + (Volume of cylinder $ABPM$ of height r and radius r - volume of the hemisphere MNP)



or, Volume of the liquid column

$$= \pi r^2 h + \left(\pi r^2 \times r - \frac{2}{3} \pi r^3 \right)$$

$$\text{or Volume of the liquid column} = \pi r^2 h + \frac{1}{3} \pi r^3$$

$$\text{or Volume of the liquid column} = \pi r^2 \left(h + \frac{1}{3} r \right)$$

$$\therefore \text{Weight of the liquid column} = \pi r^2 \left(h + \frac{1}{3} r \right) \rho g$$

In equilibrium,

$$2\pi r T \cos \theta = \pi r^2 \left(h + \frac{1}{3} r \right) \rho g$$

Neglecting the term $\frac{r}{3}$ compared to h , since $h \gg \frac{r}{3}$

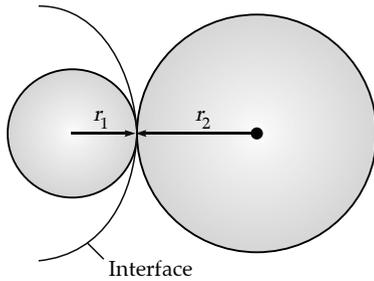
$$\frac{r}{3}$$

$$2\pi r T \cos \theta = \pi r^2 h \rho g$$

$$\therefore T = \frac{rh\rho g}{2\cos\theta}$$

This is the required formula.

Q.3. Derive an expression for the radius of interference when two soap bubbles of different radii are in contact. □



Ans. Consider two soap bubbles of radii r_1 and r_2 in contact with each other as shown in figure.

Let r be the radius of the common boundary. If P_1 and P_2 are the excess pressure on the two sides of one interface then the resultant excess pressure **1**

$$P = P_1 - P_2 \quad \mathbf{1}$$

$$\frac{4S}{r} = \frac{4S}{r_1} - \frac{4S}{r_2} \quad \mathbf{1}$$

or $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2} \quad \mathbf{1}$

$$r = \frac{r_1 r_2}{r_2 - r_1} \quad \mathbf{1}$$

Topic-3 Viscosity & Bernoulli's Theorem



Revision Notes

- **Bernoulli's theorem** : Bernoulli's theorem state that the total energy per unit volume (pressure energy, P.E. and K.E.) per unit volume of an incompressible non-viscous liquid in steady flow remain constant throughout the flow of the liquid $P + \rho gh + \frac{1}{2} \rho v^2 = \text{constant}$.
- **Torricelli's theorem** : According to this theorem, velocity of efflux i.e., the velocity with which the liquid flows out of an orifice is equal to that which a freely falling body would acquire in falling through a vertical distance equal to the depth of orifice below the free surface of liquid. R E E K I T A A B



Key Words

- **Viscosity** is the property of liquid due to which a backward dragging force called viscous force act tangentially on the layer of the liquid in motion.
- **Terminal velocity** is the maximum constant velocity acquired by the body while falling freely in a viscous medium.
- **Streamline flow of a liquid** is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.
- **Streamline** is the actual path followed by the procession of particles in a steady flow which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at that point.
- **Tube of flow** is the bundle of streamlines having the same velocity of the liquid particle over any cross-section perpendicular to the direction of flow of liquid.
- **Laminar flow** is a flow in which the liquid moves in layers.
- **Turbulent flow** is a flow when a liquid moves with a velocity greater than its critical velocity, the motion of particles of liquid becomes disorderly or irregular.
- **Critical velocity** is that velocity of liquid flow, upto which its flow is streamlined and above which its flow becomes turbulent.
- **Reynold number** is a pure number which determines the nature of flow of liquid through a pipe.

Scan to know more about this topic



Streamline flow and Reynold's number

Key Formulae

➤ **Newton's viscous drag force :**

$$F = \pm \eta A \frac{dv}{dx}$$

η = coeff. of viscosity, A = area of layer of liquid, dv/dx = velocity gradient.

➤ **Poiseuille's Formula :**

$$V = \frac{\pi p r^4}{8 \eta l}$$

p = Pressure difference across length l of horizontal tube of radius r & V = volume.

➤ **Stoke's Law :**

$$F = 6 \pi \eta r v$$

➤ **Terminal velocity :**

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where,

ρ = density of spherical body

σ = density of medium

r = radius of spherical body

η = coefficient of viscosity

➤ **Reynold's Number :**

$$R_N = \frac{\rho D v}{\eta}$$

➤ **Bernoulli's Theorem :**

$$\frac{P}{\rho} + gh + \frac{1}{2} v^2 = \text{constant} \quad \text{or} \quad \frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$

$\frac{P}{\rho}$ = pressure energy per unit mass $\frac{P}{\rho g}$ = Pressure head, h = gravitational head

gh = P. E. per unit mass $\frac{v^2}{2g}$ = Velocity head

$\frac{1}{2} v^2$ = K.E. per unit mass

Velocity of efflux : $v = \sqrt{2gh}$



Very Short Answer Type Questions

(1 mark each)

Q. 1. Where and why high viscosity liquids are used in trains ? [U]

Ans. High viscosity liquids are used as buffers in trains so that the shocks may be absorbed easily. 1

Q. 2. What is the terminal velocity of a body in a freely falling system ? [R]

Ans. The terminal velocity of a body v_c in a freely falling system is zero, because $g = 0$ for this system and $v_c \propto g$. 1

Q. 3. How does the viscosity of gases depend upon temperature ? [U]

Ans. For gases $\eta \propto T^{1/2}$. 1

Q. 4. The diameter of ball X is twice of that of Y. What will be the ratio of their terminal velocities in water ? [A]

Ans. As terminal velocity is proportional to (radius)², therefore the ratio of terminal velocities of X and Y will be 4 : 1. 1

Q. 5. If water in a flask and castor oil in other flask are violently shaken and kept on a table, then which one will come to rest earlier ? [A]

Ans. Castor oil will come to rest earlier as its viscosity is more as compared to water. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. (a) Define stream-line.

(b) Write any two properties of streamlines.

[R] [KVS 2008]

Ans. (a) Refer to Topic 3 'Know the Terms', 1

(b) Important properties of stream lines :

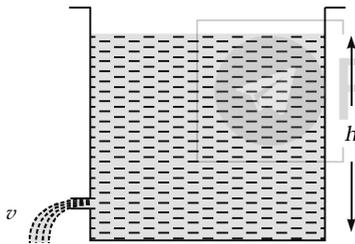
(i) In a stream-line flow, no two stream lines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will destroy the steady nature of the liquid flow. ½

(ii) The greater is the crowding of stream lines at a place, the greater is the velocity of liquid particles at that place and vice-versa. ½

Q. 2. What is Torricelli's law ? [R]

Ans. It states that the speed of a freely falling body is given by $v = \sqrt{2gh}$, where g is acceleration due

to gravity and h is the height of fall. 1



The speed of efflux, i.e., fluid flow through a narrow hole of a container open from top is given by the application of Bernoulli's equation as

$$v = \sqrt{2gh} \quad 1$$

Q. 3. Bernoulli's theorem holds for incompressible, non-viscous fluids. How is this relationship changed when the viscosity of the fluid is not negligible? [A]

Ans. If the viscosity of the fluid is not negligible, a part of the mechanical energy of the fluid is spent in doing work against forces of viscosity. 1
So, the total energy :

$P + \rho gh + \frac{1}{2} \rho v^2$ of the fluid goes on decreasing along the direction of the flow of the fluid. 1

Q. 4. Why are the cars and aeroplanes given stream line shape ? [AI] [A]

Ans. Cars and aeroplanes are given streamline shape to minimise the backward drag of atmosphere. 2

Q. 5. Why do cloud float in the sky ? [A]

Ans. Clouds are composed of tiny water droplets. Since, terminal velocity is proportional to the square of radius, the terminal velocity of tiny droplets is too small and hence they remain in sky. 2



Short Answer Type Questions-II

(3 marks each)

Q. 1. State and prove Bernoulli's principle for the flow of non-viscous fluids and give its limitations.

[U] [MSE Chandigarh 2009] [NCT 2008; KVS 2013]

OR

A liquid is in stream lined flow through a tube of non-uniform cross-section then prove that sum of its kinetic energy, pressure energy and potential energy per unit volume is constant. [NCT 2011]

Ans. Statement : Refer to 'Revision Notes',

Proof : Consider an incompressible non-viscous liquid entering at A at height h_1 having cross-section A_1 with a velocity v_1 and coming out at B at a height h_2 having area of cross-section A_2 with velocity v_2 .

The P.E. and K.E. increase since h_2 and v_2 are more than h_1 and v_1 respectively. Let P_1 and P_2 are the pressure at A and B.

The work done per second on the liquid at A

$$= -P_1 A_1 v_1$$

The work done per second by the liquid at B,

$$= -P_2 A_2 v_2 \Delta t \quad ½$$

$A_1 v_1$ and $A_2 v_2$ are the volumes of liquid entering at A and leaving at B per second respectively. These volumes must be equal and so

$$A_1 v_1 = A_2 v_2 = m/\rho$$

Net work done on the liquid

$$= (P_1 - P_2) m/\rho$$

where m = mass of liquid entering at A or leaving from B in 1 second.

Kinetic energy of the liquid at A is $\frac{1}{2} m v_1^2$

Kinetic energy of the liquid at B is $\frac{1}{2} m v_2^2$

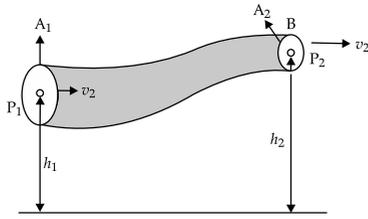
So, change in kinetic energy

$$= \frac{1}{2} m (v_2^2 - v_1^2)$$

Change in potential energy

$$= mg(h_2 - h_1)$$

Net work done = Net change in energy



$$(P_1 - P_2) = \rho g(h_2 - h_1) + \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$\text{i.e., } P_1 + \rho g h_1 + \frac{\rho}{2} v_1^2 = P_2 + \rho g h_2 + \frac{\rho}{2} v_2^2$$

$$\text{i.e., } \frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant} \quad \frac{1}{2}$$

Bernoulli's Theorem Limitations :

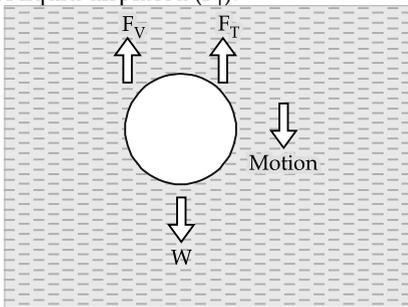
1. While deriving Bernoulli's theorem it is assumed that velocity of every particle of liquid across any cross section of tube is uniform. Practically it is incorrect. $\frac{1}{2}$
2. The viscous drag of the liquid which comes into play when liquid is in motion has not been taken into account. $\frac{1}{2}$
3. While deriving the equation, it is assumed that there is no loss of energy when liquid is in motion. $\frac{1}{2}$

Q. 2. By using Stokes law, derive an expression for terminal velocity. On what factors does it depend ? [NCT 2009; MSE Chandigarh 2009]

Ans. Terminal velocity : It is maximum constant velocity acquired by the body while falling freely in a viscous medium.

When a small spherical body falls freely through a viscous medium, three forces act on it.

- (i) Weight of the body acting vertically downwards (W).
- (ii) Upward thrust due to buoyancy equal to weight of liquid displaced (F_T).



- (iii) Viscous drag (F_v) acting in the direction opposite to the motion of body. According to Stoke's law, $F_v \propto v$, i.e., the opposing viscous drag goes on increasing with the increasing velocity of the body. **1**

As the body falls through a medium, its velocity goes on increasing due to gravity. Therefore, the opposing viscous drag which acts upwards also goes on increasing. A stage reaches when the true

weight of the body is just equal to the sum of the upward thrust due to buoyancy and the upward viscous drag. At this stage, there is no net force to accelerate the body. Hence, it starts falling with a constant velocity, which is called terminal velocity.

Let ρ be the density of the material of the spherical body of radius r and ρ_0 be the density of the medium.

$$\begin{aligned} \therefore \text{ True weight of the body,} \\ W &= \text{Volume} \times \text{Density} \times g \\ &= \frac{4}{3} \pi r^3 \rho g \end{aligned}$$

Upward thrust due to buoyancy,

$$\begin{aligned} F_T &= \text{Weight of the medium displaced} \\ \therefore F_T &= \text{Volume of the medium displaced} \\ &\quad \times \text{Density} \times g \\ &= \frac{4}{3} \pi r^3 \rho_0 g \end{aligned}$$

If v is the terminal velocity of the body, then according to Stoke's law, upward viscous drag,

$$F_v = 6\pi\eta r v$$

When body attains terminal velocity, then

$$F_T + F_v = W$$

$$\therefore \frac{4}{3} \pi r^3 \rho_0 g + 6\pi\eta r v = \frac{4}{3} \pi r^3 \rho g$$

$$\text{or } 6\pi\eta r v = \frac{4}{3} \pi r^3 (\rho - \rho_0) g$$

$$\text{or } v = \frac{2r^2(\rho - \rho_0)g}{9\eta} \quad \mathbf{1}$$

It depends directly as the square of the radius of the body and inversely as the coefficient of viscosity of the medium. It also depends upon densities of the body and the medium. **1**

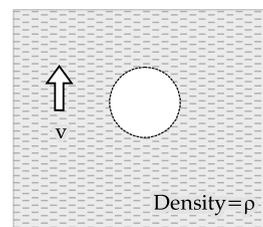
Q. 3. An air bubble of radius r rises steadily through a liquid of density ρ at the rate of v . Neglecting density of air, find the coefficient of viscosity of liquid. [A]

$$\begin{aligned} \text{Ans. Buoyant force} &= \text{weight of liquid displaced} \\ &= \frac{4}{3} \pi r^3 \rho g \quad \mathbf{1} \end{aligned}$$

Viscous force = Stoke's drag force

$$\begin{aligned} &= 6\pi\eta r v \\ 6\pi\eta r v &= \frac{4}{3} \pi r^3 \rho g \end{aligned}$$

$$\eta = \frac{2r^2 \rho g}{9v}$$



👁️ Long Answer Type Question

(5 marks)

- Q. 1. (a) Liquid drops are spherical in shape. Why ?
 (b) Derive the expression of excess pressure inside the liquid drop.
 (c) Give two similarities and dissimilarities between friction and viscosity. [A] [KVS 2014]

Ans. (a) **Liquid drops** : One consequence of surface tension is that free liquid drops and bubbles are spherical if effects of gravity can be neglected.

A liquid air interface has energy, so for a given value the surface with minimum energy is the one with the least area. The sphere has this property. So, if gravity and outer forces were ineffective, liquid drops would be spherical and the pressure inside a spherical drop is more than that the pressure outside. 1

- (b) Do yourself. Refer Q1 of long answer type questions of Topic 2.
 (c) **Similarities between friction & viscosity** :
 (1) Viscosity is due to the friction between neighbouring particles in a fluid those are moving at different velocities. Fluid friction also called as viscosity.

- (2) Viscosity and friction both depends on the size and shape of its particles and the attractions between the particles. 1

Dissimilarities :

- (1) Viscosity is the measure of the resistance of a fluid, which is deformed by either shear stress or tensile stress. While friction is caused by the contact of two rough surfaces.
 (2) The viscosity of a fluid depends on the temperature. It decreases as the temperature is increased. While friction depends on the surface area. 1



Commonly Made Error

- ▶ Students can not enlist similarities and dissimilarities between friction and viscosity.



Answering Tip

- ▶ Students should be familiarized with the terms 'friction' and 'viscosity' in detail.

👁️ Objective Type Question

(1 mark each)

A Multiple Choice Questions

- Q. 1. A particle falling through a viscous liquid reaches its terminal velocity. The acceleration then is
 (A) g (B) > g
 (C) < g (D) 0

Ans. Option (D) is correct.

Explanation: After achieving terminal velocity the particle moves with a constant velocity. Hence the acceleration is 0.

- Q. 2. Bernoulli's principle is based on principle of conservation of _____ .
 (A) Energy
 (B) Mass
 (C) Linear momentum
 (D) Angular momentum

Ans. Option (A) is correct.

- Q. 3. Water in a capillary tube rises to a height 6 cm. The area of cross-section of the capillary is made one fourth. Water will raise upto
 (A) 6 cm (B) 12 cm
 (C) 3 cm (D) 24 cm

Ans. Option (B) is correct.

Explanation: $h = \frac{2T}{r\rho g}$

Initial area = $A_1 = \pi r_1^2$
 Final area = $A_2 = \pi r_2^2$

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

Or, $4 = \frac{r_1^2}{r_2^2}$

∴ $\frac{r_1}{r_2} = 2$

$$h = \frac{2T}{r\rho g}$$

∴ $h \propto \frac{1}{r}$

or $\frac{h_1}{h_2} = \frac{r_2}{r_1}$

or $\frac{6}{h_2} = \frac{1}{2}$

∴ $h_2 = 12 \text{ cm}$

- Q. 4. The excess pressure in a soap bubble is

(A) $\frac{4T}{R}$ (B) $\frac{T}{4R}$

(C) $\frac{2T}{R}$ (D) $\frac{R}{2T}$

Ans. Option (A) is correct.

- Q. 5. 1 torr of pressure is the pressure exerted by
 (A) 1 m high column of water
 (B) 1 cm high column of water
 (C) 1 mm high column of water
 (D) None of the above

Ans. Option (D) is correct.

Explanation: 1 torr = pressure exerted by 1 mm high column of mercury.

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
 (B) Both A and R are true but R is NOT the correct explanation of A
 (C) A is true but R is false
 (D) A is false and R is true

- Q. 1. Assertion (A): Liquid does not overflow due to rise of liquid in a capillary tube of insufficient length.
 Reason (R): Rise of liquid in a capillary tube is inversely proportional to the radius of area of cross-section of the tube.

Ans. Option (B) is correct.

Explanation: Curvature of liquid surface at the top of a capillary tube increases but liquid does not overflow due to rise of liquid in a capillary tube of insufficient length. So, the assertion is true.

Rise of liquid in a capillary tube $= h = \frac{2T}{r\rho g}$. So

$h \propto \frac{1}{r}$. So, the reason is also true. But it does not explain the assertion.

- Q. 2. Assertion (A): An aircraft can gain height due to pressure difference between the upper and lower surface of its wings.

Reason (R): The velocity of air on the upper surface of the wings of an aircraft is higher than that on the lower surface of the wings.

Ans. Option (A) is correct.

Explanation: The shape of the wings of an aircraft is such that its surface area is greater on the top than on the bottom. As the air flows over the wings, its velocity increases according to continuity law.

According to Bernoulli's equation as the velocity increases, pressure decreases. Thus the higher is the velocity of the flow, the lower is the pressure. Air flowing over the wings decreases the pressure. The pressure over the top surface is less than that of the bottom surface. This results in a net pressure force in the upward direction. Thus an aircraft gains a lift.

So, the assertion and reason both are true and the reason explains the assertion.

- Q. 3. Assertion (A): Water does not wet wax.

Reason (R): Angle of contact for water-wax is obtuse.

Ans. Option (A) is correct.

Explanation: When angle of contact is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface and liquid then does not wet the solid. This is what happens with water on wax and so water does not wet wax.

So, the assertion and reason both are true and the reason explains the assertion.



NCERT Corner

- Q. 1. Explain why

- (a) The blood pressure in human is greater at the feet than at the brain.
 (b) Atmospheric pressure at a height of about 6 km decreases to nearly half its value at the sea level, though the 'height' of the atmosphere is more than 100 km.
 (c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area, and force is a vector.

Ans. (a) The height of the blood column in the human body is more at the feet than at the brain. That is why, the blood exerts more pressure at the feet than at the brain (\therefore Pressure = $h\rho g$).

- (b) The density of air is maximum near the surface of earth and decreases rapidly with height and at a height of about 6 km, it decreases to nearly half its value at the sea level. Beyond 6 km height the density of air decreases very slowly with height. Due to this reason the atmospheric pressure at the height of about 6 km decreases to nearly half of its value at sea level.

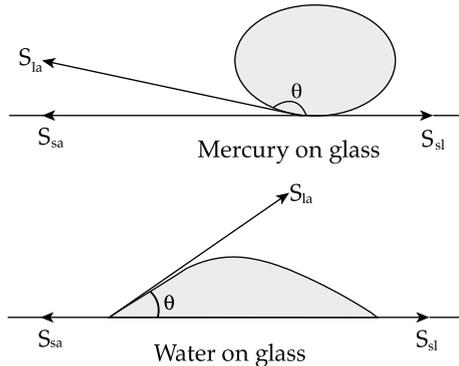
- (c) Due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for pressure due to liquid. Hence, hydrostatic pressure is scalar quantity.

- Q. 2. Explain why

- (a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
 (b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)
 (c) Surface tension of a liquid is independent of the area of the surface.
 (d) Water with detergent dissolved in it should have small angles of contact.
 (e) A drop of liquid under no external forces is always spherical in shape.

Ans. (a) The angle between the tangent to the liquid

surface at the point of contact and the surface inside the liquid is called the angle of contact (θ), as shown in the given figure.



S_{la} , S_{sa} and S_{sl} are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos \theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

The angle of contact θ , is obtuse if $S_{sa} < S_{sl}$ (as in the case of mercury on glass). This angle is acute if $S_{sa} > S_{sl}$ (as in the case of water on glass).

- (b) Mercury molecules (which make an obtuse angle with glass) have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, they tend to form drops. On the other hand, water molecules make acute angles with glass. They have a weak force of attraction between themselves and a strong force of attraction toward solids. Hence, they tend to spread out.
- (c) Surface tension is the force acting per unit length at the interface between the plane of a liquid and any other surface. This force is independent of the area of the liquid surface. Hence, surface tension is also independent of the area of the liquid surface.
- (d) Water with detergent dissolved in it has small angles of contact (θ). This is because for a small θ , there is a fast capillary rise of the detergent in the cloth. The capillary rise of a liquid is directly proportional to the cosine of the angle of contact (θ). If θ is small, then $\cos \theta$ will be large and the rise of the detergent water in the cloth will be fast.
- (e) A liquid tends to acquire the minimum surface area because of the presence of surface tension. The surface area of a sphere is minimum for a given volume. Hence, under no external forces, liquid drops always take spherical shape.

Q. 3. Fill in the blanks using the words from the list appended with each statement :

- (a) Surface tension of liquids generally _____ with

temperature. (increases / decreases)

- (b) Viscosity of gases _____ with temperature, whereas viscosity of liquids _____ with temperature. (increases / decreases)
- (c) For solids with elastic modulus of rigidity, the shearing force is proportional to _____ while for fluids it is proportional to _____. (shear strain / rate of shear strain)
- (d) For a fluid in steady flow, the increases in flow speed at a constriction follows form _____ while the decrease of pressure there follows from _____. (conservation of mass / Bernoulli's principle)
- (e) For the model of a plane in a wind tunnel, turbulence occurs at a _____ speed than the critical speed for turbulence for an actual plane. (greater / smaller)

Ans.

- (a) decreases
 (b) increases, decreases
 (c) shear strain, rate of shear strain
 (d) conservation of mass, Bernoulli's principle
 (e) greater

Q. 4. Explain why :

- (a) To keep a piece of paper horizontal, you should blow over, not under it.
 (b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.
 (c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.
 (d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.
 (e) A spinning cricket ball in air does not follow a parabolic trajectory.

Ans.

(a) When we blow over a piece of paper, velocity of air above the paper becomes more than that below it. Since, K.E. of air above the paper increases, so in accordance with Bernoulli's theorem for horizontal flow ($P + \frac{1}{2} \rho v^2 = \text{constant}$), its pressure energy and hence its pressure decreases. Because of greater value of pressure below the piece of paper it remains horizontal and does not fall.

While we blow under the paper, the pressure on the lower side decreases. The atmospheric pressure above the paper will therefore bend the paper downwards. So, the paper will not remain horizontal.

- (b) This can be explained with the equation of continuity, i.e., $a_1 v_1 = a_2 v_2$. We try to close a water tap with our fingers, the area of cross-section of the outlet of water jet is reduced considerably as the openings between our fingers provide constriction (i.e., regions of smaller area). Hence, velocity of water increases greatly and fast jets of water come through the openings between our fingers.
- (c) From Bernoulli's theorem, we know that

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant} \quad \dots(i)$$

Here, the size of the needle controls the velocity of flow and the thumb pressure controls pressure. Now P occurs with power 1 and velocity (v) occurs with power 2 in equation (i), therefore, the velocity has more influence. That is why the needle of syringe has a better control over the flow rate.

- (d) If a fluid is flowing out of a small hole in a vessel, it acquires a large velocity and hence possesses large momentum. Since, no external force is acting on the system, a backward velocity must be attained by the vessel (according to the law of conservation of momentum). As a result of it, an impulse (backward thrust) is experienced by the vessel.
- (e) This is because of Magnus effect. For a spinning ball the velocity of air at the top is higher than the velocity of air below the ball. So, according to Bernoulli's theorem, the pressure above the ball is less than the pressure below the ball. Thus there is a net upward force on the spinning ball, so the ball does not follow a curved path.

Q. 5. A 50 kg of girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor ?

Ans.
$$P = \frac{\text{Force}}{\text{Area}} = \frac{mg}{\pi(D/2)^2}$$

$$= \frac{4mg}{\pi D^2}$$

$$= \frac{4 \times 50 \times 9.8}{\left(\frac{22}{7}\right) \times (10^{-2})^2}$$

$$= 6.2 \times 10^6 \text{ Pa.}$$

Q. 6. Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m⁻³. Determine the height of the wine column for normal atmospheric pressure.

Ans.
$$P = 0.76 \times (13.6 \times 10^3) \times 9.8 = h \times 984 \times 9.8$$
 or
$$h = \frac{0.76 \times 13.6 \times 10^3 \times 9.8}{984 \times 9.8}$$

$$= 10.5 \text{ m}$$

Q. 7. A vertical off shore structure is built to withstand a maximum stress of 10⁹ Pa. Is the structure suitable for putting up on top of an oil well in Bombay High? Take the depth of the sea to be roughly 3 km, and ignore the ocean currents.

Ans. Given : Maximum stress = 10⁹ Pa

$$h = 3 \text{ km} = 3 \times 10^3 \text{ m}$$

$$\rho_{\text{water}} = 10^3 \text{ kg/m}^3, g = 9.8 \text{ m/s}^2.$$

This structure will be suitable, provided the pressure exerted by sea water is less than maximum stress it can bear.

Pressure due to sea water, $P = h\rho g$

$$= 3 \times 10^3 \times 10^3 \times 9.8$$

$$= 2.94 \times 10^7 \text{ Pa}$$

The pressure of sea water is less than the maximum stress of 10⁹Pa. So, the structure will be suitable for putting upon top of oil well.

Q. 8. A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm². What maximum pressure would the smaller piston have to bear ?

Ans. Given : The maximum force which the bigger piston can bear,

$$F = 3000 \text{ kgf}$$

$$= 3000 \times 9.8 \text{ N}$$

Area of piston, $A = 425 \text{ cm}^2$

$$= 425 \times 10^{-4} \text{ m}^2.$$

When $P =$ maximum pressure on the bigger piston

Then
$$P = \frac{F}{A} = \frac{3000 \times 9.8}{425 \times 10^{-4}}$$

$$= 6.92 \times 10^5 \text{ Pa}$$

As the liquid transmits pressure equally in all directions, hence the maximum pressure the smaller piston would have to bear is $6.92 \times 10^5 \text{ Pa}$.

Q. 9. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the relative density of spirit ?

Ans. Since, the mercury columns in the two arms are at level

$$h_1 \rho_1 g = h_2 \rho_2 g$$
 or
$$\rho_2 = \frac{h_1 \rho_1}{h_2}$$

Hence, here, $h_1 = 10 \text{ cm}$, $h_2 = 12.5 \text{ cm}$
 and $\rho_1 = 1 \text{ g cm}^{-3}$.

$$\rho_2 = \frac{10 \times 1}{12.5} = 0.8 \text{ g cm}^{-3}.$$

Relative density of spirit = 0.8

Q. 10. In the previous problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? (Relative density of mercury = 13.6.)

Ans. Here, pressure exerted by (10 + 15 = 25 cm) of water column = pressure exerted by h cm of mercury column + pressure exerted by (12.5 + 15 = 27.5 cm) of spirit column.

i.e.,
$$25 \times 1 \times g = h \times 13.6 \times g + 27.5 \times 0.8 \times g$$
 (Using $P = h\rho g$)

or
$$25 = 13.6 h + 22$$

$$\therefore h = \frac{25 - 22}{13.6} = 0.22 \text{ cm.}$$

Q. 11. Can Bernoulli's equation be used to describe the flow of water through a rapid flow in a river ? Explain.

Ans. No, Bernoulli's equation cannot be used to describe the rapid flow of water in a river (i.e.,

turbulent flow). This equation can be used for stream-line flow.



Commonly Made Error

- Students can not govern type of flow for which Bernoulli's theorem is applicable.



Answering Tip

- Students should be familiarized with streamline and turbulent flow.

Q. 12. Does it matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation ?

Ans. No, it does not matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation, provided the atmospheric pressure at the two places of consideration are not different.

Q. 13. Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3} \text{ kgs}^{-1}$, what is the pressure difference between the two ends of the tube ? (Density of glycerine = $1.3 \times 10^3 \text{ kgm}^{-3}$ and viscosity of glycerine = 0.83 Pa).

Ans. Given : Radius, $r = 1.0 \text{ cm} = 10^{-2} \text{ m}$

Length of tube, $l = 1.5 \text{ m}$

Density of glycerine,

$$\rho = 1.3 \times 10^3 \text{ kg m}^{-3}.$$

Mass of glycerine flowing per sec,

$$M = 4 \times 10^{-3} \text{ kg s}^{-1}$$

Viscosity of glycerine,

$$\begin{aligned} \eta &= 0.83 \text{ Pa} \\ &= 0.83 \text{ Nm}^{-2} \text{ s} \end{aligned}$$

Suppose P = pressure difference between two ends of the tube

Suppose V = Volume of glycerine flowing per sec.

$$= \frac{M}{\rho} = \frac{4 \times 10^{-3} \text{ kgs}^{-1}}{1.3 \times 10^3 \text{ kgm}^{-3}}$$

$$= \frac{4}{1.3} \times 10^{-6}$$

$$= 3.08 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

Now from Poiseuille's formula, we have

$$V = \frac{\pi p r^4}{8 \eta l}$$

or $p = 8 \eta l V / \pi r^4.$

or
$$p = \frac{8 \times 0.83 \times 1.5 \times \left(\frac{4}{1.3} \times 10^{-6} \right)}{3.142 \times (10^{-2})^4}$$

$$\begin{aligned} &= 9.7537 \times 10^2 \text{ Pa} \\ &= 9.8 \times 10^2 \text{ Pa} \end{aligned}$$

Q. 14. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surface of the wing are 70 ms^{-1} and 63 ms^{-1} respectively. What is the lift on the wing if its area is 2.5 m^2 . (Take the density of air to be 1.3 kg m^{-3} .)

Ans. Here $\rho = 1.3 \text{ kgm}^{-3}$, $a = 2.5 \text{ m}^2$,

$$v_1 = 70 \text{ ms}^{-1} \text{ and } v_2 = 63 \text{ ms}^{-1}$$

Using Bernoulli's equation,

$$\frac{P_1}{\rho} + \frac{1}{2} v_1^2 = \frac{P_2}{\rho} + \frac{1}{2} v_2^2$$

$$(P_2 - P_1) = \frac{1}{2} (v_1^2 - v_2^2) \rho$$

$$= \frac{1}{2} [(70)^2 - (63)^2] \times 1.3$$

$$= 605.2 \text{ Nm}^{-2}.$$

Therefore lift on the wing

$$F = (P_2 - P_1) \times a$$

$$= 605.2 \times 2.5$$

$$= 1513 \text{ N}$$



Commonly Made Error

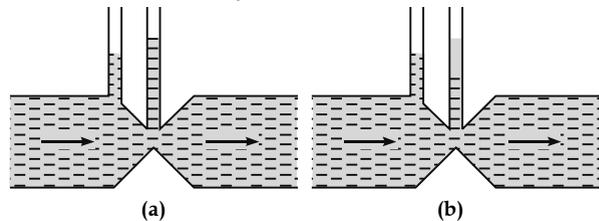
- Students can not evaluate the lift on the wing of the aeroplane.



Answering Tip

- Students should be able to apply Bernoulli's theorem to practical problems.

Q. 15. Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?



Ans. Figure (a) is incorrect. According to the equation of continuity $aV = \text{constant}$, where area of cross-section is less, flow is more. So, the velocity of liquid flow at a constriction of tube is more than other portion.

According to Bernoulli's theorem

$$P + \frac{1}{2} \rho v^2 = \text{constant}$$

So, where v is more, P is less.

Q. 16. The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm . If the liquid flow inside the tube is 1.5 m-min^{-1} , what is the speed of ejection of the liquid through the holes ?

Ans. Here, $a_1 = 8 \text{ cm}^2 = 8 \times 10^{-4} \text{ m}^2$
 $v_1 = 1.5 \text{ m min}^{-1}$
 $= 0.025 \text{ ms}^{-1}$

Diameter of one hole,
 $d = 1 \text{ mm} = 10^{-3} \text{ m}$
 Area of hole = $\frac{\pi d^2}{4} = \frac{\pi \times (10^{-3})^2}{4}$
 $= \frac{\pi}{4} \times 10^{-6} \text{ m}^2$

Therefore total cross-section of 40 holes
 $= \frac{\pi}{4} \times 10^{-6} \times 40 \text{ m}^2$

If v_2 is the speed of ejection of the liquid through the hole

$$a_1 v_1 = a_2 v_2$$

$$v_2 = \frac{a_1 v_1}{a_2} = \frac{8 \times 10^{-4} \times 0.025}{\frac{\pi}{4} \times 10^{-6} \times 40}$$

$$= 0.637 \text{ m/s}$$

Q. 17. An U-shaped wire is dipped in a soap solution and removed. The thin soap film formed between the wire and the light slider supports a weight of $1.5 \times 10^{-2} \text{ N}$ (which includes the small weight of the slider). The length of the slider is 30 cm . What is the surface tension of the film ?

Ans. A soap film has two free surfaces, total length of the film to be supported,

$$l = 2 \times 30 \text{ cm}$$

$$\Rightarrow l = 60 \text{ cm} = 0.60 \text{ m}$$

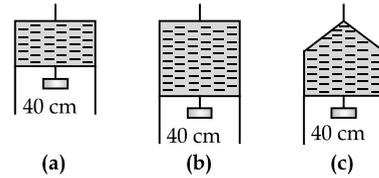
Suppose $T =$ Surface tension of the film
 When $F =$ Total force on the slider due to surface tension.
 Then $F = T \times 2l = T \times 0.6 \text{ N}$,
 $W = 1.5 \times 10^{-2} \text{ N}$.

In equilibrium position, the force F on the slider due to surface tension must be balanced by the weight (W) supported by the slider.

i.e., $F = W = mg$
 or $T \times 0.6 = 1.5 \times 10^{-2}$
 $T = \frac{1.5 \times 10^{-2}}{0.6}$
 $= 2.5 \times 10^{-2} \text{ Nm}^{-1}$.

Q. 18. Figure (a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2} \text{ N}$. What is the weight supported by a film of the same liquid at the

same temperature in figure (b) and (c) ? Explain your answer physically.



Ans. Given : Length of the film supporting the weight
 $l = 40 \text{ cm} = 0.4 \text{ m}$.

Total weight supported (i.e., force)
 $= 4.5 \times 10^{-2} \text{ N}$.
 $T =$ surface tension = ?

The film has two free surfaces, so total length,
 $L = 2l = 2 \times 0.4 \text{ m}$.

\therefore By formula $T = \frac{\text{Force}}{\text{Length}} = \frac{4.5 \times 10^{-2}}{2 \times 0.4}$
 $= 5.625 \times 10^{-2} \text{ Nm}^{-1}$.

As the liquid is same for all the cases (a), (b) and (c) and temperature is also same, therefore surface tension for cases (b) and (c) will also be same, i.e., $5.625 \times 10^{-2} \text{ Nm}^{-1}$.

In Figure (b) and (c), the length of the film supporting the weight is also the same as that of (a), hence the total weight supported in each case is $4.5 \times 10^{-2} \text{ N}$.



Commonly Made Error

Students can not evaluate surface tension of a thin liquid film of same liquid, at same temperature illustrated in different cases.



Answering Tip

Students should understand that surface tension depends on the nature of liquid and temperature.

Q. 19. What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature (20°C) is $4.65 \times 10^{-1} \text{ Nm}^{-1}$. The atmospheric pressure is $1.01 \times 10^5 \text{ Pa}$. Also give the excess pressure inside the drop.

Ans. Here, $S = 4.65 \times 10^{-1} \text{ Nm}^{-1}$
 $R = 3 \text{ mm}$
 $= 3 \times 10^{-3} \text{ m}$
 $P_{\text{outside}} = 1.01 \times 10^5 \text{ Pa}$.

Using the relation,
 Excess pressure = $\frac{2S}{r}$,
 $= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}}$
 $= 310 \text{ Pa}$
 Total pressure = $1.01 \times 10^5 + 310$
 $= 1.0131 \times 10^5 \text{ Pa}$

Q. 20. What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temp. (20° C) is $2.50 \times 10^{-2} \text{ Nm}^{-1}$? If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atmospheric pressure is $1.0 \times 10^5 \text{ Pa}$.)

Ans. In case of soap bubble, excess pressure

$$= \frac{4S}{R} = \frac{4(2.5 \times 10^{-2})}{5 \times 10^{-3}}$$

$$= 20 \text{ Pa}$$

In case of air bubble, excess pressure

$$= \frac{2S}{R} = \frac{2(2.5 \times 10^{-2})}{5 \times 10^{-3}}$$

$$= 10 \text{ Pa}$$

$$P_{\text{outside}} = \text{Pressure depth of 40 cm}$$

$$= \text{Atm. pressure} + \text{column pressure}$$

$$= (1.01 \times 10^5) + 0.4 \times 1.2 \times 10^3 \times 9.8$$

$$= 1.0570 \times 10^5 \text{ Pa} = 1.06 \times 10^5 \text{ Pa}$$

Now, excess pressure = $P_{\text{inside}} - P_{\text{outside}}$

$$P_{\text{inside}} = P_{\text{outside}} + \text{excess pressure}$$

$$= 1.06 \times 10^5 + 10$$

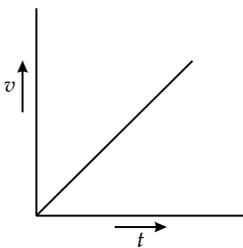
$$\approx 1.06 \times 10^5 \text{ Pa}$$



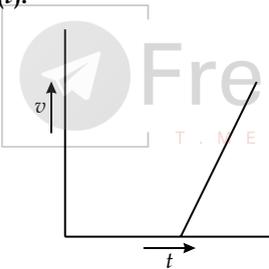
NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

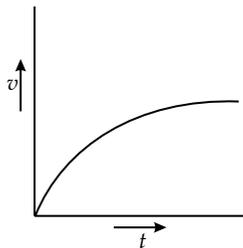
Q. 1. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in Fig. indicate the one that represents the velocity (v) of the pebble as a function of time (t).



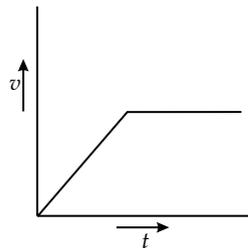
(A)



(B)



(C)



(D)

Ans. Option (C) is correct.

Explanation: As the pebble acquires terminal velocity after some time.



Commonly Made Error

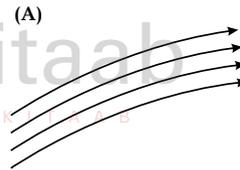
► Students lack understanding about viscosity and terminal velocity.



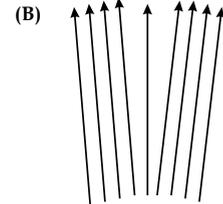
Answering Tip

► Students should understand that the terminal velocity is the maximum constant velocity attained by the body.

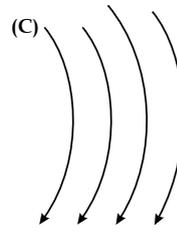
Q. 2. Which of the following diagrams does not represent a streamline flow?



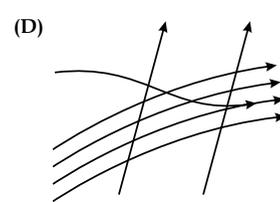
(A)



(B)



(C)



(D)

Ans. Option (D) is correct.

Explanation: In streamline flow of flow do not intersect each other.

Q. 3. Along a streamline

- (A) the velocity of a fluid particle remains constant.
- (B) the velocity of all fluid particles crossing a given position is constant.
- (C) the velocity of all fluid particles at a given instant is constant.
- (D) the speed of a fluid particle remains constant.

Ans. Option (B) is correct.

Q. 4 An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in two pipes is

- (A) 9 : 4
- (B) 3 : 2

(C) $\sqrt{3}:\sqrt{2}$

(D) $\sqrt{2}:\sqrt{3}$

Ans. Option (A) is correct.

Explanation: According to equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\text{or } \frac{v_1}{v_2} = \frac{a_2}{a_1} = \frac{\pi \frac{d_2^2}{4}}{\pi \frac{d_1^2}{4}} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{3.75}{2.50}\right)^2 = \frac{9}{4}$$

Q. 5 The angle of contact at the interface of water-glass is 0° , Ethyl alcohol- glass is 0° , Mercury-glass is 140° and methyl iodide- glass is 30° . A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is

(A) water

(B) ethyl alcohol

(C) mercury

(D) methyl iodide.

Ans. Option (C) is correct.



Commonly Made Error

► Students can not describe the shape of meniscus using angle of contact.



Answering Tip

► Students should understand that if the angle of contact is obtuse, then shape of meniscus is convex.

Very Short Answer Type Questions

Q. 6. Is viscosity a vector?

[NCERT Exemplar Q. No. 11]

Ans. Viscosity is a scalar quantity because it a property of liquid having no direction.

Q. 7. Iceberg floats in water with part of it submerged. What is the fraction of the volume of iceberg submerged if the density of ice is $\rho_i = 0.917 \text{ g cm}^{-3}$? [NCERT Exemplar Q. No. 13]

Ans. Let the volume of iceberg be V_i and volume of water displaced by iceberg be V_w in floating condition. Weight of iceberg = Weight of water displaced by submerged ice.

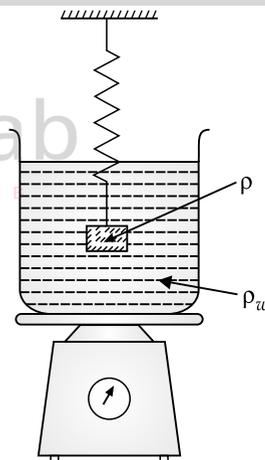
$$\rho_w V_w g = \rho_i V_i g \quad [\because W = mg = V\rho g]$$

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{0.917}{1} = 0.917.$$

Q. 8. A vessel filled with water is kept on a weighing pan and the scale adjusted to zero. A block of mass M and density ρ is suspended by a massless spring of spring constant k . This block is submerged inside the water in the vessel. What is the reading of the scale?

[NCERT Exemplar Q. No. 14]

Ans. Considering the diagram below. Beaker is placed on weighing pan which is filled with water and then scale is adjusted to zero. When block is submerged into water the buoyant force acts on block by water.



By Newton's III law, this buoyant force acts as reaction force, so block will apply force downward due to which reading on scale increases.

This is equal to buoyant force = $V\rho_w g$

$$\therefore \text{Reading of Weighing scale} = \frac{M}{\rho} \cdot \rho_w g$$

$$= \frac{\rho_w}{\rho} \cdot Mg = \rho_w V_s$$

Q. 9. A cubical block of density ρ is floating on the surface of water. Out of its height L , fraction x is submerged in water. The vessel is in an elevator accelerating upward with acceleration a . What is the fraction immersed?

[NCERT Exemplar Q. No. 15]

Ans. When cubical block submerged into water –
By principle of floatation –

$$V\rho_B g = V' \rho_w g$$

$$\left[\begin{array}{l} V = \text{volume of block} \\ V' = \text{volume of block inside water} \\ = \text{area of base of block} \times \text{height submerged} \end{array} \right]$$

$$V' = L^2 x$$

$$V = \text{volume of block} = L^3, \rho_B = \text{density of block}$$

$$\therefore L^3 \rho_B = L^2 x \rho_w$$

$$\text{or, } \frac{\rho_B}{\rho_w} = \frac{x}{L} \text{ or, } x = \frac{\rho_B}{\rho_w} L \quad \dots(i)$$

When immersed block is in lift [moving in upward direction]

Then, net acceleration = $g + a$

$$\begin{aligned} \text{Weight of block} &= m(g + a) = V\rho_B (g + a) \\ &= L^3 \rho_B (g + a) \end{aligned}$$

Let x_1 be the part of block submerged into water in moving lift.

Weight of block = Buoyant force

$$L^3 \rho_B (g + a) = x_1 L^2 \rho_w (g + a) \text{ or } \frac{\rho_B}{\rho_w} = \frac{x_1}{L}$$

$$\text{or } x_1 = L \cdot \frac{\rho_B}{\rho_w} \quad \dots(ii)$$

From (i) & (ii), we conclude that it is independent of acceleration of lift.

Short Answer Type Questions

Q. 10. The free surface of oil in a tanker, at rest, is horizontal. If the tanker starts accelerating the free surface will be tilted by an angle θ . If the acceleration is a ms^{-2} , what will be the slope of the free surface? [NCERT Exemplar Q. No. 17]

Ans. The tanker accelerates in the forward direction with acceleration a .

Let us consider a small mass m at P. When tanker is pulled by forward acceleration a then this mass also experiences the same. But due to inertia of rest it tries to remain at rest. So, with same acceleration it moves in backward direction.

Different forces acting on the mass are:

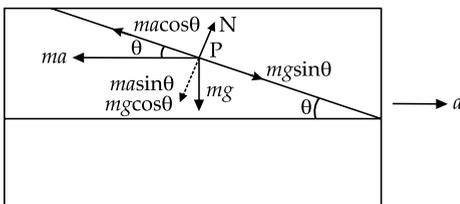
- (i) ma in horizontally backward direction. It is resolved in two components : $ma \cos \theta$ along the inclined plane and $ma \sin \theta$ along perpendicular to the inclined plane.
- (ii) mg in vertically downward direction. It is resolved in components : $mg \sin \theta$ along the inclined plane and $mg \cos \theta$ along perpendicular to inclined plane.

Normal reaction is balanced by $ma \sin \theta$.

$ma \cos \theta$ is balanced by $mg \sin \theta$.

$$ma \cos \theta = mg \sin \theta$$

$$\therefore \tan \theta = \frac{a}{g} \text{ is the required slope.}$$



Q. 11. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R , break into N

small droplets each of radius r . Estimate the drop in temperature. [NCERT Exemplar Q. No. 19]

Ans. When a big drop having radius R breaks into N droplets each of radius r , the volume remains constant.

\therefore Volume of big drop = $\Delta A = N \times$ volume of each small drop

$$\frac{4}{3} \pi R^3 = N \times \frac{4}{3} \pi r^3$$

$$\text{or, } R^3 = Nr^3 \text{ or, } N = \frac{R^3}{r^3}$$

$$\begin{aligned} \text{Change in surface area} &= 4\pi R^2 - N4\pi r^2 \\ &= 4\pi(R^2 - Nr^2) \end{aligned}$$

$$\text{Energy released} = T \times \Delta A$$

$$= T \times 4\pi(R^2 - Nr^2)$$

Released energy lowers the temperature by $\Delta\theta$, then

$$\text{Energy released} = ms\Delta\theta$$

$$T \times 4\pi(R^2 - Nr^2) = \left(\frac{4}{3} \times R^3 \times \rho \right) s \Delta\theta$$

$$\left[\begin{array}{l} s = \text{specific heat of liquid} \\ \rho = \text{density} \end{array} \right]$$

$$\text{or } \Delta\theta = \frac{T \times 4\pi(R^2 - Nr^2)}{\frac{4}{3} \pi R^3 \rho \times s} = \frac{3T}{\rho s} \left[\frac{R^2}{R^3} - \frac{Nr^2}{R^3} \right]$$

$$\Delta\theta = \frac{3T}{\rho s} \left[\frac{1}{R} - \frac{1}{r} \right]. \quad \left(\because N = \frac{R^3}{r^3} \right)$$

Q. 12. The surface tension and vapour pressure of water at 20°C is $7.28 \times 10^{-2} \text{ Nm}^{-1}$ and $2.33 \times 10^3 \text{ Pa}$, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20°C ?

[NCERT Exemplar Q. No. 20]

Ans. Surface Tension of water, $T = 7.28 \times 10^{-2} \text{ N/m}$.

Vapour pressure, $P = 2.33 \times 10^3 \text{ Pa}$.

The drop will evaporate if the water pressure is more than the vapour pressure. Let a water droplet of radius R can be formed.

Vapour pressure = Excess pressure in drop

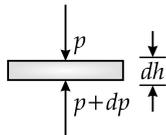
$$\begin{aligned} \therefore P &= \frac{2T}{r} = 2.33 \times 10^3 \text{ Pa} \\ \therefore P &= \frac{2T}{p} = \frac{2(7.28 \times 10^{-2})}{2.33 \times 10^3} \\ &= 6.25 \times 10^{-5} \text{ m} \end{aligned}$$

👁 Long Answer Type Question

- Q. 13.** (a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ , what is the change in pressure dp over a differential height dh ?
- (b) Considering the pressure p to be proportional to the density, find the pressure p at a height h if the pressure on the surface of the earth is p_0 .
- (c) If $p_0 = 1.03 \times 10^5 \text{ Nm}^{-2}$, $\rho_0 = 1.29 \text{ kgm}^{-3}$ and $g = 9.8 \text{ ms}^{-2}$, at what height will the pressure drop to (1/10) the value at the surface of the earth?
- (d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

[NCERT Exemplar Q. No. 21]

Ans. (a) Consider a horizontal parcel of air with cross section A and height dh . Let the pressure on the top surface and bottom surface be p and $p + dp$ respectively. If the parcel is in equilibrium, then the net upward force must be balanced by the weight.



i.e., $(p + dp)A - pA = -\rho g A dh$ [since
Weight = Density \times Volume $\times g$]
or $dp = -\rho g dh$ [here, ρ = density of air] ... (i)
(-) ve sign indicates pressure decreases with height.

- (b) Let the density of air on the earth's surface be ρ_0 , then

$$\begin{aligned} \frac{p}{p_0} &= \frac{\rho}{\rho_0} \\ \text{or } \rho &= \frac{\rho_0}{p_0} p \\ \text{Putting in equation (i)} \\ dp &= -\frac{\rho_0 g}{p_0} p dh \\ \text{or } \frac{dp}{p} &= -\frac{\rho_0 g}{p_0} dh \\ \text{or } \int_{p_0}^p \frac{dp}{p} &= -\frac{\rho_0 g}{p_0} \int_0^h dh && \text{[at } h = 0, p = p_0\text{]} \\ \text{or } \ln \frac{p}{p_0} &= -\frac{\rho_0 g}{p_0} h && \text{[at } h = h, p = p\text{]} \end{aligned}$$

Taking antilog,

$$\begin{aligned} \text{or } p &= p_0 \exp\left(-\frac{\rho_0 g}{p_0} h\right) \\ \text{(c) } \ln \frac{p}{p_0} &= -\frac{\rho_0 g}{p_0} h \\ \ln \frac{1}{10} &= -\frac{\rho_0 g}{p_0} h \\ \text{or } h &= -\frac{p_0}{\rho_0 g} \ln \frac{1}{10} = \frac{-p_0}{\rho_0 g} \ln(10)^{-1} = \frac{p_0}{\rho_0 g} \ln 10 \\ &= \frac{p_0}{\rho_0 g} \times 2.303 \\ &= \frac{1.013 \times 10^5}{1.29 \times 9.8} \times 2.303 = 0.184 \times 10^5 \text{ m} = 18.4 \text{ km} \end{aligned}$$

- (d) The assumption $p \propto \rho$ is valid only for the isothermal case which is only valid for small distances.

Temperature remains constant only near the surface of the earth, not at greater heights.



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any 4 of the following questions on the basis of the same.

I. Molecular Perspective

In water, there are two types of molecules. Some molecules are at the surface, called exterior

molecules, and some molecules are inside, called interior molecules. The interior molecules are attracted to all the molecules around them. The exterior molecules are attracted to only the other surface molecules and to those below the surface.

So that the energy state of the molecules on the interior is much lower than that of the molecules on the exterior. The molecules always try to

maintain a lower energy state and hence the exterior molecules experience a downward force. This force is known as cohesive force. As a result they try to maintain a minimum surface area, thus allowing more molecules to have a lower energy state. Thus surface tension is created.

The water molecules attract one another due to the water molecule's polar property. The hydrogen ends, which are positive in comparison to the negative ends of the oxygen cause water to "stick" together. This is why there is surface tension.

Water has very high surface tension. It is 72.8 milli newton per meter at 20°C.

Q. 1. The energy state of the interior molecules of a fluid is

- (A) Higher than the exterior molecules of the fluid
- (B) Lower than the exterior molecules of the fluid
- (C) Equal to the exterior molecules of the fluid
- (D) Higher than or Equal to the exterior molecules of the fluid

Ans. Option (B) is correct.

Explanation: In water, there are two types of molecules. Some molecules are at the surface, called exterior molecules, and some molecules are inside, called interior molecules. The interior molecules are attracted to all the molecules around them. The exterior molecules are attracted to only the other surface molecules and to those below the surface. So that the energy state of the molecules on the interior is much lower than that of the molecules on the exterior.

Q. 2. The tendency of water to maintain surface area is known as surface tension.

- (A) Maximum
- (B) Minimum
- (C) Fixed
- (D) Energetic

Ans. Option (B) is correct.

Explanation: The energy state of the molecules on the interior is much lower than that of the molecules on the exterior. The molecules always try to maintain a lower energy state. As a result, they try to maintain a minimum surface area, thus allowing more molecules to have a lower energy state. Thus, surface tension is created.

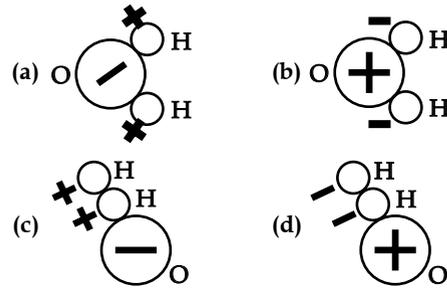
Q. 3. The water molecules attract one another due to the water molecule's

- (A) Polar covalent bond
- (B) Non-polar covalent bond
- (C) Ionic bond
- (D) Metallic bond

Ans. Option (A) is correct.

Explanation: The water molecules attract one another due to the water molecule's polar property. The hydrogen ends, which are positive in comparison to the negative ends of the oxygen cause water to "stick" together.

Q. 4. Which one of the following is the correct molecular structure of water molecule?



- (A) Figure (a)
- (B) Figure (b)
- (C) Figure (c)
- (D) Figure (d)

Ans. Option (A) is correct.

Explanation: Water is a polar molecule having positively charged sides where the two hydrogen atoms are found and a negatively charged end where the oxygen atom is located.

Q. 5. Surface tension of water is

- (A) 72.8 mN
- (B) 72.8 N/m
- (C) 72.8 mN/m
- (D) 72.8 mN/m²

Ans. Option (C) is correct.

Explanation: Water has very high surface tension. It is 72.8 milli newton per metre at 20°C.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same.

We observe several flights every day. Birds fly. But they fly not only by flapping their wings. They also glide with their wings outstretched.

Hot air balloon which is lighter than air fly on Archimedes principle. The density of hot air inside the balloon is less than that of the air at sea level, so the balloon rises. It continues its rise until the air outside of the balloon is of the same density as the air inside.

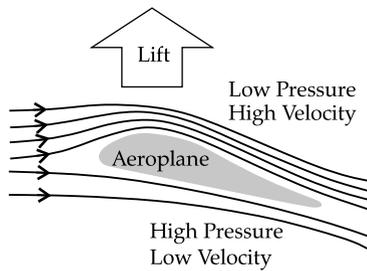
Aeroplanes which are heavier than air fly by carefully balancing four physical forces: lift, drag, weight and thrust. For aeroplane, the lift must balance its weight and its thrust must exceed its drag. Aeroplane uses its wings for lift and its engine provides the thrust. Drag is reduced by aeroplane's smooth shape and its weight is controlled by the materials with which it is constructed.

Bernoulli equation and the continuity equation show how air flowing over and below an aeroplane creates a lift. Imagine air is flowing over stationary wings of aeroplane. Far ahead the air is travelling with uniform velocity. It splits in two parts of flow – one travelling on top and other travelling on the bottom.

The shape of wing is asymmetrical. Its surface area is greater on the top than on the bottom. As

the air flows over the wings, it is displaced more by the top surface than the bottom. According to the continuity law, this leads to an increase in velocity.

The Bernoulli's equation states that an increase in velocity leads to a decrease in pressure. Thus the higher is the velocity of the flow, the lower is the pressure. Air flowing over the wings decreases the pressure. The pressure loss over the top surface is greater than that of the bottom surface. The result is a net pressure force in the upward direction. This pressure force is lift.



Every physical body that is propelled through the air experiences resistance of the air flow. This resistance is called air drag. A rough surface experiences more frictional drag than a smooth surface. An aeroplane's wings are designed to be smooth to reduce drag.

The weight of an aeroplane is a limiting factor of its design. A heavy aeroplane requires more lift than a light plane. Weight is calculated using a form of Newton's second law: $W = mg$

Thrust itself is a force which is described by Newton's second law: $F = ma$

Q. 1. How does hot air balloon fly without any engine?

Ans. Hot air balloon fly on Archimedes principle. The density of hot air inside the balloon is less than that of the air at sea level, so the balloon rises since, the buoyancy is higher than its weight. It continues its rise until the density of air outside

of the balloon becomes equal to the density of the inside air.

Q. 2. Name the forces act on a flying aeroplane.

Ans. Four forces act on an aeroplane: lift, drag, weight and thrust. The lift balances its weight and its thrust exceeds its drag.

Q. 3. How does an aeroplane get a lift?

Ans. The shape of the wings of aeroplane is such that its surface area is greater on the top than on the bottom. As the air flows over the wings, its velocity increases according to continuity law. According to Bernoulli's equation as the velocity increases, pressure decreases. Thus the higher is the velocity of the flow, the lower is the pressure. Air flowing over the wings decreases the pressure. The pressure over the top surface is less than that of the bottom surface. This results in a net pressure force in the upward direction. This pressure force is the lift.

Q. 4. What is air drag?

Ans. When a solid object moves through air, it interacts with the air molecules generating a number of miniature forces.

Air drag is the force needed to overcome these miniature forces when an object moves through air at a certain velocity. It depends on the speed of the object.

Q. 5. It has been observed that the shower curtain normally remains vertical but bulges towards the shower when the shower is on. Explain the phenomenon.

Ans. When shower is not on, there is atmospheric pressure on both sides of the curtain and the curtain remains vertical. When shower is on, due to the high-velocity stream of water and air a low pressure zone is created at the inner-side of the curtain and standard atmospheric pressure remains at the outer-side. Due to this pressure difference the curtain bulges toward the shower.

Study Time
Maximum Time: 3 : 15 hours
Maximum Questions: 88

CHAPTER

10

THERMAL PROPERTIES OF MATTER



Syllabus

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water, specific heat capacity; C_p , C_v - calorimetry, change of state - latent heat capacity, Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law.

Topic-1

Thermal Expansion & Heat Capacities

TOPIC - 1

Thermal Expansion & Heat Capacities

.... P. 217

TOPIC - 2

Heat Transfer

.... P. 221



Revision Notes

➤ Four Scales of Temperature :

S. No.	Scale	Ice point	Steam point	No. of divisions	Smallest division
1.	Centigrade scale	0°C	100°C	100	1°C
2.	Fahrenheit scale	32°F	212°F	180	1°F
3.	Thermodynamical scale or Absolute Kelvin scale	273 K	373 K	100	1 K

➤ Thermal Expansion of solids. : It is the phenomenon of expansion of solids on heating. It is of three types :

- (a) **Linear Expansion** : It is the increase in length of a solid on heating. α is called coefficient of linear expansion.
- (b) **Area Expansion** : It is the increase in surface area of a solid on heating. β is called coefficient of area expansion.
- (c) **Volume Expansion** : It is the increase in volume of a solid on heating. γ is called coefficient of volume expansion.

➤ Expansion of liquids :

- (a) Coefficient of real expansion of a liquid is defined as the real increase in volume of the liquid per unit original volume per °C rise in temperature. If γ_r is the coefficient of real expansion of a liquid, then

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

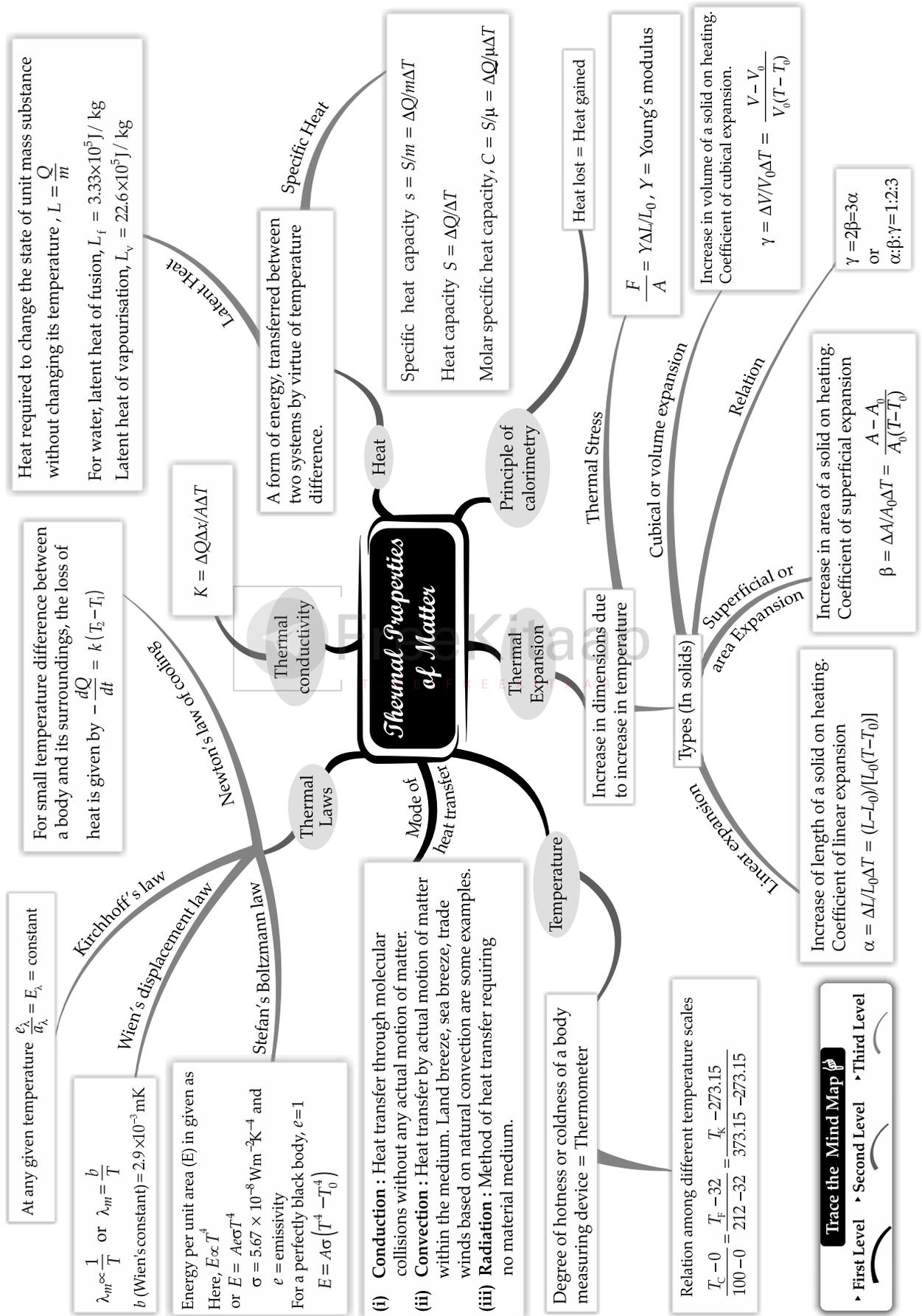
- (b) Coefficient of apparent expansion of a liquid is defined as the apparent increase in volume per unit original volume per °C rise in temperature. If γ_a is the coefficient of apparent expansion of a liquid, then

$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Original volume} \times \text{Rise in temperature}}$$

Scan to know more about this topic



Thermal expansion



Trace the Mind Map

- First Level
- Second Level
- Third Level

Key Words

- **Heat** is a form of energy, which produces the sensation of warmth. The thermal energy in matter is present in the form of translational, rotational and vibrational energy of its atoms/molecules.
- **Temperature** of a body is a measure of degree of hotness/coldness of the body. This macroscopic property determines the direction of flow of heat, when the given body is placed in contact with some other body.
- Anomalous expansion of water is the decrease of volume of water with increase in temperature from 0°C to 4°C.
- **Specific heat of a substance** is the amount of heat required to raise the temperature of unit mass of substance through unit degree.
- **Molar specific heat of a substance** is the amount of heat required to raise the temperature of 1 g mole of substance through 1°C.
- **Heat capacity or thermal capacity** of a body is the amount of heat required to raise the temperature of whole body through 1°C or 1 K.
- **Water equivalent** is the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature & represented by W.
- **Change of state** is the conversion of one of the states of matter to another.
- **Latent heat** of a substance is the amount of heat required to change the state of unit mass of the substance at constant temperature ($Q = ML$). Its units are cal/g or joule/kg and its dimensions are $[L^2T^{-2}]$.

Key Formulae

➤ Temperature

(a) Relation between °C and °F is

$$\frac{C}{5} = \frac{F - 32}{9}$$

(b) $T K = (t^{\circ}C + 273)$ or $t^{\circ}C = (T K - 273)$

(c) Temperature. diff. of 1°C = Temp. diff. of 1 K.

(d) Normal body temperature of a person is 98.6°F or 37°C.

(e) The temp. of -40° is same in °C and °F.

➤ Thermal expansion

(a) Coefficient of linear expansion $\alpha = \frac{\Delta L}{L\Delta T}$

(b) Coefficient of Area expansion $\beta = \frac{\Delta S}{S(\Delta T)}$

(c) Coefficient of volume expansion $\gamma = \frac{\Delta V}{V(\Delta T)}$

(d) $\beta = 2\alpha$; $\gamma = 3\alpha$

(e) In liquids. $\gamma_r = \gamma_a + \gamma_g$

where,

γ_r = Coefficient of real expansion of liquid

γ_a = Coefficient of apparent expansion of liquid

γ_g = Coefficient of expansion of vessel

➤ Specific heat.

$$\Delta Q = ms\Delta T$$

➤ Molar specific heat

$$C = M \times s$$

➤ Latent heat

$$\Delta Q = ML$$

L = Latent heat

➤ Specific heat of gases

$$C_p - C_v = R, \text{ Here } R = \frac{PV}{T}$$

- **Calorimetric principle:** The heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings.

Scan to know
more about
this topic



Coefficient of
expansion



Mnemonics

Concept: Relation between coefficients of thermal expansion.

Mnemonics: Atal and Vikie stood 2nd and 3rd respectively in last exam.

Interpretation:

A - Areal thermal expansion

V - Volume thermal expansion

2nd - 2

3rd - 3

last - Linear thermal expansion

Coefficient of Areal Expansion = 2 × Coefficient linear expansion
Coefficient of Volume Expansion = 3 × Coefficient linear expansion



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is the thermal capacity of a body ? R

Ans. Thermal capacity of a body is the energy required to raise its temperature by 1°C.

$$c = \frac{Q}{\Delta T} \text{ J/K or cal } ^\circ\text{C}^{-1}. \quad 1$$

Q. 2. What is specific heat capacity ? R

Ans. Specific heat capacity is defined as the amount of energy required to raise the temperature of 1 kg of a substance per unit mass by 1°C. In SI system it is expressed in J kg⁻¹ K⁻¹.

$$s = \frac{Q}{m\Delta T}, \text{ where, } m = 1\text{Kg, } \Delta T = 1^\circ\text{C} \quad 1$$



Commonly Made Error

- Students can not express the term 'specific heat capacity' explicitly.



Answering Tip

- Student should be familiarized with the related terms 'heat capacity', 'molar heat capacity' and 'specific heat capacity' in detail to avoid confusion.

Q. 3. What is latent heat energy ? R

Ans. Latent heat energy is the heat energy required to change the state of 1 kg of substance from one to another at constant temperature. 1

Q. 4. Do water and ice have the same specific heats ? U

Ans. No, water and ice do not have the same values of specific heats :

For water $c = 1 \text{ cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$

For ice $c = 0.5 \text{ cal g}^{-1} \text{ } ^\circ\text{C}^{-1}. \quad 1$

Q. 5. Why is a gap left between the ends of two railway lines in a railway track ? A [KVS 2014]

Ans. It is done to accommodate the linear expansion of railway line during summer. If the gap is not left, in summer, the lines will bend causing a threat of derailment. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. What is absolute or Kelvin scale in modern thermometry. R [KVS 2014]

Ans. Modern thermometry uses only one reference point called the triple point of water. This scale is called absolute scale or Kelvin scale and has its zero at - 273.15°C. At this temperature all molecular motion is said to cease. 2

Q. 2. Write the relation among α , β and γ . R

Ans. $\beta = 2\alpha$

and $\gamma = 3\alpha$

$\alpha : \beta : \gamma = 1 : 2 : 3$

where,

$\alpha \rightarrow$ Coefficient of linear expansion,

$\beta \rightarrow$ Coefficient of superficial expansion,

$\gamma \rightarrow$ Coefficient of cubical or volume expansion. 2

Q. 3. Two identical rectangular strips one of copper and the other of steel are riveted to form a bimetallic strip. What will happen on heating ? A

Ans. Coefficient of linear expansion of copper is more than steel. On heating, the expansion in copper strip is more than the steel strip. 1

The bimetallic strip will bend with steel strip on inner. 1

Q. 4. Why is the specific heat at constant pressure greater than specific heat at constant volume ? U A I

Ans. When the gas is heated at constant volume, the heat is consumed only in increasing kinetic energy of the molecules. Less heat is needed for same rise in temperature. 1

On heating at constant pressure, the gas expands and does external work. It needs more heat for same rise in gas temperature. It makes C_p more than C_v by the amount required to do external work. 1



Commonly Made Error

- Students can not reason out for the greater specific heat at constant pressure.



Answering Tip

- Student should understand that C_p is greater than C_v because at constant pressure extra work is done.

Q. 5. Water is used as an effective coolant. Why? U

Ans. It is because, the specific heat of water is very high. When water runs through the hot part of the engine or machinery, it absorbs a large amount of heat from it. For this reason, the temperature of the hot engine decreases. 2



Long Answer Type Questions

(5 marks each)

Q. 1. Derive an expression to find the specific heat with the help of calorimeter. U

Ans. Let

W = Water equivalent of calorimeter.

T_1 = Initial temperature of water and calorimeter

m_1 = Mass of water.

m_2 = Mass of substance

c = Specific heat of the substance

T_2 = Temperature of the substance

T = Final temperature

Rise in temperature of water and calorimeter

$$= (T - T_1)$$

Fall in temperature of substance = $(T_2 - T)$ 1

Heat gained by water and calorimeter

$$= (m_1 + W)(T - T_1) \quad \dots(i) \quad 1$$

Heat lost by the substance

$$= c.m_2(T_2 - T) \quad \dots(ii) \quad 1$$

If we assume that there is no stray loss of heat then

Heat lost = Heat gained

$$c m_2 (T_2 - T) = (m_1 + W)(T - T_1)$$

$$c = \frac{(m_1 + W)(T - T_1)}{m_2(T_2 - T)} \quad 1$$

Topic-2 Heat Transfer



Revision Notes

➤ **Thermal Conductivity :**

(i) **Coefficient of Thermal Conductivity :** It is equal to rate of flow of heat per unit area per unit temperature gradient across the solid at steady state. It is represented by K and its value depends on nature of material of solid.

$$K = \frac{\Delta Q \Delta x}{\Delta T A}$$

$$K = \Delta Q, \text{ when } \left(\frac{\Delta x}{\Delta T} \right) = 1, A = 1$$

(ii) **Thermal resistance** corresponds to electrical resistance (V/i) and is given by the ratio of temperature difference and rate of flow of heat i.e.,

$$R_{Th} = \frac{T_1 - T_2}{dQ/dt}$$

$$= \frac{x}{KA}$$

➤ **Total emittance or emissive power** of a body at a certain temperature is the total amount of thermal energy emitted per unit time per unit area of the body for all possible wavelengths. It is represented by e' .

$$e = \int_0^\infty e_\lambda d\lambda$$

Scan to know more about this topic



Thermal conductivity

- **Emissivity** (ϵ) of a body at a given temperature is the ratio of emissive power of the body (e) to the emissive power of perfectly black body (E) at that temperature,

i.e.,
$$\epsilon = \frac{e}{E}$$

Similarly, we can define monochromatic absorptance or spectral absorptive power. Total absorptance or absorbing power

$$a = \int_0^{\infty} a_{\lambda} d\lambda$$

- **Kirchhoff's Law.** From this law, at a given temperature and for a given wavelength, the ratio of spectral emissive power (e_{λ}) to spectral absorptive power (a_{λ}) for all bodies is constant which is equal to spectral emissive power of a perfectly black body (E_{λ}) at the same temperature and for the same wavelength, i.e., $\frac{e_{\lambda}}{a_{\lambda}} = E_{\lambda}$ clearly, $e_{\lambda} \propto a_{\lambda}$ it

means good emitters are good absorbers. The law implies that at a particular temperature, a body can absorb only those wavelengths, which it is capable of emitting.

- **Wien's law :** From this law, the wavelength (λ_m) corresponding to which energy emitted/sec/ unit area by a perfectly black body is maximum, is inversely proportional to the absolute temperature (T) of the black body.

$$\lambda_m \propto \frac{1}{T}$$

or,
$$\lambda_m = \frac{b}{T}$$

where b is a constant of proportionality and is known as Wien's constant $b = 2.898 \times 10^{-3}$ mK.

- **Newton's law of cooling.** According to this law, when difference in temperature of a liquid and its surroundings is small ($\sim 30^{\circ}\text{C}$), then the rate of loss of heat of the liquid is directly proportional to difference in temperatures of the liquid and the surroundings, i.e.,

or,
$$\begin{aligned} -\frac{dQ}{dt} &\propto (\theta - \theta_0) \\ -\frac{dQ}{dt} &= K(\theta - \theta_0) \end{aligned}$$

where K is constant of proportionality.

- **Stefan's law :** From this law, the total energy (E) emitted/sec/ unit area by a perfectly black body corresponding to all wavelengths is directly proportional to fourth power of the absolute temperature (T) of the body i.e.

$$E \propto T^4$$

or,
$$E = \sigma T^4$$

There σ is a constant of proportionality and is called *Stefan's constant*. Its value is

$$\sigma = 5.67 \times 10^{-8} \text{ watt m}^{-2}\text{K}^{-4}$$

If Q is the total amount of heat energy emitted by the black body, then by definition,

$$E = \frac{Q}{At}$$

$\therefore Q = At \times E = At (\sigma T^4)$

If the body is not perfectly black and has an emissivity e , then $Q = eAt (\sigma T^4)$

- **Stefan Boltzman law :** From this law, the net amount of radiation emitted per second per unit area of a perfectly black body at temperature T is equal to difference in the amounts of radiation emitted/sec/unit area by the body and by the black body enclosure at T_0 .

$\Rightarrow E' = E - E_0$

As $E = \sigma T^4$

and $E_0 = \sigma T_0^4$

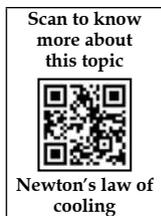
$\therefore E' = \sigma T^4 - \sigma T_0^4$
 $= \sigma(T^4 - T_0^4)$

Proceeding as above, total energy lost

$$\begin{aligned} Q' &= E'At \\ &= At\sigma(T^4 - T_0^4) \end{aligned}$$

When the body and enclosure are not perfectly black and have emissivity ϵ , then

$$Q' = \epsilon At\sigma(T^4 - T_0^4)$$



Key Words

- **Conduction** is the mode of transfer of heat from one part of the body to another, from particle to particle in the direction of fall of temperature without any actual movement of heated particles.
- **Thermal convection** is the phenomenon of transfer of heat by actual mass motion of the medium. All liquids and gases are heated by convection.
- **Radiation** is the phenomenon of transfer of heat from source to the receiver without any actual movement of source or receiver and without heating the intervening medium. For example, heat comes to us from the Sun through radiation.
- **Perfectly black body** is that which absorbs all the radiations incident upon it. Thus absorptive power of a perfectly black body is unity (*i.e.*, 100%). When such a body is heated to high temperature, it would emit radiations of all wavelengths.

Key Formulae

- Rate of conduction of Heat, $\frac{\Delta Q}{\Delta t} = KA \frac{\Delta T}{\Delta x}$
- Thermal resistance, $R_{Th} = \frac{T_1 - T_2}{dQ/dt}$
- Emissive power, $e' = \int_0^\infty e_\lambda d\lambda$
- Emissivity, $\varepsilon = \frac{e}{E}$

Very Short Answer Type Questions (1 mark each)

- Q. 1. Why are the bottoms of cooking vessels painted black ? 1
- Ans. Black paint absorbs more heat. The black painted bottoms absorb more heat and help in cooking meals faster. 1
- Q. 2. Give the expression for the net energy radiated per second. 1
- Ans. When an object at temperature T_1 is placed in a surrounding of temperature T_2 the net energy radiated per second is 1
- $E = \varepsilon A \sigma (T_1^4 - T_2^4)$
- Q. 3. Name various ways of heat transmission ? 1
- Ans. There are three ways of heat transmission namely, 1
- 1) Conduction 2) Convection 3) Radiation
- Q. 4. In how many ways the heat energy can be transferred from one body to another body ? 1
- Ans. Heat can be transferred on account of difference in temperatures of bodies by conduction and radiation. 1

Short Answer Type Questions-I (2 marks each)

- Q. 1. Two rods A and B are of equal lengths. Each rod has its ends at temperature T_1 and T_2 . What is the condition that will ensure equal rates of flow of heat through the rods A and B ? 1
- Ans. Since, $Q = KA(\theta_1 - \theta_2)t/x$
- $Q_A/t = K_A A_A (\theta_1 - \theta_2)/x$
- $Q_B/t = K_B A_B (\theta_1 - \theta_2)/x$ 1
- For equal rates of flow
- $Q_A/t = Q_B/t$
- or $K_A A_A = K_B A_B$ 1
- Q. 2. Determine the surface area of the filament of a 100 W incandescent lamp radiating out its labelled power at 3000 K. (Given $\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$, and emissivity ' e ' of the material of the filament = 0.3.) 1
- Ans. We have $H = eA\sigma T^4$
- $\therefore A = \frac{H}{e\sigma T^4}$ 1
- $= \frac{100 \text{ W}}{0.3 \times 5.7 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4} \times (3000 \text{ K})^4}$
- $= 7.22 \times 10^{-5} \text{ m}^2$ 1
- Q. 3. A man, the surface area of whose skin is 2 m^2 , is sitting in a room where air temperature is 20°C . If his skin temperature is 28°C and emissivity of his skin equals 0.97, find the rate at which his body loses heat. (Given $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$) 1
- Ans. We have $H = \sigma \varepsilon A (T^4 - T_0^4)$
- [A] [CBSE Supplement Question]

Here, $T = (28 + 273)K = 301K$
 $T_0 = (20 + 273)K = 293K$
 $H = (5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}) \times 0.97 \times 2 \text{ m}^2 [(301K)^4 - (293K)^4]$
 $= 92.2 \text{ W}$ 1

Here, since $(T - T_0)$ is small, let us also calculate H by using the approximate form of Stefan Boltzmann law. We have

$$H = \sigma \varepsilon A (T - T_0)$$

$$= 5.67 \times 10^{-8} \times 0.97 \times 2 \times 17 \text{ W}$$

$$= 88 \text{ W}$$

Q. 4. Can we boil water inside an Earth satellite by convection? U AI

Ans. No. Convection currents are set only in presence of gravity. In a satellite, in absence of gravity convection currents will not be set up. 2

Q. 5. Light, from the Moon, is found at the peak (or wavelength of maximum emission) at $\lambda = 14 \mu\text{m}$. Given that the Wien's constant 'b' equals $2.898 \times 10^{-3} \text{ mK}$, estimate the temperature of the Moon.

[A] [CBSE Supplement Question]

Ans. Using Wien's law ($\lambda_m T = b$), we get 1/2

$$T = \frac{b}{\lambda_m} = \frac{2.898 \times 10^{-3} \text{ mK}}{14 \times 10^{-6} \text{ m}}$$

$$= 207 \text{ K}$$

Thus the surface temperature of Moon is about 1
 207 K. 1/2



Short Answer Type Questions-II

(3 marks each)

Q. 1. State and prove Stefan-Boltzmann law. U

Ans. It states that the power (E) emitted per unit area by a body at absolute temperature T, is given by 1/2

$$E = \varepsilon \sigma T^4$$

where

$E =$ Energy being emitted per second per unit area of surface 1/2

$\varepsilon =$ Emissivity or emissive power of that body (which is equal to 1 for black body) 1/2

$\sigma =$ Stefan's constant 1/2
 $= 5.67 \times 10^{-8} \text{ Jm}^{-2} \text{ K}^{-4} \text{ s}^{-1}$

If temperature of the surrounding is to be taken into consideration then the net rate of loss of energy as heat per unit area per second is given by

$$E = \varepsilon \sigma (T^4 - T_0^4)$$

If the total area or surface is A, then the total energy radiated in t time

$$E = A \varepsilon \sigma (T^4 - T_0^4)$$
 1

Q. 2. Derive the Newton's law of cooling from Stefan's law. U

Ans. Consider a hot body with a temperature T_1 placed in a uniform temperature enclosure with temperature T_2 .

According to Stefan's law.

$$E = \varepsilon \sigma (T_1^4 - T_2^4)$$
 1

where, $E =$ Rate of cooling

$\varepsilon =$ The emissivity of the surface of the hot body.

$$E = \varepsilon \sigma (T_1 - T_2)(T_1^3 + T_1^2 T_2 + T_1 T_2^2 + T_2^3)$$
 1

As $(T_1 - T_2)$ is small, T_1 can be taken approximately equal to T_2 .

Then $E = \varepsilon \sigma (T_1 - T_2)(T_2^3 + 2T_2^3 + T_2^3)$

$$E = 4\varepsilon \sigma T_2^3 (T_1 - T_2)$$

As $4\varepsilon \sigma T_2^3 = \text{Constant}$

$$E \propto (T_1 - T_2)$$
 1



Answering Tip

► Student should memorize the formula $a^4 - b^4 = (a - b)(a^3 + ba^2 + ab^2 + b^3)$

Q. 3. Find the coefficient of thermal conductivity. U

Ans. Consider a cube of side x and area of each face A. The opposite faces of the cube are maintained at temperature, θ_1 and θ_2 where $\theta_1 > \theta_2$. Heat gets conducted in the direction of the fall of temperature. The flow of heat depends upon following factors

(i) $Q \propto A$...(i)

(ii) $Q \propto \frac{d\theta}{dx}$...(ii)

(iii) $Q \propto t$...(iii)

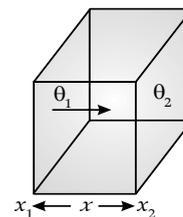
Combining equations (i),(ii) & (iii)

$$Q \propto A \frac{d\theta}{dx} \times t$$

or $Q = KA \frac{d\theta}{dx} t$

or $Q = K \frac{A(\theta_1 - \theta_2)}{x} t$

Here, K is a constant called the coefficient of thermal conductivity of the material of the cube and t stands for time interval.



Commonly Made Error

► Students can not evaluate the term of the form $(a^4 - b^4)$.

If $A = 1 \text{ m}^2$, $x = 1 \text{ m}$, $t = 1 \text{ s}$, $\Delta\theta = 1^\circ\text{C}$ then $Q = K$
Thus coefficient of thermal conductivity is defined as the amount of heat required to flow through a solid having unit area of cross section, unit length to rise 1°C in 1s.

We can also write as

$$H = KA \left[\frac{\Delta\theta}{\Delta x} \right]$$

where $H =$ Heat flow per second

$$\frac{\Delta\theta}{\Delta x} = \text{Temperature gradient}$$

$$T = \theta$$

when, $A = 1 \text{ m}^2$

$$(\theta_1 - \theta_2) = 1^\circ\text{C}$$

$$t = 1 \text{ s}$$

$$x = 1 \text{ cm}$$

then $H = K$. 1



Long Answer Type Questions

(5 marks each)

Q.1. What is temperature gradient. Find the dimension of thermal conductivity K .

Ans. The quantity $\frac{\theta_1 - \theta_2}{x}$ or $\frac{d\theta}{dx}$ represents the rate of fall of temperature w.r.t. distance. 1

The quantity $\frac{d\theta}{dx}$ is called temperature gradient.

$\frac{1}{2}$

$$Q = -KA \left[\frac{d\theta}{dx} \right] t \quad 1$$

Q represents energy and its dimensions are :

$$[Q] = [ML^2T^{-2}] \quad \frac{1}{2}$$

$$[dx] = [L] \quad \frac{1}{2}$$

$$[A] = [L^2] \quad \frac{1}{2}$$

$$[d\theta] = [\theta] \quad \frac{1}{2}$$

$$[t] = [T] \quad \frac{1}{2}$$

Dimensions of K

$$[K] = \frac{[ML^2T^{-2}][L]}{[L^2][\theta][T]} = [MLT^{-3}\theta^{-1}] \quad 1$$

Let K_1 and K_2 be the coefficients of thermal conductivity and θ_1 and θ_2 are the temperatures of the end faces ($\theta_1 > \theta_2$) and θ is the temperature of the surface in contact. 1

For material A,

$$Q_1 = \frac{K_1 A_1 (\theta_1 - \theta)}{d_1} \quad \dots(i) \quad 1$$

For the material B,

$$Q_2 = \frac{K_2 A_2 (\theta - \theta_2)}{d_2} \quad \dots(ii) \quad 1$$

From eqn. (i) and (ii), $Q_1 = Q_2$

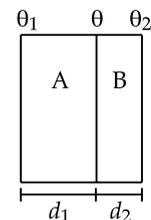
$$\frac{K_1 A_1 (\theta_1 - \theta)}{d_1} = \frac{K_2 A_2 (\theta - \theta_2)}{d_2}$$

$$\theta = \frac{\frac{K_1 \theta_1}{d_1} + \frac{K_2 \theta_2}{d_2}}{\frac{K_1}{d_1} + \frac{K_2}{d_2}} \quad 1$$

Considering $A_1 = A_2$

Substituting the value of θ in eqn. (i)

$$Q_1 = \frac{A(\theta_1 - \theta)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$



In general for any no. of walls

$$Q = \frac{A(\theta_1 - \theta_2)}{\Sigma \left(\frac{d}{K} \right)} \quad 1$$



Commonly Made Error

- Students can not find the dimensions of coefficient of thermal conductivity K .



Answering Tip

- Students should be familiarized with the dimensions of base and derived quantities.

Q.2. Derive the expression for heat flow through a compound wall.

Ans. Consider a compound wall (or a slab) made of two materials A and B of thickness d_1 and d_2 .



Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. 100g ice at 0°C is mixed with 100g water at 20°C. The final temperature will be

- (A) 0°C (B) 10°C
(C) 20°C (D) -10°C

Ans. Option (A) is correct.

Explanation: Heat required by ice to melt completely = $80 \times 100 = 8000$ calorie.

If the temperature of mixture becomes 0°C, then the heat released by water is 20°C

$$\begin{aligned} &= ms\Delta\theta \\ &= 100 \times 1 \times 20 \\ &= 2000 \text{ calorie} \end{aligned}$$

This heat will not be capable to melt the ice completely.

So, the temperature of the mixture will remain 0°C.

Q. 2. When temperature of water decreases from 4°C to 0°C its density _____, volume _____.

- (A) Increases, increases.
(B) Decreases, decreases.
(C) Increases, decreases.
(D) Decreases, increases.

Ans. Option (D) is correct.

Explanation: Due to anomalous expansion of water if the temperature of water decreases from 4°C to 0°C its density decreases, volume increases.

Q. 3. Which of following process of transmission of heat will not work in space station?

- (A) Conduction
(B) Convection
(C) Radiation
(D) Both conduction and convection

Ans. Option (B) is correct.

Explanation: Convection process depends on gravity. So, where there is no gravity the convection process does not work.

Q. 4. Two ends of a metal rod are maintained at temperatures 200°C and 210°C. The rate of heat flow was found to be 10 J/s. If the ends of the rod are maintained at 400°C and 410°C, the rate of heat flow will be

- (A) 10J/s (B) 20J/s
(C) 80J/s (D) 30J/s

Ans. Option (A) is correct.

Explanation: Since, the temperature difference remains same, the rate of heat flow will also remain same.

Q. 5. When a metal sphere is heated, the largest percentage increase will occur in its

- (A) Volume (B) Diameter
(C) Area (D) Mass

Ans. Option (A) is correct.

Explanation: Since, volume = $\frac{4}{3}\pi r^3$ and area = $4\pi r^2$, the percentage volume increase will be maximum. Density decreases with temperature. Mass is a constant quantity.

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is true

Q. 1. Assertion (A): Temperature at sea coast does not rise too much due to sea breeze.

Reason (R): Specific heat capacity of water is very high.

Ans. Option (B) is correct.

Explanation: During day time, the ground heats up more quickly than the sea. The air in contact with the warm ground is heated by conduction. It expands, becomes light and goes up. Comparatively cooler air over sea rushes to fill in the space. This is called sea breeze. The process repeats. Thus the temperature at sea coast does not rise too much due to sea breeze. So, the assertion is true.

Specific heat capacity is approximately 4200J kg⁻¹K⁻¹. It is too high. So, the reason is also true. But it does not explain the assertion.

Q. 2. Assertion (A): Water in boiled in satellite by convection process.

Reason (R): In convection process heat is transmitted by enhanced vibration of particles.

Ans. Option (D) is correct.

Explanation: Convection process does not work where there is no gravity. Hence, it does not work in satellite. So, the assertion is false.

In convection process of heat transmission particles move from high temperature zone to low temperature zone. So, the reason is also false.

Q. 3. Assertion (A): A bimetallic strip made of metals two different metals A and B bends in form an arc.

Reason (R): A and B are two different metals but having same coefficient of linear expansion.

Ans. Option (C) is correct.

Explanation: Bimetallic strip consists of the strips of different metals having different linear coefficient of expansion. So they expand at different rates when heated and bend.



NCERT Corner

Q. 1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperature on the Celsius and Fahrenheit scales.

Ans. Relation between Kelvin and Celsius scale

$$T_C = T_K - 273.15$$

where T_C , T_K are temperatures in Celsius and Kelvin respectively.

$$\text{For Neon : } T_C = 24.57 - 273.15 \\ = -248.58^\circ\text{C}$$

$$\text{For CO}_2 : T_C = 216.55 - 273.15 \\ = -56.60^\circ\text{C}$$

Relation between Kelvin and Fahrenheit

$$\frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

$$T_F = \frac{180}{100} (T_K - 273.15) + 32$$

$$\text{For Neon : } T_F = \frac{180}{100} (24.57 - 273.15) + 32 \\ = -415.44^\circ\text{F}$$

$$\text{For CO}_2 : T_F = \frac{180}{100} (216.55 - 273.15) + 32 \\ = -69.88^\circ\text{F}$$

Q. 2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?

Ans. Given :

$$\text{Triple point of water} = 200A = 350B = 273.16\text{K}$$

$$\therefore 1A = \frac{273.16}{200}\text{K}, \quad 1B = \frac{273.16}{350}\text{K}$$

If T_A and T_B represent the triple point of water on scales A & B,

$$\frac{273.16}{200} T_A = \frac{273.16}{350} T_B$$

$$\text{or} \quad \frac{T_A}{T_B} = \frac{200}{350} = \frac{4}{7}$$

$$\therefore T_A = \frac{4}{7} T_B$$

Q. 3. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$R = R_0 [1 + \alpha(T - T_0)]$. The resistance is 101.6 Ω at the normal melting point of lead 600.5K. What is the temperature when the resistance is 123.4 Ω ?

Ans. $R_0 = 101.6 \Omega$, $T_0 = 273.16 \text{K}$, $R_1 = 165.5 \Omega$,

$$T_1 = 600.5 \text{K}, R_2 = 123.4 \Omega,$$

$$\text{As,} \quad R = R_0 [1 + \alpha(T - T_0)]$$

$$165.5 = 101.6 [1 + \alpha(600.5 - 273.16)] \\ = 101.6 [1 + \alpha \times 327.34]$$

$$\text{or} \quad \alpha = \frac{165.5 - 101.6}{101.6 \times 327.34} \quad (1)$$

Also,

$$123.4 = 101.6 [1 + \alpha(T_2 - 273.16)]$$

$$123.4 = 101.6 \left[1 + \frac{63.9}{101.6 \times 327.34} (T_2 - 273.16) \right] \\ \text{[from (1) putting } \alpha]$$

$$\therefore T_2 = \frac{(123.4 - 101.6) \times 327.34}{63.9} + 273.16$$

$$T_2 = 384.83\text{K}$$

Q. 4. Answer the following :

- The triple point of water is a standard fixed point in modern thermometer. Why? What is wrong in taking melting point of ice and the boiling point of water as standard fixed points (as was originally done in Celsius scale)?
- There were two fixed points in the original Celsius scale as mentioned above which were assigned the numbers 0°C and 100°C respectively. On the absolute scale one of the fixed points is the triple point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) Scale?
- The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$. Why do we have 273.15 in this relation and not 273.16?

- What is the temperature of triple point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?

Ans. (a) This is because the triple point of water is unique condition of temperature and pressure, i.e., it occurs only at one particular set of values of pressure and temperature. Hence, it is easily reproducible. Hence it is used as a standard temperature in thermometry.

The melting point of ice and boiling point of water are pressure dependent which means they change with change in the value of pressure. They are also very sensitive to the dissolved impurities. Therefore these have been discarded as a reference point and triple of water has been taken as a fixed point.

- The 0 K or absolute zero itself is the other fixed point on the absolute scale of temperature. It corresponds to the temperature when the volume and pressure of a gas become zero.
- Triple point is 0.01°C and not 0°C . Thus to make a Celsius degree equal to Kelvin degree 273.15 is assigned instead of 273.16. Moreover ice point on Kelvin scale is 273.15 K and the corresponding is

point on celsius scale is 0°C which is clear from the relation, i.e., $t_c = T - 273.15$. When we have 273.16 instead of 273.15 in the above relation, then the ice point on celsius scale will be 0 which is not the case.

- (d) The unit interval size of Fahrenheit scale is $212 - 32 = 180$.

So, the required temperature of triple point of water =

$$T = \frac{273.16}{100} \times 180$$

$$= 491.69$$

- Q. 5. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made:**

Temperature	Pressure thermometer A	Pressure thermometer B
Triple-point of water	$1.250 \times 10^5 \text{ Pa}$	$0.200 \times 10^5 \text{ Pa}$
Normal melting point of sulphur	$1.797 \times 10^5 \text{ Pa}$	$0.287 \times 10^5 \text{ Pa}$

- (a) What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?
- (b) What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?

Ans. (a) Triple point of water, $T = 273.16 \text{ K}$

At this temperature, pressure in thermometer A, $P_A = 1.250 \times 10^5 \text{ Pa}$

Let T_1 be the normal melting point of sulphur.

At this temperature, pressure in thermometer, $P_1 = 1.797 \times 10^5 \text{ Pa}$

According to Charles' law, we have the relation:

$$\frac{P_A}{T} = \frac{P_1}{T_1}$$

$$\therefore T_1 = \frac{P_1 T}{P_A} = \frac{1.797 \times 10^5 \times 273.16}{1.25 \times 10^5}$$

$$= 392.69 \text{ K}$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer A is 392.69 K.

At triple point 273.16 K, the pressure in thermometer B,

$$P_B = 0.200 \times 10^5 \text{ Pa}$$

At temperature T_1 , the pressure in thermometer B,

$$P_2 = 0.287 \times 10^5 \text{ Pa}$$

According to Charles' law, we can write the relation:

$$\frac{P_B}{T} = \frac{P_2}{T_1}$$

$$\frac{0.200 \times 10^5}{273.16} = \frac{0.287 \times 10^5}{T_1}$$

$$\therefore T_1 = \frac{0.287 \times 10^5}{0.200 \times 10^5} \times 273.16 = 391.98 \text{ K}$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer B is 391.98 K.

- (b) The oxygen and hydrogen gas present in thermometers A and B respectively are not perfect ideal gases. Hence, there is a slight difference between the readings of thermometers A and B.

To reduce the discrepancy between the two readings, the experiment should be carried under low pressure conditions. At low pressure, these gases behave as perfect ideal gases.

- Q. 6. A steel tape 1m long is correctly calibrated for a temperature of 27.0°C . The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0°C . What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C ? Coefficient of linear expansion of steel = $1.20 \times 10^{-5} \text{ K}^{-1}$.**

Ans. The length of the steel tape at temperature

$$T = 27^{\circ}\text{C}, l = 1 \text{ m} = 100 \text{ cm}$$

At temperature $T_1 = 45^{\circ}\text{C}$, the length of the steel rod, $l_1 = 63 \text{ cm}$

Coefficient of linear expansion of steel, $\alpha = 1.20 \times 10^{-5} \text{ K}^{-1}$

Let l_2 be the actual length of the steel rod and l' be the length of the steel tape at 45°C .

$$l' = l + \alpha l (T_1 - T)$$

$$\therefore l' = 100 + 1.20 \times 10^{-5} \times 100(45 - 27)$$

$$= 100.0216 \text{ cm}$$

Hence, the actual length of the steel rod measured by the steel tape at 45°C can be calculated as:

$$l_2 = \frac{100.0216}{100} \times 63 = 63.0136 \text{ cm.}$$

Therefore, the actual length of the rod at 45.0°C is 63.0136 cm. Its length at 27.0°C is 63.0 cm.

- Q. 7. A large steel wheel is to be fitted on a shaft of the same material. At 27°C , the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume the coefficient of linear expansion of the steel to be constant over the required temperature range.**

$$\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{ K}^{-1}.$$

Ans. Suppose l_1 and l_2 be the outer diameter dimensions of a steel at temperatures T_1 and T_2 respectively.

Given $\alpha_{\text{steel}} = 1.20 \times 10^{-5} \text{ K}^{-1}$.

$$l_1 = 8.70 \text{ cm}$$

$$l_2 = 8.69 \text{ cm}$$

$$T_1 = 27^\circ\text{C} = 273 + 27 = 300\text{K}$$

$$T_2 = ?$$

When the steel shaft is cooled, its linear dimension, *i.e.*, its diameter decreases according to the formula.

$$l_2 = l_1[1 + \alpha(T_2 - T_1)] \quad \dots(i)$$

When the shaft is cooled to temperature T_2 such that $l_2 = 8.69$ cm, the wheel can slip on the shaft.

\therefore By putting all values in eq. (i), we get

$$8.69 = 8.70[1 + 1.20 \times 10^{-5}(T_2 - 300)]$$

$$\text{or, } T_2 - 300 = \frac{8.69 - 8.70}{8.70 \times 1.20 \times 10^{-5}}$$

$$= -95.78 \text{ K}$$

$$\text{or, } T_2 = 300 - 95.78 = 204.22 \text{ K}$$

$$= 204.22 - 273.15$$

$$= -68.93^\circ\text{C}$$

$$\therefore T_2 = -69^\circ\text{C}.$$

Q. 8. A hole is drilled in a Cu sheet. The diameter of the hole is 4.24 cm at 27°C . What is the change in diameter of the hole when the sheet is heated to 227°C ? (Coefficient of linear expansion of Cu = $1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.)

Ans. Using, $S_2 = S_1(1 + \beta\Delta T)$ when

$$S_1 = \frac{\pi d_1^2}{4}$$

$$= \frac{\pi}{4} (4.24)^2$$

$$= \pi \times 4.494 \text{ cm}^2$$

$$\Delta T = 227 - 27$$

$$= 200^\circ\text{C}$$

$$\beta = 2\alpha$$

$$= 2 \times 1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}.$$

$$\text{we get, } S_2 = (\pi \times 4.94) [1 + (2 \times 1.7 \times 10^{-5}) (200)]$$

$$= \pi \times 4.525 \text{ cm}^2$$

$$\text{or } \frac{\pi d_2^2}{4} = \pi \times 4.525$$

$$\text{or } d_2 = \sqrt{4 \times 4.525} = 4.254 \text{ cm}$$

$$\text{Change in diameter}$$

$$= d_2 - d_1$$

$$= 4.254 - 4.24 = 0.014 \text{ cm}$$

Q. 9 A brass wire 1.8 m long at 27°C is held tight with little tension between two rigid supports. If the wire is cooled to a temperature of -39°C . What is the tension developed in the wire, if its diameter is 2.0 mm?

Ans. Here, $l_1 = 1.8 \text{ m,}$
 $\Delta T = (-39) - 27$
 $= -66^\circ\text{C}$
 $\alpha = 2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$
 $l_2 = l_1(1 + \alpha\Delta T)$
 $= 1.8[1 + (2 \times 10^{-5})(-66)]$
 $= 1.798 \text{ m}$

$$\text{Change in length} = 1.8 - 1.798$$

$$= 0.002 \text{ m}$$

$$\text{Here } A = \frac{\pi d^2}{4} = \frac{\pi}{4} (2 \times 10^{-3})^2$$

$$= 3.14 \times 10^{-6} \text{ m}^2$$

$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

$$\text{or, } F = \frac{YA\Delta l}{l}$$

$$= (0.91 \times 10^{11}) \times (3.14 \times 10^{-6}) \times \left(\frac{0.002}{1.8}\right)$$

$$= 3.81 \times 10^2 \text{ N}$$

Q. 10. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter, what is the change in length of the combined rod at 250°C , if the original lengths are at 40.0°C ? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand.

Ans. For brass rod $l_2 = l_1(1 + \alpha\Delta T)$,
 $l_2 = 50[1 + (2 \times 10^{-5})(250 - 40)]$
 $= 50.21 \text{ cm}$

For steel Rod $l_2' = l_1'(1 + \alpha\Delta T)$,
 $l_2' = 50[1 + (1.2 \times 10^{-5})(250 - 40)]$
 $= 50.126 \text{ cm}$

$$\text{Combined length} = 50.21 + 50.126$$

$$= 100.336$$

$$\text{Change in combined length at } 250^\circ\text{C}$$

$$= 100.336 - 100$$

$$= 0.336 \text{ cm}$$

Since, the rods are free to expand at both ends, no thermal stress is developed at the junction.

Q. 11. The coefficient of volume expansion of glycerine is $49 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$. What is the fractional change in its density for a 30°C rise in temperature? **[AI]**

Ans. Here, $\Delta T = 30^\circ\text{C}$
 and $\gamma = 49 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$
 $V = V_0 [1 + \gamma\Delta T]$,

we get

$$V = V_0 [1 + (49 \times 10^{-5})(30)]$$

$$= 1.015 V_0$$

$$\text{Initial density, } \rho_0 = \frac{m}{V_0}$$

$$\text{Final density, } \rho = \frac{m}{V} = \frac{m}{1.015V_0} = \frac{\rho_0}{1.015}$$

$$= 0.986 \rho_0$$

$$\text{Fractional change in density}$$

$$= \frac{\rho_0 - \rho}{\rho_0} = \frac{\rho_0 - 0.986\rho_0}{\rho_0} = 0.014$$

$$= 0.014$$

Q. 12. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 min, assuming 50% of power is used up heating the machine itself or lost to the surrounding? (Specific heat of aluminium = $0.91 \text{ Jg}^{-1} \text{ }^\circ\text{C}^{-1}$.)

Ans. Here $t = 2.5 \text{ min} = 2.5 \times 60 = 150 \text{ s}$
 and $P = 10^4 \text{ W}$

Energy used in drilling

$$Q' = P \times t$$

$$= 10^4 \times 150 = 1.5 \times 10^6 \text{ J}$$

Useful energy $Q = Q' \times \frac{50}{100} = \frac{1.5 \times 10^6 \times 50}{100}$

$$= 7.5 \times 10^5 \text{ J}$$

Using $Q = mc\Delta T$,

$$7.5 \times 10^5 = 8 \times 0.91 \times 10^3 \Delta T$$

$$7.5 \times 10^5 = 7.28 \times 10^3 \Delta T$$

$$\therefore \Delta T = \frac{7.5 \times 10^5}{7.28 \times 10^3} = 103^\circ\text{C}$$

Q. 13. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of Copper = $0.39 \text{ J/g}^\circ\text{C}$, Latent heat of fusion of water = 335 J/g .) AI

Ans. Here $M = 2.5 \text{ kg}$, $\Delta t = 500^\circ\text{C}$
and $c = 0.39 \times 10^3 \text{ Jkg}^{-1} \text{ }^\circ\text{C}^{-1}$
Using $Q = Mc\Delta t$,
Heat absorbed by Cu block,
 $Q = 2.5 \times 0.39 \times 10^3 \times 500 \text{ J}$
Heat absorbed by the ice block to melt,
 $Q = ML$
 $= M \times 335 \times 10^3 \text{ J}$
Then $(3.35 \times 10^3) M = 2.5 \times 0.39 \times 10^3 \times 500$
 $M = \frac{2.5 \times 0.39 \times 10^3 \times 500}{335 \times 10^3}$
 $= 1.455 \text{ kg}$

Q. 14. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150°C is dropped in Cu calorimeter of water equivalent 0.025 kg containing 150 cm^3 of water at 27°C . The final temperature is 40°C . Compute the specific heat of the metal.

Ans. Here $T_1 = 150^\circ\text{C}$,
 $T_2 = 27^\circ\text{C}$
and $T = 40^\circ\text{C}$
and $M_1 = 0.20 \text{ kg}$
and $W = 0.25 \text{ kg}$
Volume of water in calorimeter
 $= 150 \text{ cm}^3$
 $= 150 \times 10^{-6} \text{ m}^3$
Mass of water in calorimeter,
 $M_2 = (150 \times 10^{-6}) \times 10^3$
 $= 150 \times 10^{-3} \text{ kg}$
Using,
Heat gained = Heat Lost
 $(M_2 + W) c_2(T - T_2) = M_1 c_1(T_1 - T)$
 $(150 \times 10^{-3} + 0.025) \times (4.2 \times 10^{-3})(40 - 20)$
 $= 0.20 c_1(150 - 40)$
or $9.55 \times 10^{-3} = 22 c_1$
 $c_1 = 434 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$



Commonly Made Error

- ▶ While calculating the heat gain by calorimeter students use the specific heat of copper.



Answering Tip

- ▶ In this case water equivalent of calorimeter is given. So, the specific heat of water is to be used to find the heat gain by the calorimeter.

Q. 15. Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar specific heat (C_v) (cal mol^{-1}) K^{-1}
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K . Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

Ans. The gases listed in the given table are diatomic. Besides the translational degree of freedom, they have other degrees of freedom.

Heat must be supplied to increase the temperature of these gases. This increases the average energy of all the modes of motion. Hence, the molar specific heat of diatomic gases is more than that of monatomic gases.

If only rotational mode of motion is considered, then the molar specific heat of a diatomic

$$\text{gas} = \frac{5}{2} R$$

$$= \frac{5}{2} \times 1.98 = 4.95 \text{ cal mol}^{-1} \text{ } K^{-1}$$

With the exception of chlorine, all the observations

in the given table agree with $\left(\frac{5}{2} R\right)$

This is because at room temperature, chlorine also has vibrational modes of motion besides rotational and translational modes of motion.

Q. 16. A child running a temperature of 101°F is given an antipyryn (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 min, what is the average rate of extra evaporation caused, by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of

the child is 30 kg. The specific heat of human body is approximately the same as that of water, and latent heat of evaporation of water at that temperature is about 580 cal g⁻¹.

Ans. Initial temperature of the body of the child, $T_1 = 101^\circ\text{F}$
Final temperature of the body of the child, $T_2 = 98^\circ\text{F}$.

Change in temperature, $\Delta T = \left[(101 - 98) \times \frac{5}{9} \right]^\circ\text{C}$

Time taken to reduce the temperature, $t = 20$ min

Mass of the child, $m = 30$ kg

Specific heat of the human body = Specific heat of water = $c = 1000$ cal / kg / °C

Latent heat of evaporation of water, $L = 580$ cal g⁻¹

The heat lost by the child is given as:

$$\begin{aligned} \Delta Q &= mc\Delta T \\ &= 30 \times 1000 \times (101 - 98) \times \frac{5}{9} \\ &= 50000 \text{ cal} \end{aligned}$$

Let m_1 be the mass of the water evaporated from the child's body in 20 min.

Loss of heat through water is given by:

$$\begin{aligned} \Delta Q &= m_1 L \\ \therefore m_1 &= \frac{\Delta Q}{L} \\ &= \frac{50000}{580} = 86.2 \text{ g} \end{aligned}$$

\therefore Average rate of extra evaporation caused by the drug = $\frac{m_1}{t}$

$$= \frac{86.2}{20} = 4.3 \text{ g / min}$$

Q. 17. A 'thermocool' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h.

The outside temperature is 45 °C, and co-efficient of thermal conductivity of thermocol is 0.01 J s⁻¹m⁻¹K⁻¹. [Heat of fusion of water = 335 × 10³ J kg⁻¹]

Ans. Side of the given cubical ice box, $s = 30$ cm = 0.3 m

Thickness of the ice box, $l = 5.0$ cm = 0.05 m

Mass of ice kept in the ice box, $m = 4$ kg

Time gap, $t = 6$ h = 6 × 60 × 60 s

Outside temperature, $T = 45^\circ\text{C}$

Coefficient of thermal conductivity of thermocol, $K = 0.01$ J s⁻¹m⁻¹K⁻¹

Heat of fusion of water, $L = 335 \times 10^3$ J kg⁻¹

Let m' be the total amount of ice that melts in 6 h.

The amount of heat lost:

$$Q = \frac{KA(T_1 - T_2)t}{l}$$

where, $A =$ Surface area of the box = $6s^2$
 $= 6 \times (0.3)^2$
 $= 0.54$ m²

$$\begin{aligned} Q &= \frac{0.01 \times 0.54 \times (45) \times 6 \times 60 \times 60}{0.05} \\ &= 104976 \text{ J} \end{aligned}$$

Also, $Q = m'L$ [m' = amount of ice melt]

$$\begin{aligned} \therefore m' &= \frac{Q}{L} \\ &= \frac{104976}{335 \times 10^3} = 0.313 \text{ kg} \end{aligned}$$

Mass of ice left = $4 - 0.313 = 3.687$ kg

Hence, the amount of ice remaining after 6 h is 3.687 kg

Q. 18. A brass boiler has a base area 0.15 m² and thickness 1.0 cm. It boils water at the rate of 6.0 kg min⁻¹ when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. (Thermal conductivity of brass = 109 Js⁻¹ m⁻¹ °C⁻¹, heat of vaporisation of water = 2256 × 10³ J kg⁻¹.)

Ans. Using

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{d}$$

we get

$$\begin{aligned} \frac{Q}{t} &= \frac{109 \times 0.15}{10^{-2}} (T_1 - 100) \\ &= 1635(T_1 - 100) \end{aligned}$$

$$\begin{aligned} \frac{Q}{t} &= \frac{ML}{t} \\ &= \frac{6}{60} \times 2256 \times 10^3 \end{aligned}$$

$$\text{i.e., } 1635(T_1 - 100) = \frac{6}{60} \times 2256 \times 10^3$$

$$\begin{aligned} T_1 &= \frac{100 \times 2256}{1655} + 100 \\ &= 236.31^\circ\text{C.} \end{aligned}$$

Q. 19. Explain why :

- a body with large reflectivity is a poor emitter
- a brass tumbler feels much colder than a wooden tray on a chilly day
- an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace
- the earth without its atmosphere would be inhospitably cold
- heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

Ans. (a) A body which has large reflectivity is a poor absorber of heat. So, it is poor emitter of heat also.

- (b) When a brass tumbler or a wooden tray is touched on a chilly day heat flows from body to the brass tumbler or the wooden tray. Brass being a good conductor of heat good amount of heat from our body and we feel colder. Wood being a bad conductor of heat not much amount of heat flow from our body and we do not feel that much cold.
- (c) When the red hot iron is in furnace then the energy radiated = $E_1 = \sigma T_1^4$
When it is brought outside the energy radiated is $E_2 = \sigma (T_4 - T_0^4)$
So, the pyrometer shows low value for the temperature in the open.
- (d) Atmosphere traps the heat reaching earth from sun. In absence of atmosphere, almost the total heat reaching earth will be reflected back. So, the earth will be inhospitably cold.
- (e) Steam at 100°C contains 2.27×10^6 J amount of heat (as latent heat) per kg more than water at 100°C. So, steam is more efficient in warming than water.

Q. 20. A body cools from 80 °C to 50 °C in 5 minutes. Calculate the time it takes to cool from 60 °C to 30 °C. The temperature of the surroundings is 20 °C.

Ans. In 1st case:

$$\text{Initial temperature} = T_i = 80^\circ\text{C}$$

$$\begin{aligned} \text{Final temperature} &= T_f = 50^\circ\text{C} \\ \text{Surrounding temperature} &= T_0 = 20^\circ\text{C} \\ \text{Time} &= t = 5 \text{ minutes} \end{aligned}$$

Applying Newton's law of cooling,

$$\frac{dT}{dt} = K \left[\frac{T_i + T_f}{2} - T_0 \right]$$

$$\text{or, } \frac{(80 - 50)}{5} = K \left[\frac{80 + 50}{5} - 20 \right]$$

$$\therefore K = \frac{2}{25}$$

In 2nd case:

$$\begin{aligned} \text{Initial temperature} &= T_i = 60^\circ\text{C} \\ \text{Final temperature} &= T_f = 30^\circ\text{C} \\ \text{Surrounding temperature} &= T_0 = 20^\circ\text{C} \\ \text{Time} &= t \end{aligned}$$

Applying Newton's law of cooling,

$$\frac{dT}{dt} = K \left[\frac{T_i + T_f}{2} - T_0 \right]$$

$$\text{or, } \frac{(60 - 30)}{t} = \left(\frac{2}{15} \right)$$

$$\times \left[\frac{60 + 30}{2} - 20 \right]$$

$$\therefore t = 9 \text{ minutes}$$



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

- Q. 1. A bimetallic strip is made of aluminium and steel ($\alpha_{AL} > \alpha_{steel}$). On heating, the strip will**
- (A) remain straight
(B) get twisted
(C) will bend with aluminium on concave side.
(D) will bend with steel on concave side.

Ans. Option (D) is correct.

Explanation: $\because \alpha_{AL} > \alpha_{steel}$

\therefore Aluminium will expand more than that of steel strip. Due to it, this steel strip will bend on concave side.

- Q. 2. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly**
- (A) its speed of rotation increases.
(B) its speed of rotation decreases.
(C) its speed of rotation remains same.
(D) its speed increases because its moment of inertia increases.

Ans. Option (B) is correct.

Explanation: Its M.I increases.

Angular momentum, $L = I\omega$,

ω is angular speed which decreases to conserve L.



Commonly Made Error

- Students can not link the increase in temperature to its speed of rotation.



Answering Tip

- Student should understand that with increase in temperature, moment of inertia increases which brings about a decrease in angular velocity to conserve angular momentum.

Q. 3. The graph between two temperature scales A and B is shown in Fig. Between upper fixed point and lower fixed point there are 150 equal division on scale A and 100 on scale B. The relationship for conversion between the two scales is given by

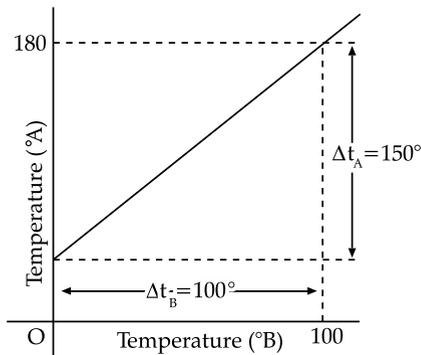
(A) $\frac{t_A - 180}{100} = \frac{t_B}{150}$

(B) $\frac{t_A - 30}{150} = \frac{t_B}{100}$

(C) $\frac{t_B - 180}{150} = \frac{t_A}{100}$

(D) $\frac{t_B - 40}{100} = \frac{t_A}{180}$

A1



Ans. Option (B) is correct.

Explanation: For the scale A,

Lower fixed point = 30°A

Upper fixed point = 180°A

For the scale B,

Lower fixed point = 0°B

Upper fixed point = 100°B

∴ The relationship between the two scales A & B is given by

$$\frac{T_A - 30}{150} = \frac{T_B - 0}{100}$$

Q. 4. An aluminium sphere is dipped into water. Which of the following is true?

- (A) Buoyancy will be less in water at 0°C than that in water at 4°C.
- (B) Buoyancy will be more in water at 0°C than that in water at 4°C.
- (C) Buoyancy in water at 0°C will be same as that in water at 4°C.
- (D) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere.

Ans. Option (A) is correct.

Explanation: Density of water at 4°C is more than that at 0°C.

Q. 5. As the temperature is increased, the time period of a pendulum

- (A) increases as its effective length increases even though its centre of mass still remains at the centre of the bob.
- (B) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob.

(C) increases as its effective length increases due to shifting of centre of mass below the centre of the bob.

(D) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob.

Ans. Option (A) is correct.

Explanation: Effective length increases with temperature. As length increases, time period increases.

Q. 6. Heat is associated with

- (A) kinetic energy of random motion of molecules.
- (B) kinetic energy of orderly motion of molecules.
- (C) total kinetic energy of random and orderly motion of molecules.
- (D) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other.

Ans. Option (A) is correct.

Q. 7. The radius of a metal sphere at room temperature T is R, and the coefficient of linear expansion of the metal is α. The sphere is heated a little by a temperature ΔT so that its new temperature is T + ΔT. The increase in the volume of the sphere is approximately

- (A) $2\pi R\alpha\Delta T$
- (B) $\pi R^2\alpha\Delta T$
- (C) $4\pi R^3\alpha\Delta T/3$
- (D) $4\pi R^3\alpha\Delta T$

Ans. Option (D) is correct.

Explanation:

$$\therefore \frac{\Delta V}{V \times \Delta T} = \gamma \quad \text{and } \gamma = 3\alpha$$

$$\therefore 3\alpha = \frac{\Delta V}{\left(\frac{4}{3}\pi R^3\right)\Delta T}$$

$$\text{or } \Delta V = 4\pi R^3\alpha\Delta T$$

Q. 8. A sphere, a cube and a thin circular plate, all of same material and same mass are initially heated to same high temperature.

- (A) Plate will cool fastest and cube the slowest
- (B) Sphere will cool fastest and cube the slowest
- (C) Plate will cool fastest and sphere the slowest
- (D) Cube will cool fastest and plate the slowest.

Ans. Option (C) is correct.

Explanation: Surface area of plate is more compared to sphere and cube.

Very Short Answer Type Questions

Q. 9. Is the bulb of a thermometer made of diathermic or adiabatic wall? [NCERT Exemplar Q. No. 13]

Ans. The bulb of a thermometer is made of diathermic because diathermic walls allow exchange of heat energy between two systems while adiabatic do not.

Q. 10. A student records the initial length l, change in temperature ΔT and change in length Δl of a rod as follows :

S. No	l(m)	ΔT (°C)	Δl (m)
1.	2	10	4×10^{-4}
2.	1	10	4×10^{-4}
3.	2	20	2×10^{-4}
4.	3	10	6×10^{-4}

If the first observation is correct, what can you say about observations 2, 3 and 4 ?

[NCERT Exemplar Q. No. 14]

Ans. $\alpha = \frac{\Delta l}{l\Delta T}$

From 1st, observation, $\alpha = \frac{4 \times 10^{-4}}{2 \times 10} = 2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$

For 2nd, observation $\Delta l = \alpha l \Delta T$
 $= 2 \times 10^{-5} \times 1 \times 10$
 $= 2 \times 10^{-4} \text{ m} \neq 4 \times 10^{-4} \text{ m}$

For 3rd, observation $\Delta l = \alpha l \Delta T$
 $= 2 \times 10^{-5} \times 2 \times 20$
 $= 8 \times 10^{-4} \text{ m} \neq 2 \times 10^{-4} \text{ m}$

For 4th, observation $\Delta l = \alpha l \Delta T$
 $= 2 \times 10^{-5} \times 3 \times 10$
 $= 6 \times 10^{-4} \text{ m}$

Therefore, 4th observation is correct and 2nd, 3rd are wrong.

Q. 11. Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.

[NCERT Exemplar Q. No. 15]

Ans. Metals have higher conductivity compared to wood. When metal bar and wooden bar are at higher temperature, then by touching with a finger, heat from the metal bar flows faster to the finger and so one feels it hotter. Similarly, when both are at lower temperature, on touching by finger, the heat from the finger flows away to the metal bar faster. So, one feels it colder.

Q. 12. Calculate the temperature which has same numeral value on celsius and Fahrenheit scale.

[NCERT Exemplar Q. No. 16]

Ans. Let T be the value of temperature which has same numeral value on Celsius and Fahrenheit scale.

Now, $\frac{^\circ\text{F} - 32}{180} = \frac{^\circ\text{C}}{100}$

Let $F = C = T$

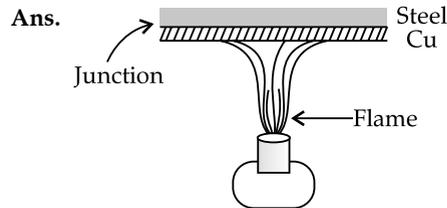
$\frac{T - 32}{180} = \frac{T}{100}$

or $T = -40^\circ\text{C} = -40^\circ\text{F}$

Q. 13. These days people use steel utensils with copper bottom. This is supposed to be good for uniform heating of food. Explain this effect using the fact that copper is the better conductor.

[AI]

[NCERT Exemplar Q. No. 17]



Since, copper has a high conductivity in comparison to steel it absorbs heat more quickly than steel and give it to the food in utensils. Therefore, food inside utensils is heated uniformly, quickly and cooked faster.

Short Answer Type Questions

Q. 14. Find out the increase in moment of inertia I of a uniform rod (coefficient of linear expansion α) about its perpendicular bisector when its temperature is slightly increased by ΔT .

[NCERT Exemplar Q. No. 18]

Ans. M.I. about its axis along perpendicular bisector

$$I = \frac{1}{12} Ml^2$$

When temperature increased by ΔT , length of rod increases.

$$\Delta l = l\alpha\Delta T$$

$$\therefore \text{New M.I.}, I_1 = \frac{M}{12} (l + \Delta l)^2 = \frac{M}{12} (l^2 + \Delta l^2 + 2l\Delta l)$$

Neglecting $(\Delta l)^2$ (very very small quantity) –

$$I_1 = \frac{M}{12} (l^2 + 2l\Delta l)$$

$$= \frac{Ml^2}{12} + \frac{Ml\Delta l}{6} = I + \frac{Ml\Delta l}{6}$$

Therefore, increase of moment of inertia

$$\Delta I = I_1 - I = \frac{Ml\Delta l}{6}$$

$$= 2 \left(\frac{Ml^2}{12} \right) \frac{\Delta l}{l}$$

$$\Delta I = 2I \alpha \Delta T.$$



Commonly Made Error

- Students can not find the increase in moment of inertia of a uniform rod.



Answering Tip

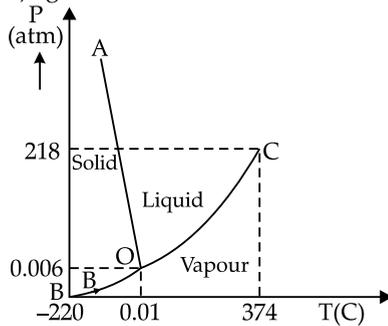
- Student should be familiarized with the concept of moment of inertia and should be able to find the moment of inertia along different axis.

Q. 15. During summers in India, one of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavoured sugar syrup and sip it. For this a stick is inserted into crushed ice and is squeezed in the palm to make it into the ball. Equivalently in winter, in those areas where it snows, people make snow balls and throw around. Explain the formation of ball out of crushed ice or snow in the light of P-T diagram of water.

[NCERT Exemplar Q. No. 19]

Ans. From the P-T graph we observe that at 0°C as pressure increases, the solid (crushed ice or

snow) is converted into liquid (water) and when pressure is released it gets converted into solid (ice) again.



When crushed ice is squeezed, some of it melts, filling up gap between ice flakes. Upon releasing pressure this water freezes, binding all ice flakes making an ice ball ball more stable.

- Q. 16.** 100 g of water is supercooled to -10°C . At this point, due to some disturbance mechanized or otherwise some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze?

[$S_w = 1 \text{ cal/g}^{\circ}\text{C}$ and $L^w_{\text{fusion}} = 80 \text{ cal/g}$]

[NCERT Exemplar Q. No. 20]

- Ans.** Given: Mass of water = 100gm
Change in temperature = $0 - (-10) = 10^{\circ}\text{C}$
Specific heat of water $S_w = 1 \text{ Cal/g}^{\circ}\text{C}$
Latent heat of fusion $L^w_{\text{fusion}} = 80 \text{ cal/g}$
Heat required by water for temperature rise from -10°C to 0°C .

$$Q = ms\Delta T$$

$$= 100 \times 1 \times 10$$

$$Q = 1000 \text{ cal}$$

Let ice melted be $m \text{ g}$,

$$Q = mL$$

or
$$m = \frac{Q}{L} = \frac{1000}{80} = 12.5 \text{ gm.}$$

- Q. 17.** One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some time, say 5–10 minutes before he could take bath. Now he had two options : (i) fill the remaining bucket completely by cold water and then attend to the work. (ii) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? [NCERT Exemplar Q. No. 21]

- Ans.** According to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and the surrounding. In the first case the temperature difference is less, so rate of loss of heat will be less. But in second case, the temperature difference being high, the loss of heat will be more. So, Ramesh should opt for the first option.

Long Answer Type Questions

- Q. 18.** We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length (both components) would change in such a way that difference between their lengths remain constant. If $\alpha_{\text{iron}} = 1.2 \times 10^{-5} / \text{K}$ and $\alpha_{\text{brass}} = 1.8 \times 10^{-5} / \text{K}$, what should we take as length of each strip?

[NCERT Exemplar Q. No. 22]

- Ans.** From question.

$$l_{\text{iron}} - l_{\text{brass}} = 10 \text{ cm} = \text{constant at all temperatures.}$$

Let l° be length at temperature 0°C and length l after change in temperature.

$$l_{\text{iron}}^{\circ} - l_{\text{brass}}^{\circ} = 10 \text{ cm at all temperature}$$

$$\therefore l_{\text{iron}}^{\circ} (1 + \alpha_{\text{iron}} \Delta t) - l_{\text{brass}}^{\circ} (1 + \alpha_{\text{brass}} \Delta t) = 10 \text{ cm}$$

$$l_{\text{iron}}^{\circ} \alpha_{\text{iron}} = l_{\text{brass}}^{\circ} \alpha_{\text{brass}}$$

$$\therefore \frac{l_{\text{iron}}^{\circ}}{l_{\text{brass}}^{\circ}} = \frac{1.8 \times 10^{-5}}{1.2 \times 10^{-5}} = \frac{1.8}{1.2} = \frac{3}{2}$$

Then,
$$l_{\text{iron}}^{\circ} - l_{\text{brass}}^{\circ} = 10$$

or
$$l_{\text{iron}}^{\circ} - \left(\frac{2}{3}\right) l_{\text{brass}}^{\circ} = 10$$

or
$$\left(\frac{1}{3}\right) l_{\text{iron}}^{\circ} = 10$$

$\therefore l_{\text{iron}}^{\circ} = \text{Length of iron rod} = 30 \text{ cm}$

and $l_{\text{brass}}^{\circ} = \text{Length of brass rod} = 20 \text{ cm}$

- Q. 19.** We would like to make a vessel whose volume does not change with temperature (take a hint from the problem above). We can use brass and iron ($\beta_{\text{brass}} = 6 \times 10^{-5} / \text{K}$ and $\beta_{\text{iron}} = 3.55 \times 10^{-5} / \text{K}$) to create a volume of 100 cc. How do you think you can achieve this?

[NCERT Exemplar Q. No. 23]

- Ans.** From question –

The difference in volume is constant.

Let V_i and V_b be the volume of iron and brass vessel at 0°C respectively, and V_i' and V_b' be the volume after $\Delta\theta$ rise of temperature.

γ_i & γ_b be the volume expansion coefficients of iron & brass respectively.

$$V_i - V_b = 100 \text{ cc} = V'_i - V'_b \quad \text{(I)}$$

$$V'_i = V_i (1 + \gamma_i \Delta\theta)$$

$$V'_b = V_b (1 + \gamma_b \Delta\theta)$$

$$V_i - V_b = (V_i - V_b) + \Delta\theta (V_i \gamma_i - V_b \gamma_b)$$

As, $V_i - V_b = \text{constant}$

$\therefore V_i \gamma_i = V_b \gamma_b$

$$\Rightarrow \frac{V_i}{V_b} = \frac{\gamma_b}{\gamma_i}$$

$$= \frac{\frac{3}{2} \beta_{\text{vbrass}}}{\frac{3}{2} \beta_{\text{viron}}} = \frac{\beta_{\text{vbrass}}}{\beta_{\text{viron}}}$$

$$= \frac{6 \times 10^{-5}}{3.55 \times 10^{-5}} = \frac{6}{3.55}$$

$$\frac{V_i}{V_b} = \frac{6}{3.55} \quad \text{(II)}$$

From eqn (s), (I) & (II)

$$V_i = 244.9 \text{ cc}$$

$$V_b = 144.9 \text{ cc}$$

Q. 20. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}/^\circ\text{C}$, bulk modulus for copper = $140 \times 10^9 \text{ N/m}^2$.
[NCERT Exemplar Q. No. 24]

Ans. Given:

Change in temperature

$$\Delta T = 57^\circ\text{C} - 37^\circ\text{C} = 20^\circ\text{C}$$

Linear expansion, $\alpha = 1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$

Cubical expansion, $\gamma = 3\alpha$

$$= 3 \times 1.7 \times 10^{-5}$$

$$= 5.1 \times 10^{-5} \text{ K}^{-1}$$

Let V be the volume of cavity, due to increase in temperature ΔT , volume increased by ΔV ,

$$\Delta V = \gamma V \Delta T$$

or $\frac{\Delta V}{V} = \gamma \Delta T$

Thermal stress produced = $B \times \text{volumetric strain}$

$$= B \times \frac{\Delta V}{V}$$

$$= B \times \gamma \Delta T$$

$$\text{Thermal stress} = 140 \times 10^9 \times 1.7 \times 10^{-5} \times 20$$

$$= 14280 \times 10^4$$

$$= 1.428 \times 10^8 \text{ N/m}^2$$



Commonly Made Error

Students can not evaluate the stress developed inside the tooth cavity.

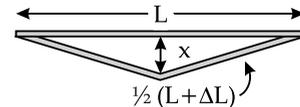


Answering Tip

Students should be familiarized with the concept of bulk modulus to calculate stress.

Q. 21. A rail track made of steel having length 10 m is clamped on a railway line at its two ends. On a summer day due to rise in temperature by 20°C , it is deformed as shown in figure.

Find x (displacement of the centre) if $\alpha_{\text{steel}} = 1.2 \times 10^{-5}/^\circ\text{C}$. [NCERT Exemplar Q. No. 25]



Ans. Given: $\alpha = 1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, $L = 10\text{m}$, $\Delta T = 20^\circ\text{C}$.

By Pythagoras theorem,

$$x^2 = \left[(L + \Delta L) \frac{1}{2} \right]^2 - \left(\frac{L}{2} \right)^2$$

$$= \frac{1}{4} [L^2 + \Delta L^2 + 2L\Delta L] - \frac{L^2}{4}$$

$$x = \frac{1}{2} \sqrt{\Delta L^2 + 2L\Delta L}$$

$\Delta L^2 \ll L$, \therefore neglecting ΔL^2

$$x = \frac{1}{2} \times \sqrt{2L\Delta L}$$

But $\Delta L = L\alpha\Delta t$

$$\therefore x = \frac{1}{2} \sqrt{2L \times L\alpha\Delta t}$$

$$= \frac{1}{2} L \sqrt{2\alpha\Delta t}$$

$$= \frac{10}{2} \times \sqrt{2 \times 1.2 \times 10^{-5} \times 20}$$

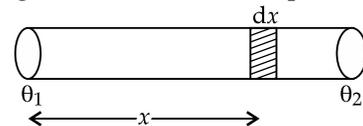
$$= 5 \times \sqrt{4 \times 1.2 \times 10^{-4}}$$

$$= 5 \times 2 \times 1.1 \times 10^{-2}$$

$$= 0.11\text{m}$$

$$x = 11\text{cm}$$

Q. 22. A thin rod having length L_0 at 0°C and coefficient of linear expansion α has its two ends maintained at temperatures θ_1 and θ_2 , respectively. Find its new length. [NCERT Exemplar Q. No. 26]



Ans.

A small length dx_0 is considered at a distance x from one end (where temperature is θ_1) of the rod.

If temperature gradient is linear, then temperature θ at this point is

$$\theta = \theta_1 + \frac{x}{L_0} (\theta_2 - \theta_1)$$

The new length be dx .

So, $dx = dx_0 (1 + \alpha\theta)$

$$\text{or } dx = dx_0 + dx_0 \alpha \left[\theta_1 + \frac{x}{L_0} (\theta_2 - \theta_1) \right]$$

Integrating,

$$L = L_0 + L_0 \alpha \theta_1 + \frac{\alpha(\theta_2 - \theta_1)}{L_0} \int_0^{L_0} x dx_0$$

$$\text{or } L = L_0 + L_0 \alpha \theta_1 + \frac{\alpha(\theta_2 - \theta_1)}{L_0} \times \frac{1}{2} L_0^2$$

$$\text{or } L = L_0 \left[1 + \alpha \theta_1 + \frac{\alpha(\theta_2 - \theta_1)}{2} \right]$$

$$\therefore L = L_0 \left[1 + \frac{\alpha(\theta_2 - \theta_1)}{2} \right]$$

Q. 23. According to Stefan's law of radiation, a black body radiates energy σT^4 from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of 10^6K and can be treated as a black body.

- (a) Estimate the power it radiates.
 (b) If surrounding has water at 30°C , how much water can 10% of the energy produced evaporate in 1 s?
 [S_w = 4186.0 J/kg K and L_v = 22.6 × 10⁵ J/kg]
 (c) If all this energy U is in the form of radiation, corresponding momentum is $P = U/c$. How much momentum per unit time does it impart on unit area at a distance of 1 km?

[NCERT Exemplar Q. No. 27]

Ans. (a) Power radiated,

$$P = \sigma AT^4 = \sigma(4\pi R^2)T^4 \\ = 5.67 \times 10^{-8} \times 4 \times 3.14 \times (0.5)^2 \times (10^6)^4$$

$$= 1.78 \times 10^{17} \text{ J/s} \\ \approx 1.8 \times 10^{17} \text{ J/s}$$

(b) Energy available per second,

$$U = 1.8 \times 10^{17} \text{ J/s} \\ = 18 \times 10^{16} \text{ J/s}$$

Energy available to evaporate water

$$= 10\% \text{ of } U = 1.8 \times 10^{16} \text{ J/s}$$

Energy required (per second) to raise temperature $30^\circ\text{C} \rightarrow 100^\circ\text{C}$ and then into vapour (at 100°C)-

$$= mS_w \Delta\theta + mL_v \\ = m \times 4186 \times (100 - 30) + m \times 22.6 \times 10^5 \\ = 2.93 \times 10^5 m + 22.6 \times 10^5 m \\ = m(25.53 \times 10^5 \text{ J/s})$$

From question,

$$m(25.53 \times 10^5) = 1.8 \times 10^{16}$$

$$\text{or } m = \frac{1.8 \times 10^{16}}{25.53 \times 10^5} \\ = 7.0 \times 10^9 \text{ kg.}$$

(c) Momentum imparted per unit time,

$$P = \frac{U}{c} \\ = \frac{18 \times 10^{16}}{3 \times 10^8} \\ = 6 \times 10^8 \text{ kg.m/s}^2$$

Momentum per unit time per unit area,

$$= \frac{P}{4\pi R^2} = \frac{6 \times 10^8}{4 \times 3.14 \times (10^3)^2} \\ = 47.7 \text{ N/m}^2$$



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any four of the following questions on the basis of the same:

I. Triple Point:

The temperature of a substance remains constant during its change of state (phase change). A graph between the temperature T and the Pressure P of the substance is called a phase diagram or P - T diagram.

The following figure shows the phase diagram of water and CO_2 . Such a phase diagram divides the P - T plane into a solid-region, the vapour-region and

the liquid-region. The regions are separated by the curves such as sublimation curve (BO), fusion curve (AO) and vaporisation curve (CO).

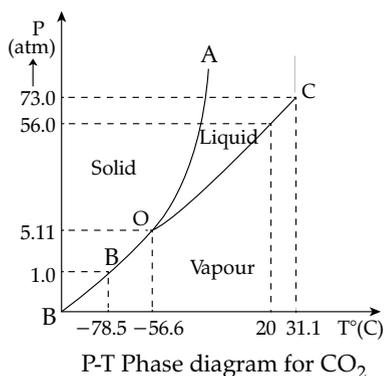
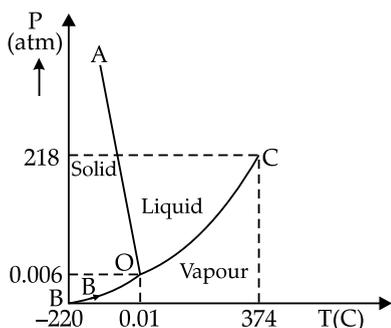
The points on sublimation curve represent states in which solid and vapour phases coexist. The point on the sublimation curve BO represent states in which the solid and vapour phases coexist.

Points on the fusion curve AO represent states in which solid and liquid phase coexist.

Points on the vaporisation curve CO represent states in which the liquid and vapour phases coexist.

The temperature and pressure at which the fusion curve, the vaporisation curve and the sublimation

curve meet and all the three phases of a substance coexist is called the triple point of the substance.



Q. 1. Sublimation curve represents the coexistence of

- (A) Solid and liquid state
- (B) Solid and vapour state
- (C) Liquid and vapour state
- (D) Liquid, solid and vapour state

Ans. Option (B) is correct.

Explanation: The points on sublimation curve represent states in which solid and vapour phases coexist. The point on the sublimation curve BO represent states in which the solid and vapour phases coexist.

Q. 2. The temperature and pressure at which all the three phases of a substance coexist is called

- (A) Boiling point
- (B) Freezing point
- (C) Triple point
- (D) Melting point

Ans. Option (C) is correct.

Explanation: The temperature and pressure at which the fusion curve, the vaporisation curve and the sublimation curve meet and all the three phases of a substance coexist is called the triple point of the substance.

Q. 3. During phase change of a substance

- (A) The temperature remains constant
- (B) The temperature changes very slowly
- (C) The temperature changes very fast
- (D) Both temperature and pressure change very slowly

Ans. Option (A) is correct.

Explanation: The temperature of a substance remains constant during its change of state (phase change).

Q. 4. The triple point of water is

- (A) 0.01°C at 0.006 atmosphere
- (B) 0.01K at 0.006 atmosphere
- (C) - 0.01K at 0.006 atmosphere
- (D) - 0.01°C at 0.006 atmosphere

Ans. Option (A) is correct.

Explanation: From graph, the triple pint of water is 0.01° C at 0.006 atmosphere pressure.

Q. 5. Triple point of CO₂ is

- (A) - 78.5° C at pressure of 1 atm.
- (B) - 56.4° C at pressure of 5.11 atm.
- (C) - 56.4 K at pressure of 5.11 atm.
- (D) - 78.5 K at pressure of 1 atm.

Ans. Option (b) is correct.

Explanation: From graph, the triple pint of CO₂ is -56.4° C and a pressure of 5.11 atm.

II. Railway track expansion joint:

Expansion and contraction of steel and concrete structure due to seasonal heating and cooling is a common problem found in civil engineering. To combat this problem, engineers put expansion joints to absorb these changes.



This problem is compounded on railway tracks. This could lead to rail buckling, known in the industry as "sun kink", as shown below, and cause the derailment of train.

When exposed to temperature variations, the rail tends to vary its length. If this tendency is freely allowed, for a temperature variance Δt , the rail length L will vary by ΔL . This length variance can be computed as:

$$\Delta L = \alpha L \Delta t$$

where α = expansion coefficient of steel
= $11.5 \times 10^{-6} / ^\circ\text{C}$.

The coefficient of thermal expansion is defined as the fractional increase in length per unit rise in temperature.

Traditional railway tracks are of standard lengths. When the tracks are laid, the lengths are joined end to end using "fishplates"—short lengths of steel plate overlapping the joint, and bolted to the ends of the rails.



At each joint there must be a short gap ($\approx 1/8''$) between the rail ends, to allow for longitudinal thermal expansion of the rails on hot days.

Q. 1. "Sun kink" is the

- (A) Buckling of railway tracks due to seasonal heating and cooling.
- (B) Fracture of railway tracks due to seasonal heating and cooling.
- (C) Buckling of railway track arising from the high pressure of loaded wagons.
- (D) None of the above.

Ans. Option (A) is correct.

Explanation: Expansion and contraction of steel and concrete structure due to seasonal heating and cooling is a common problem found in civil engineering. This problem is predominant on railway tracks. This could lead to rail buckling, known in the industry as "sun-kink", and cause the derailment of train.

Q. 2. What is the value of thermal expansion coefficient of steel?

- (A) $1.15 \times 10^{-6} / ^\circ\text{C}$.
- (B) $0.115 \times 10^{-6} / ^\circ\text{C}$
- (C) $11.5 \times 10^{-6} / ^\circ\text{C}$.
- (D) $11.5 \times 10^6 / ^\circ\text{C}$

Ans. Option (C) is correct.

Explanation: Expansion coefficient of steel
 $= 11.5 \times 10^{-6} / ^\circ\text{C}$.

Q. 3. What will be the expansion of a 20 m long railway steel track for 30°C variation of temperature?

- (A) 6.9 m
- (B) 6.9 cm
- (C) 6.9 mm
- (D) 0.69 m

Ans. Option (C) is correct.

Explanation:

$$\begin{aligned} \Delta L &= \alpha L \Delta t \\ \alpha &= 11.5 \times 10^{-6} / ^\circ\text{C}. \\ L &= 20 \text{ m} \\ \Delta t &= 30^\circ\text{C} \end{aligned}$$

Putting the values in the expression:

$$\begin{aligned} \Delta L &= 11.5 \times 10^{-6} \times 20 \times 30 \\ &= 6.9 \text{ mm} \end{aligned}$$

Q. 4. At each railway track joint a short gap of (approximately) is left.

- (A) 1/8 mm
- (B) 1/8 cm
- (C) 1/8 m
- (D) 1/8 inch

Ans. Option (D) is correct.

Explanation: At each joint of railway track there must be a short gap ($\approx 1/8''$) between the rail ends, to allow for longitudinal thermal expansion of the rails on hot days.

Q. 5. Which of the following statement is true?

- (A) Expansion joint maintain $1/8''$ gap whatever be the temperature change.
- (B) Expansion joint increases when temperature decreases and decreases when temperature increases.
- (C) Expansion joint decreases when temperature decreases and increases when temperature increases.
- (D) Expansion joints expands with the rise of temperature.

Ans. Option (B) is correct.

Explanation: Normally $1/8''$ gap is left when two railway track ends are joined with fishplate. When temperature rises, the rail tracks expand and this gap accommodates the expansion of the steel track. So, the gap contracts. When temperature decreases, the rail tracks contract and this gap increases.

B Subjective Questions

I. Read the following text and answer any four of the following questions on the basis of the same:

One day in the morning, Ramesh filled up $\frac{1}{3}$

bucket of hot water from geyser, to take bath.

Remaining $\frac{2}{3}$ was to be filled by cold water

(at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh received a phone call. He noticed that it was a call from his tutor. He realized that it would take about 10-15 minutes at least minutes before he could take bath.

Now he had two options: (i) fill the remaining bucket completely by cold water and then attend to the phone call, (ii) first attend to the phone call and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer ?

Q. 1. If the mass of $\frac{1}{3}$ bucket of water is 5kg,

temperature of hot water is 80°C , temperature of cold water is 20°C , then what will be the final

temperature when the remaining $\frac{2}{3}$ bucket is filled with cold water?

Ans. Mass of $\frac{1}{3}$ bucket of water is 5kg.

So, mass of $\frac{2}{3}$ bucket of water is 10kg.

Let T be the final temperature.

So, applying principle of calorimetry,

$$m_1s(80 - T) = m_2s(T - 2)$$

or $5 \times (80 - T) = 10 \times (T - 2)$

or $80 - T = 2T - 40$

$\therefore T = 40^\circ\text{C}$

Q. 2. Ramesh had two options :

- (i) fill the remaining bucket completely by cold water and then attend the phone.
- (ii) first attend the phone and fill the remaining bucket just before taking bath.

Which option do you think would have kept water warmer?

Ans. The first option would have kept water warmer because according to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and the surrounding. In the first case the temperature difference is less, so rate of loss of heat will be less.

Q. 3. State Newton's law of cooling.

Ans. For small temperature differences, the rate of cooling, due to conduction, convection and radiation combined, is proportional to the difference in temperatures between the body and its environment.

Q. 4. A pan filled with hot milk cools from 95°C to 85°C in 2 minutes when the room temperature is at 20°C . How long will it take to cool from 85°C to 75°C ?

Ans. In 1st case:

$$\text{Average temperature} = \frac{(95 + 85)}{2} = 90^\circ\text{C}$$

$$K\Delta T = \frac{\text{Change in temperature}}{\text{Time}}$$

or $K(90 - 20) = \frac{95 - 85}{2}$

or $K \times 70 = 5$
 $\therefore K = \frac{5}{70}$

In 2nd case:

$$\text{Average temperature} = \frac{(85 + 75)}{2} = 80^\circ\text{C}$$

$$K\Delta T = \frac{\text{Change in temperature}}{\text{Time}}$$

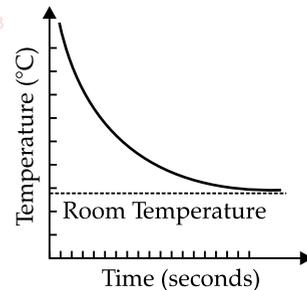
or $K(80 - 20) = \frac{85 - 75}{t}$

or $\left(\frac{5}{70}\right) \times 60 = \frac{10}{t}$

$\therefore t = \frac{7}{3} \text{ m}$

Q. 5. If Ramesh opted the option (ii) how the temperature of hot water would come down to room temperature according to Newton's law of cooling?

Ans.



UNIT-VIII THERMODYNAMICS

Study Time
Maximum Time: 2 : 30 hours
Maximum Questions: 54

CHAPTER

11

THERMODYNAMICS



Syllabus

Thermal equilibrium and definition of temperature (zeroth law of thermodynamics), heat, work and internal energy, first law of thermodynamics, isothermal and adiabatic processes; Second law of thermodynamics: gaseous state of matter, change of condition of gaseous state - isothermal, adiabatic, reversible, irreversible and cyclic processes.



Revision Notes

- **Thermodynamics:** Thermodynamics is the branch of physics that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy. Thermodynamics is a macroscopic science. It deals with bulk systems and does not go into the molecular constitution of matter.
- **Thermal equilibrium:** A system is in equilibrium if the macroscopic variables (pressure, volume, temperature, mass and composition) those characterise the system do not change with time.
- **Adiabatic wall:** Adiabatic wall is an insulating wall that does not allow flow of energy (heat) from one to another.
- **Diathermic wall:** Diathermic wall is a conducting wall that allows flow of energy (heat) from one to another.
- **Zeroth Law of Thermodynamics:** Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other.
- **Internal energy:** Every bulk system consists of a large number of molecules. Internal energy is the sum of the kinetic energies and potential energies of these molecules.
- **Equivalence of work and heat:** Work is a form of heat energy. 4.18×10^3 Joule of work is equivalent to 1 kilocalorie of heat.

$$W = JQ$$

J is the mechanical equivalent of heat.

- **Sign convention of heat:**
 - (i) Negative when heat is given from a system to its surroundings.
 - (ii) Positive when heat is taken from the surroundings by the system.
- **First law of thermodynamics:** If an amount of heat ΔQ is given to a system, a part of it increases the internal energy ΔU of the system and the rest is utilized in doing work ΔW by the system.

$$\Delta Q = \Delta U + \Delta W$$

- **In cyclic process:** In cyclic process, a system is taken from one initial state to other different states and finally brought back to its initial state.

So, there is no change in internal energy i.e.

$$\Delta U = 0$$

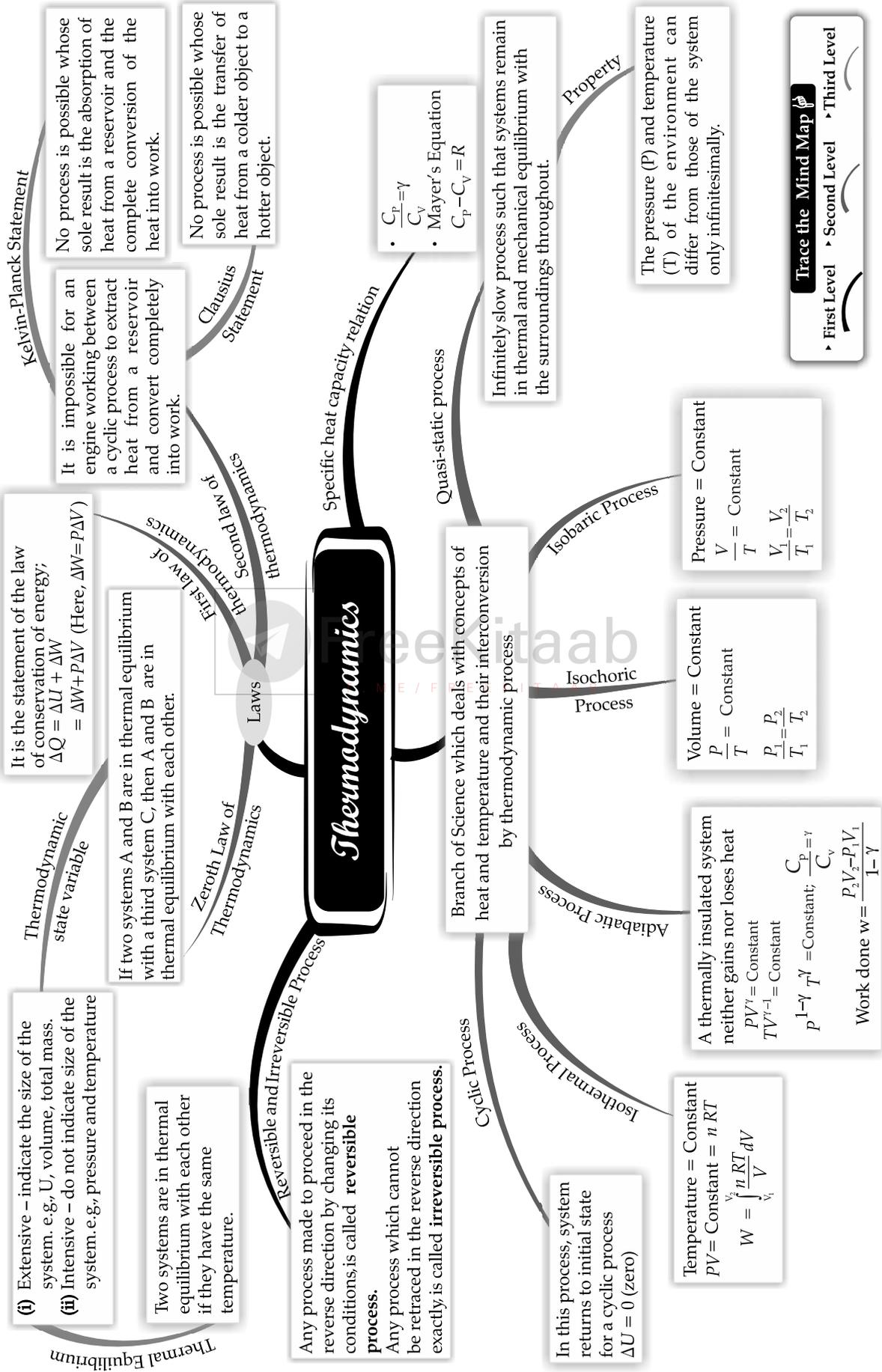
So,

$$\Delta Q = \Delta W$$

Scan to know more about this topic



Thermal expansion



- **In isobaric process:** In isobaric process the pressure remains constant.

$$\text{Work done} = \Delta W = P\Delta V$$

So,
$$\Delta Q = \Delta U + P\Delta V$$

- **In isochoric process:** In isochoric process the volume remains constant.

So,
$$\text{work done} = \Delta W = P\Delta V = 0$$

So,
$$\Delta Q = \Delta U$$

- **In isothermal process:** In isothermal process the temperature remains constant.

$$\Delta Q = \Delta U + \Delta W$$

(For ideal gas, the internal energy depends on temperature only. As temperature is constant, then $\Delta U = 0$.)

So,
$$\Delta Q = \Delta W$$

- **In adiabatic process:** In adiabatic process heat neither enters nor leaves the system.

Hence,
$$\Delta Q = 0$$

So,
$$\Delta U = -\Delta W$$

➤ **Thermodynamic processes :**

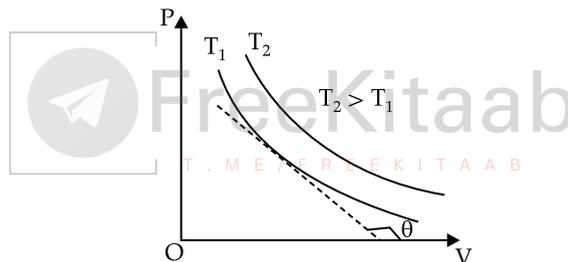
- (a) **Isothermal process**, where the temperature remains constant. The pressure and volume of a given mass of gas changes.

Essential conditions :

(i) Walls of container must be perfectly conducting.

(ii) Changes must be slow.

Isothermal process obeys Boyle's law i.e., $PV = \text{constant}$.



Variation of P with V at constant temperature is represented by Isothermal curves.

Slope of an isothermal graph is negative.

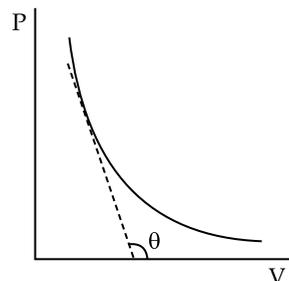
- (b) **Adiabatic process**, where the heat content of a gaseous system remains constant. The pressure and volume of given mass of gas change with consequent change in temperature.

Essential conditions are :

(i) Walls of container must be perfectly insulating.

(ii) Changes must be sudden.

Adiabatic process obeys Poission's law i.e., $PV^\gamma = \text{constant}$.



The variation of P with V at constant heat content is represented by an Adiabatic curve.

$$\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}} = \gamma$$

Since, γ is always greater than 1, adiabatic curve is steeper than isothermal curve.

(c) **Isobaric process**, where pressure is kept constant.

Since, temperature changes, so does internal energy. The heat absorbed goes partly to increase internal energy and partly to do work. The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant pressure.

(d) **Isochoric process**, where volume is kept constant.

In an isochoric process, V is constant. No work is done on or by the gas. The heat absorbed by the gas goes entirely to change its internal energy and its temperature. The change in temperature for a given amount of heat is determined by the specific heat of the gas at constant volume.

➤ **Work done in isothermal and adiabatic process:**

Area under the curve is the measurement of work done.

- For same expansion,
Work done by isothermal process > work done in adiabatic process.
- For same compression:
Work done by adiabatic process > work done in isothermal process.

➤ **2nd law of thermodynamics:**

There are two statements:

- **Kelvin-Planck statement:** No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.
 - **Clausius statement:** No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.
- **Reversible process:** The process which can be reversed in such a way that all changes taking place in the direct process are exactly repeated in reverse order and opposite sense is called reversible process.
- **Irreversible process:** The process which cannot be reversed is called irreversible process.



Key Words

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➤ **Thermodynamical system & thermodynamical parameters.**

A gaseous system is called a thermodynamical system. The state of the system is represented in terms of pressure (P), volume (V), temperature (T) and heat content (Q) of the gas. These four quantities are called thermodynamical parameters of the system.

- **Open system :** System which exchanges both energy & matter with surroundings.
- **Closed system :** System which exchanges only energy with surroundings.
- **Isolated system :** System which Exchanges neither energy nor matter with surroundings.
- **Equation of state** is the equation connecting pressure, volume and temperature of the gas.
- **Isothermal process**, where the temperature remains constant.
- **Adiabatic process**, where the heat content of a gaseous system remains constant.
- **Isobaric process**, where pressure is kept constant.
- **Isochoric process**, where volume is kept constant.
- **Reversible process** is a process which can be reversed back to initial state.
- **Irreversible process** is a process which cannot be traced back in opposite direction.



Key Formulae

➤ **Equation of state for:**

- | | | |
|-----|---------------------|----------------------------------|
| (a) | Isothermal process: | $PV = \text{Constant.}$ |
| (b) | Isobaric process: | $\frac{V}{T} = \text{Constant.}$ |
| (c) | Isochoric process: | $\frac{P}{T} = \text{Constant.}$ |

(d) Adiabatic process: $PV^\gamma = \text{constant}$;
 $TV^{\gamma-1} = \text{constant}$ and
 $\frac{P^{\gamma-1}}{T^\gamma} = \text{constant}.$

➤ **Work done during expansion of gas:** $dW = PdV$ (for constant pressure)
 or $W = \int_{V_1}^{V_2} PdV$ (for variable pressure)

(a) In an isothermal process :

$$W = 2.3026 RT \log_{10} \left(\frac{V_2}{V_1} \right)$$

$$W = 2.3026 RT \log_{10} \left(\frac{P_1}{P_2} \right)$$

(b) In an adiabatic process :

$$W = \frac{R}{1-\gamma} (T_2 - T_1)$$

$$= \frac{1}{1-\gamma} (P_2V_2 - P_1V_1)$$



Mnemonics

- **Concept:** Four thermodynamic processes.
 - **Mnemonics:** Today **I**ncome **T**ax was **p**aid in **I**ndian **B**ank by **V**ijoy in **I**ndian **C**urrency, **a**djacent to **N**ational **h**ighway.
- Interpretation:**
- | | | |
|-----------------------------|---|---|
| T → temperature is constant | } | Temperature remains constant in isothermal process. |
| I → Iso | | |
| T → Thermal | } | Pressure remains constant in isobaric process. |
| P → pressure | | |
| I → Iso | | |
| B → baric | } | Volume remain constant in isochoric process. |
| V → volume | | |
| I → Iso | } | No heat transfer between system and surrounding takes place in adiabatic process. |
| C → Choric | | |
| Ad → adiabatic | | |
| N → No | } | |
| h → heat transfer | | |



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is an isothermal process ? [R] [NCT 2010]

Ans. Isothermal process is the process in which temperature variation does not exist. Such processes are to be carried in (i) conducting cylinders, (ii) at a slow pace. 1

Q. 2. When two bodies are said to be in thermal equilibrium ? [U]

Ans. When the two bodies are at the same temperature. 1

Q. 3. What is the change in internal energy of a system over one complete cycle of a cyclic process ? [U]

Ans. Zero. It is because when a system is moved over a complete cycle, it returns to its original state. 1



Commonly Made Error

- Students can not estimate the change in internal energy in a cyclic process.



Answering Tip

- Students can be familiarized with state and path functions.

Q. 4. What is the relation between heat energy, work done and change in internal energy ? [U]

Ans. The relation is :

$$dQ = dU + dW \quad 1$$

Q. 5. What is a diathermic material ? [R]

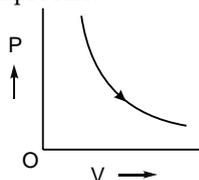
Ans. A material which allows the flow of heat is called diathermic material. 1

Short Answer Type Questions-I

(2 marks each)

Q. 1. What is an 'Indicator diagram' ? What is its importance ? [R]

Ans. Indicator diagram is the graphical representation of a thermodynamic system using two thermodynamic variables, e.g., P and V diagram. Area under PV diagram gives work done in the thermodynamic process. 1



Q. 2. On removing the valve, the air escaping from a cycle tube becomes cool. Why ? [A]

Ans. Tube is a thermal insulator. The removal of valve makes the pressurised air inside the tube come out of the tube suddenly. 1

There are the conditions suitable for adiabatic change which ensure decrease in temperature during expansion. Thus the air escaping from the cycle tube becomes cool. 1

Q. 3. Is it possible to increase the temperature of a gas without adding heat to it ? If yes, then explain how ? [U][A][I]

Ans. For an adiabatic change, first law of thermodynamics may be expressed as :

$$dU + PdV = 0$$

or $dU = -PdV$ 1

The temperature of a gas will increase, if dU is

positive. For this, dV has to be negative. Therefore temperature of a gas can be increased without adding heat, if it is compressed adiabatically. 1

Q. 4. By applying the first law of thermodynamics to isobaric process, obtain relation between two specific heats of a gas. [A]

Ans. In an isobaric process, pressure remains constant. If an amount of heat dQ is supplied to one mole of a gas at constant pressure and its temperature increases by dT , then ½

$$dQ = C_p dT$$

Here, C_p is molar specific heat of the gas at constant pressure. Therefore, for an isobaric process, the first law of thermodynamics becomes :

$$C_p dt = dU + PdV \quad \dots(i) \quad \frac{1}{2}$$

From perfect gas equation it follows that

$$PdV = RdT \quad \frac{1}{2}$$

In the eqn. (i), substituting PdV and dU , we have

$$C_p dT = C_v dT + RdT$$

$$C_p = C_v + R. \quad \frac{1}{2}$$

Q. 5. State second law of thermodynamics. [R][NCT 2008, MSE Chandigarh, 2009]

Ans. Kelvin's statement : No process is possible whose sole result is the absorption of heat from a reservoir and complete conversion of the heat into work 1

Clausius statement : It is not possible to transfer heat from a body at lower temperature to another at higher temperature without the help of some external energy. 1

Long Answer Type Questions

(5 marks each)

Q. 1. Derive an expression for work done in isothermal process. [U][KVS 2013; NCT 2008]

Ans. Suppose 1 g mole of an ideal gas enclosed in a cylinder of conducting walls. Let P_1, V_1, T be initial pressure, volume, and temperature. Let gas expand to volume V_2 where pressure reduces to P_2 and temperature remains constant. 1
If A is the area of piston

$$F = P \times A \quad 1$$

$$dW = F \times dx \quad 1$$

$$= P \times A \times dx \quad 1$$

$$W = \int_{V_1}^{V_2} PdV \quad [\because Adx = dV]$$

But, $PV = RT$

$$W = \int_{V_1}^{V_2} \frac{RT}{V} dV \quad 1$$

$$W = RT [\log_e V]_{V_1}^{V_2}$$

$$W = RT [\log_e V_2 - \log_e V_1]$$

$$W = 2.303 RT \log_{10} \frac{V_2}{V_1} \quad 1$$



Commonly Made Error

► Students can not find the expression for work done in an isothermal process.



Answering Tip

► Step by step derivation for finding the work done in each type of processes should be understood.

Q. 2. Derive the equation of adiabatic changes. □

Ans. For 1 mole ideal gas,

$$PV = RT$$

Differentiating,

$$PdV + VdP = RdT$$

or $PdV + VdP = (C_p - C_v)dT$

or $C_v dT + PdV = C_p dT - VdP$... (1) 1

From the 1st equation of thermodynamics,

$$dQ = dU + dW$$

Putting $dU = C_v dT$ and $dW = PdV$

$$dQ = C_v dT + PdV = C_p dT - VdP$$

(from equation 1)(2) 1

In adiabatic process, $dQ = 0$

∴ $0 = C_p dT - VdP$

(Putting in equation 2)

∴ $C_p dT = VdP$... (3)

Also $C_v dT + PdV = 0$

(Putting in equation 2)

∴ $C_v dT = -PdV$... (4) 1

Dividing equation 3 by equation 4,

$$\frac{C_p}{C_v} = - \frac{V dP}{P dV}$$

or $\gamma = - \frac{V dP}{P dV}$ FreeKItaab

or $\frac{dP}{P} + \gamma \frac{dV}{V} = 0$ 1

Integrating,

ln $P + \gamma \ln V = \ln c$
(ln c is an integration constant)

or, ln $PV^\gamma = \ln c = \text{constant}$

∴ $PV^\gamma = \text{constant}$ 1

Q. 3. Derive an expression for work done in adiabatic expansion. □

Ans. A cylinder thermally insulated from surroundings contains ideal gas which is compressed (adiabatic compression) or expanded (adiabatic expansion).

If A is the area of piston then force is given by

$$F = P \times A \quad \frac{1}{2}$$

where P = pressure of the gas

$$dW = F \times dx$$

$$= P \times Adx$$

$$dW = P \times dV \quad 1$$

$$dV = Adx$$

For small increase in volume of the gas,

total work done by the gas in adiabatic expansion from volume V_1 to V_2

$$W = \int_{V_1}^{V_2} PdV \quad 1$$

The equation of adiabatic change is

$$PV^\gamma = K, \text{ a constant}$$

$$\gamma = C_p/C_v$$

$$W = \int_{V_1}^{V_2} KV^{-\gamma} dV \quad \frac{1}{2}$$

$$= K \left[\frac{V^{1-\gamma}}{1-\gamma} \right]_{V_1}^{V_2}$$

$$W = \frac{K}{1-\gamma} [V_2^{1-\gamma} - V_1^{1-\gamma}]$$

$$W = \frac{1}{1-\gamma} [KV_2^{1-\gamma} - KV_1^{1-\gamma}] \quad 1$$

Equation of adiabatic change

$$P_2 V_2^\gamma = P_1 V_1^\gamma = K$$

∴ $W = \frac{1}{1-\gamma} [P_2 V_2^\gamma V_2^{1-\gamma} - P_1 V_1^\gamma V_1^{1-\gamma}]$

$$W = \frac{1}{1-\gamma} [P_2 V_2 - P_1 V_1]$$

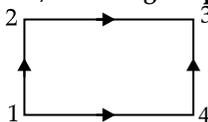
$$W = \frac{1}{1-\gamma} [T_2 - T_1] \quad 1$$

Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. When a system is taken from state 1 to state 3 along the path 1 → 2 → 3, it is found that $\Delta Q = 60$ cal and $\Delta W = 10$ cal. Along the path 1 → 4 → 3, $\Delta Q = 50$ cal, ΔW along the path 1 → 4 → 3 is



- (A) 10 cal (B) 40 cal
(C) 0 cal (D) 60 cal

Ans. Option (A) is correct.

Explanation: From 1st law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Along 1 → 2 → 3 path,

$$60 = \Delta U + 10$$

∴ $\Delta U = 40$ cal

Along 1 → 4 → 3 path

$$50 = 40 + \Delta W$$

∴ $\Delta W = 10$ cal

Q. 2. A monoatomic gas at a pressure P_1 , having a volume V_1 expands isothermally to volume $2V_1$ and then adiabatically to volume $16V_1$. The final pressure of the gas is

(assume $\gamma = \frac{5}{3}$)

- (A) $\frac{P_1}{64}$ (B) $64P_1$
 (C) $32P_1$ (D) $\frac{P_1}{32}$

Ans. Option (A) is correct

Explanation: When expanding isothermally,

$$P_1V_1 = P_2V_2$$

or $P_1V_1 = P_2 \times 2V_1$

or $P_1 = 2P_2$

When expanding adiabatically

$$(P_2V_2)^\gamma = (P_3V_3)^\gamma$$

or $\left(\frac{P_1 \times 2V_1}{2}\right)^{5/3} = (P_3 \times 16V_1)^{5/3}$

$\therefore P_3 = \frac{P_1}{64}$

Q. 3. An ideal gas is compressed to half of its initial volume by means of isobaric, isochoric, isothermal and adiabatic process separately. Work done is maximum in

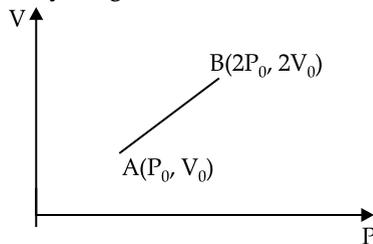
- (A) Adiabatic process
 (B) Isobaric process
 (C) Isochoric process
 (D) Isothermal process

Ans. Option (A) is correct.

Explanation: $W_{\text{isochoric}} = 0$

$$W_{\text{adiabatic}} > W_{\text{isothermal}} > W_{\text{isobaric}}$$

Q. 4. 2 mole monoatomic ideal gas is taken from state A to state B as shown in the figure. State A and B are characterized by (P_0, V_0) and $(2P_0, 2V_0)$. Heat absorbed by the gas is



- (A) $3P_0V_0$ (B) $6P_0V_0$
 (C) $9P_0V_0$ (D) $4P_0V_0$

Ans. Option (B) is correct.

Explanation: Let T_1 and T_2 be the temperature at A and B.

\therefore At A, $P_0V_0 = nRT_1$

At B, $4P_0V_0 = nRT_2$

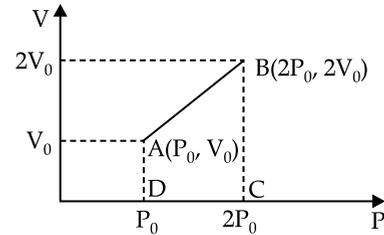
$\therefore T_2 - T_1 = \frac{4P_0V_0}{nR} - \frac{P_0V_0}{nR} = \frac{3P_0V_0}{nR}$

Increase in internal energy = $\Delta U = nC_V(T_2 - T_1)$

or $\Delta U = n \times \left(\frac{3R}{2}\right) \times \frac{3P_0V_0}{nR}$

$\therefore \Delta U = \frac{9P_0V_0}{2}$

Work done = $\Delta W = \text{Area of } ABCD$



or $\Delta W = \frac{1}{2}(2V_0 + V_0)(2P_0 - P_0)$

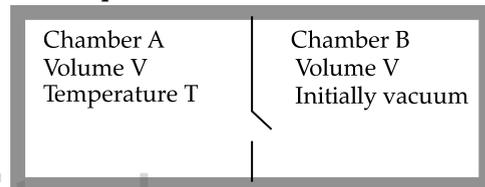
$\therefore \Delta W = \frac{3P_0V_0}{2}$

Heat absorbed = $\Delta Q = \Delta U + \Delta W$

Or $\Delta Q = \frac{9P_0V_0}{2} + \frac{3P_0V_0}{2}$

$\therefore \Delta Q = 6P_0V_0$

Q. 5. An ideal gas is in chamber A of volume V and absolute temperature T. It is allowed to rush freely into chamber B of volume V which was initially vacuum. The whole system is thermally isolated. The final temperature of the system after equilibrium is



- (A) $\frac{T}{4}$ (B) $\frac{T}{2}$
 (C) T (D) 2T

Ans. Option (C) is correct.

Explanation: For free expansion,

$$\Delta U = 0$$

\therefore Initial temperature = Final temperature = T

B Assertion Reason Type Questions

Directions: These questions consist of two statements - Assertion and Reason.

Choose any one of the following four responses.

- (A) If both Assertion and Reason are true and the Reason is a correct explanation of the Assertion.
 (B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.
 (C) If the Assertion is true but Reason is false.
 (D) If both the Assertion and Reason are false.

Q. 1. **Assertion:** If systems A and B are separately in equilibrium with system C, then the systems A and B may not be in thermal equilibrium.

Reason: First Law of Thermodynamics states that 'two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other'.

Ans. Option (D) is correct.

Explanation: Zeroth Law of Thermodynamics states that 'two systems in thermal equilibrium with a third system separately are in thermal

equilibrium with each other'. So, accordingly if systems A and B are separately in equilibrium with system C, then the systems A and B will also be in thermal equilibrium.

Hence, the assertion and reason both are false.

- Q. 2. Assertion:** When a box containing gas is moving as a whole with some velocity, the kinetic energy of the box is not to be included in the internal energy of the gas.

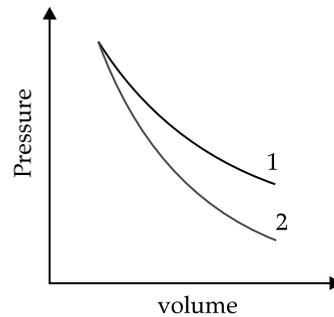
Reason: Internal energy includes only the (disordered) energy associated with the random motion of molecules of the system.

Ans. Option (A) is correct.

Explanation: Internal energy of a gas is the sum of molecular kinetic and potential energies in the frame of reference relative to which the centre of mass of the system is at rest. Thus, it includes only the (disordered) energy associated with the random motion of molecules of the system. We denote the internal energy of a system by U .

Hence, the assertion and reason both are true and the reason explains the assertion.

- Q. 3. Assertion:** In the following P-V diagram, plot 1 is for isothermal process and plot 2 is for adiabatic process.



Reason: γ is always greater than 1.

Ans. Option (B) is correct.

Explanation:
$$\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}} = \gamma$$

Since, γ is always greater than 1, adiabatic curve is steeper than isothermal curve. Curve 2 is steeper than curve 1. Hence, curve 2 corresponds to adiabatic process and curve 1 corresponds to isothermal process.

So, assertion and reason both are true but the reason does not explain the assertion.



NCERT Corner

- Q. 1.** A geyser heats water flowing at the rate of 3 litres per minute from 27°C to 77°C . If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heating of combustion is $4.0 \times 10^4 \text{ J/g}$?

Ans. The mass of water flowing per unit time
 $= 3000 \text{ g per min.}$
 $= \frac{3000}{60} = 50 \text{ g/s}$
 $= \frac{50}{1000} \text{ kg/s}$

Heat required to heat this water from 27°C to 77°C
 $= mc \Delta T$
 $= \frac{50}{1000} \times 4.2 \times 10^3 \times 50 \text{ J/s}$
 $= 1.05 \times 10^4 \text{ J/s}$

Heat generated by the combustion of 1 g of fuel
 $= 4.0 \times 10^4 \text{ J}$

\therefore Fuel required per second

$$= \frac{1.05 \times 10^4}{4.0 \times 10^4}$$

$$= 0.2625 \text{ g}$$

Fuel required to be burnt per min.
 $= 0.2625 \times 60$
 $= 15.75 \text{ g}$

- Q. 2.** What amount of heat must be supplied to $2.0 \times 10^{-2} \text{ kg}$ of nitrogen (at room temperature) to raise its temperature by 45°C at constant pressure? (Molecular mass of $\text{N}_2 = 28$, $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$)

Ans. Mass of nitrogen, $m = 2 \times 10^{-2} \text{ kg} = 20 \text{ g}$
 Rise in temperature, $\Delta T = 45^\circ\text{C}$
 Molecular mass of N_2 , $M = 28$
 Universal gas constant, $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$
 Number of moles, $n = m/M$
 $= (2 \times 10^{-2} \times 10^3) / 28$
 $= 0.714$

Molar specific heat at constant pressure for nitrogen, $C_p = (7/2)R$
 $= (7/2) \times 8.3$
 $= 29.05 \text{ J mol}^{-1} \text{ K}^{-1}$

The total amount of heat to be supplied is given by the relation:

$$\Delta Q = nC_p \Delta T$$

$$= 0.714 \times 29.05 \times 45$$

$$= 933.38 \text{ J}$$

Therefore, the amount of heat to be supplied is 933.38 J

Q. 3. Explain why

- Two bodies at different temperatures T_1 and T_2 , if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2) / 2$?
- The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat. Comment
- Air pressure in a car tyre increases during driving. Why?
- The climate of a harbour town is more temperate than that of a town in a desert at the same latitude. Why?

- Ans.** (a) When two bodies at different temperatures T_1 and T_2 are brought in thermal contact, heat flows from the body at higher temperature to the body at lower temperature till equilibrium is achieved, i.e., the temperatures of both the bodies becomes equal. The equilibrium temperature becomes equal to the mean temperature $(T_1 + T_2) / 2$ only when thermal capacities of the two bodies are equal.
- (b) The coolant in a chemical or nuclear plant should have a high specific heat. This is because higher the specific heat of the coolant, higher is its heat-absorbing capacity and vice-versa. So, it absorbs more heat for 1 degree rise of temperature. Hence, cooling is better.
- (c) When the car is in motion, the temperature of air inside the tyre increases due to motion of the air molecules. According to Charles's law, pressure is directly proportional to the temperature, $P \propto T$. Hence, if the temperature inside a tyre increases, then the air pressure inside the tyre will also increase.
- (d) A harbour town has a more temperate climate than a town located in a desert at the same latitude. This is because in a harbour town, the relative humidity is more than in a desert town.

Q. 4. A cylinder with a movable piston contains 3 moles of hydrogen at constant temperature and pressure. The walls of a cylinder are made up of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of a gas increases if the gas is compressed to half its original volume ?

Ans. Since, the process is adiabatic

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma$$

But
$$\frac{V_1}{V_2} = \frac{2}{1}$$

$$\gamma = 7/5 \text{ for hydrogen}$$

\therefore Factor by which the pressure of the gas increases

or
$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\frac{P_2}{P_1} = \left(\frac{2}{1}\right)^{7/5} = (2)^{7/5}$$

so,
$$P_2 = 2^{7/5} P_1 = 2.64 P_1$$

Q. 5. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state B to A via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the later case ? (Take 1 cal = 4.19 J)

Ans. Since, the system is going from A to B and then back to A, it is undergoing a cyclic change. Now in cyclic change there is no change in internal energy ($\Delta U = 0$). From first law of thermodynamic.

$$\Delta Q = \Delta U + \Delta W$$

For first case, $\Delta U = 22.3 \text{ J}$

$$[\therefore \Delta Q = 0, \Delta W = -22.3 \text{ J}]$$

In the second case, $\Delta Q = 9.35 \text{ cal}$

$$= 9.35 \times 4.19 = 39.18 \text{ J}$$

$$\Delta W = ?$$

As, $\Delta U + \Delta W = \Delta Q$

$$\Delta W = \Delta Q - \Delta U$$

$$= 39.18 - 22.3$$

$$= 16.9 \text{ J}$$

Q. 6. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :

- What is the final pressure of the gas in A and B ?
- What is change in the internal energy of the gas ?
- What is the change in the temperature of the gas ?
- Do the intermediate states of the system (before setting to the final equilibrium state) lie on its P-V-T surface ?

Ans. (a) The gas would rush from A to B. Now the volume becomes double.

So, the pressure becomes half. Earlier pressure was 1 atmosphere. So, the present pressure will be 0.5 atmosphere.

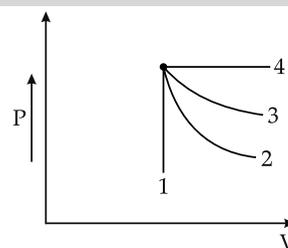
- The change in internal energy of the gas will be zero. (if gas is ideal)
- The change in temperature will be zero.
- Since the process is rapid, the intermediate states are not equilibrium states and hence do not satisfy the gas equation. So, the intermediate states of the system do not lie on the P-V-T surface.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

Q. 1. An ideal gas undergoes four different processes from same initial state. Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1,2,3 and 4 which one is adiabatic.



- (A) 4 (B) 3
(C) 2 (D) 1

Ans. Option (C) is correct.

Q. 2. If an average person jogs, he produces 14.5×10^3 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires 580×10^3 cal for evaporation) is

- (A) 0.25 kg (B) 2.25 kg
(C) 0.05 kg (D) 0.20 kg

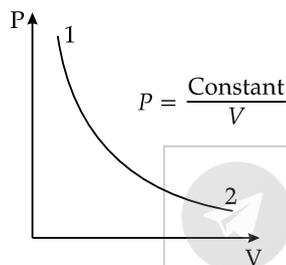
Ans. Option (A) is correct.

Explanation: Amount of sweat evaporated per min

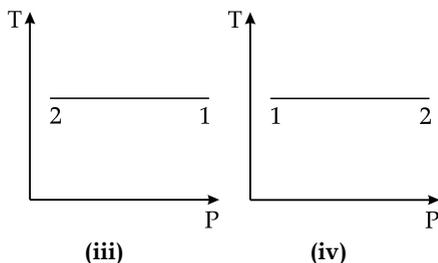
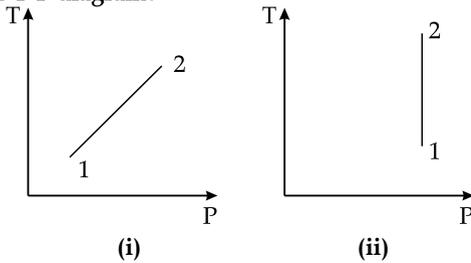
$$= \frac{\text{calories produced per minute}}{\text{no. of calories required for evaporation per kg}}$$

$$= \frac{14.5 \times 10^4}{580 \times 10^3} = 0.25 \text{ kg}$$

Q. 3. Consider P-V diagram for an ideal gas shown in figure



Out of the following diagrams, which represents the T-P diagram?



- (A) (iv) (B) (ii)
(C) (iii) (D) (i)

Ans. Option (C) is correct.

Explanation: Temperature remains constant. Pressure decreases from state 2 to state 1.



Commonly Made Error

Students can not find the correct T-P curve.

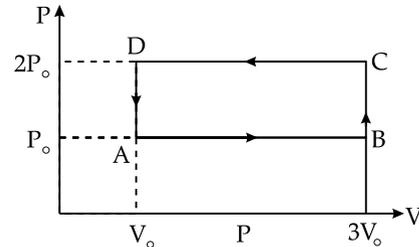


Answering Tip

In order to find the curve, firstly the type of process (adiabatic, isothermal, isochoric, isobaric) should be determined. Moreover, initial and final states should be carefully looked into.

Q. 4. An ideal gas undergoes cyclic process ABCDA as shown in given P-V diagram.

The amount of the work done by the gas is



- (A) $6P_0V_0$ (B) $-2P_0V_0$
(C) $+2P_0V_0$ (D) $+4P_0V_0$

Ans. Option (B) is correct.

Explanation: Work done = $\Delta P \times \Delta V$
 $= (2P_0 - P_0)(3V_0 - V_0) = 2P_0V_0$

\therefore Cyclic process is anticlockwise, work done by the gas is negative.



Commonly Made Error

Students can not find the correct value of work done using PV diagram.



Answering Tip

Student should be familiarized with the fact that the work done is given by the area under the PV curve. Moreover, in a cyclic process, work done is negative for anticlockwise cycle and positive for clockwise cycle.

Q. 5. Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isothermally while the gas in container B is compressed to half of its original value adiabatically. The ratio of final pressure of gas in B to that of gas in A is

- (A) $2^{\gamma-1}$ (B) $\left(\frac{1}{2}\right)^{\gamma-1}$
(C) $\left(\frac{1}{1-\gamma}\right)^2$ (D) $\left(\frac{1}{\gamma-1}\right)^2$

Ans. Option (A) is correct.

Explanation:

For cylinder A $P_1V_1 = P_2V_2$

$$P_2 = \frac{P_1V_1}{V_2} = P_1 \frac{V_1}{V_2} = 2P_1$$

Gas in container B is compresses adiabatically

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P_o(2V_o)^\gamma = P_2(V_o)^\gamma$$

$$P_2 = P_o \left(2 \frac{V_o}{V_o} \right)^\gamma = P_o(2)^\gamma$$

Hence, ratio of final pressure

$$= \frac{(P_2)_B}{(P_2)_A} = \frac{P_o(2)^\gamma}{P_o} = 2^{\gamma-1}$$

- Q. 6.** Three copper blocks of masses M_1, M_2 and M_3 kg respectively are brought into thermal contact till they reach equilibrium. Before contact, they were at T_1, T_2, T_3 ($T_1 > T_2 > T_3$). Assuming there is no heat loss to the surroundings, the

equilibrium temperature T is (s is specific heat of copper)

- (A) $T = \frac{T_1 + T_2 + T_3}{3}$
 (B) $T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$
 (C) $T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{3(M_1 + M_2 + M_3)}$
 (D) $T = \frac{M_1T_1s + M_2T_2s + M_3T_3s}{M_1 + M_2 + M_3}$

Ans. Option (B) is correct.

Explanation: Copper block with mass M_1 will lose heat and copper block with mass M_2 and M_3 will gain heat.

If T is the final temperature, then

$$M_1s(T_1 - T) = M_2s(T_2 - T) + M_3s(T_3 - T)$$

$$\text{or } T(M_1 + M_2 + M_3) = M_1T_1 + M_2T_2 + M_3T_3$$

$$\therefore T = (M_1T_1 + M_2T_2 + M_3T_3) / (M_1 + M_2 + M_3)$$

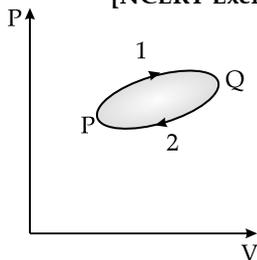
Very Short Answer Type Questions

- Q. 7.** Can a system be heated and its temperature remains constant?[NCERT Exemplar Q. No. 12]

Ans. Yes it is possible. If the system utilises the total heat supplied to do work then the temperature remains constant. The process is isothermal.

- Q. 8.** A system goes from P to Q by two different paths in the P-V diagram as shown in Fig. Heat given to the system in path 1 is 1000 J. The work done by the system along path 1 is more than path 2 by 100 J. What is the heat exchanged by the system in path 2?

[NCERT Exemplar Q. No. 13]



Ans. For path 2,

$$\Delta Q = \Delta U + \Delta W \quad (i)$$

For path 1,

$$1000 = \Delta U + (\Delta W + 100)$$

$$\text{or } \Delta U + \Delta W = 900$$

$$\text{or } \Delta Q = 900J \quad (\text{from (i)})$$



Commonly Made Error

- Students can not find the heat exchanged by the system.



Answering Tip

- Student should be familiar with the first law of thermodynamics and its application in detail.

- Q. 9.** Is it possible to increase the temperature of a gas without adding heat to it? Explain.

[NCERT Exemplar Q. No. 15]

Ans. Yes. When the gas undergoes adiabatic compression, its temperature increases, while no heat is given to it.

From First Law of Thermodynamics.

$$dQ = dU + dW$$

As $dQ = 0$ [for adiabatic process]

$$\text{so, } dU = -dW$$

In compression, work is done on the system So, work done is negative,

$$\text{i.e., } dW = -ve$$

$$\text{or } dU = +ve$$

So, internal energy of the gas increases due to which its temperature increases.

- Q. 10.** Air pressure in a car tyre increases during driving. Explain. [NCERT Exemplar Q. No. 16]

Ans. Try it yourself. Refer Q3(c) of NCERT corner.

Short Answer Type Questions

Q. 11. Consider a cycle tyre being filled with air by a pump. Let V be the volume of the tyre (fixed) and at each stroke of the pump $\Delta V (\ll V)$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2 ?

[NCERT Exemplar Q. No. 19]

Ans. Let initial volume of air in tyre be V and after pumping one stroke it becomes $(V + dV)$ and pressure increased from P to $(P + dP)$.

Then

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P(V + dV)^\gamma = (P + dP)V^\gamma$$

$$PV^\gamma \left(1 + \frac{dV}{V}\right)^\gamma = P \left[1 + \frac{dP}{P}\right] V^\gamma$$

$$PV^\gamma \left[1 + \gamma \frac{dV}{V}\right] = P V^\gamma \left[1 + \frac{dP}{P}\right]$$

or $\gamma \frac{dV}{V} = \frac{dP}{P}$

or $dV = \frac{V dP}{\gamma P}$ or $P dV = \frac{V dP}{\gamma}$

Integrating both sides

$$\int P dV = \int_{P_1}^{P_2} \frac{V dP}{\gamma} \text{ or } \int dW = \int_{P_1}^{P_2} \frac{V dP}{\gamma} \quad [V = \text{constant}]$$

$$W = \frac{(P_2 - P_1)V}{\gamma}$$

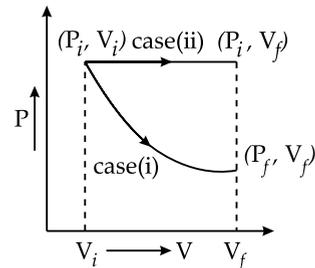
Q. 12. The initial state of a certain gas is (P_i, V_i, T_i) . It undergoes expansion till its volume becomes V_f . Consider the following two cases:

- (a) the expansion takes place at constant temperature.
- (b) the expansion takes place at constant pressure.

Plot the P-V diagram for each case. In which of the two cases, is the work done by the gas more?

[NCERT Exemplar Q. No. 22]

Ans.



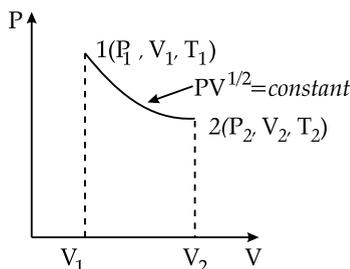
The P-V diagram for each case is shown in the figure.

- (a) In case (i) $P_i V_i = P_f V_f$; therefore process is isothermal. Work done = area under the PV curve.
- (b) The expression is at constant pressure P_i , so (ii) is isobaric process. Work done = area under the PV curve. Area enclosed by graph (i) is less than (ii), so work done is more when the gas expands at constant pressure (case ii).

Long Answer Type Questions

Q. 13. Consider a P-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in Fig.

- (a) Find the work done when the gas is taken from state 1 to state 2.
- (b) What is the ratio of temperature T_1/T_2 if $V_2 = 2V_1$?
- (c) Given the internal energy for one mole of gas at temperature T is $(3/2) RT$, find the heat supplied to the gas when it is taken from state 1 to 2, with $V_2 = 2V_1$.



[NCERT Exemplar Q. No. 23]

Ans. Let $PV^{1/2} = \text{constant} = K$

- (a) Work done by the gas for process 1 to 2

$$\Delta W = \int_{V_1}^{V_2} P dV = K \int_{V_1}^{V_2} \frac{dV}{\sqrt{V}} = K \left[\frac{\sqrt{V}}{1/2} \right]_{V_1}^{V_2} = 2K(\sqrt{V_2} - \sqrt{V_1})$$

$$= 2P_1 V_1^{1/2} [\sqrt{V_2} - \sqrt{V_1}]$$

- (b) From ideal gas equation-

$$PV = nRT$$

or $T = \frac{PV}{nR} = \frac{P\sqrt{V}\sqrt{V}}{nR}$

$$T = \frac{K\sqrt{V}}{nR} \quad [P\sqrt{V} = K]$$

$$T_1 = \frac{K\sqrt{V_1}}{nR}, \quad T_2 = \frac{K\sqrt{V_2}}{nR}$$

$$\therefore \frac{T_1}{T_2} = \frac{\frac{K\sqrt{V_1}}{nR}}{\frac{K\sqrt{V_2}}{nR}} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V_1}{2V_1}} = \frac{1}{\sqrt{2}} \quad [\because V_2 = 2V_1]$$

(c) Then, the change in internal energy = $U = \frac{3}{2}RT$

$$\Delta U = U_2 - U_1 = \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}RT_1(\sqrt{2} - 1)$$

$$\begin{aligned} \Delta W &= 2P_1V_1^{1/2}(\sqrt{V_2} - \sqrt{V_1}) \\ &= 2P_1V_1^{1/2}(\sqrt{2} \times \sqrt{V_1} - \sqrt{V_1}) \\ &= 2P_1\sqrt{V_1} \times \sqrt{V_1}(\sqrt{2} - 1) \end{aligned}$$

$$\begin{aligned} &= 2P_1\sqrt{V_1} \times \sqrt{V_1}(\sqrt{2} - 1) \\ &= 2P_1V_1(\sqrt{2} - 1) \\ &= 2RT_1(\sqrt{2} - 1) \end{aligned}$$

Such that,

$$\begin{aligned} \Delta Q &= \Delta U + \Delta W = \frac{3}{2}RT_1(\sqrt{2} - 1) + 2RT_1(\sqrt{2} - 1) \\ &= (\sqrt{2} - 1)RT_1\left(\frac{3}{2} + 2\right) \end{aligned}$$

Amount of heat supplied, $\Delta Q = \frac{7}{2}RT_1(\sqrt{2} - 1)$



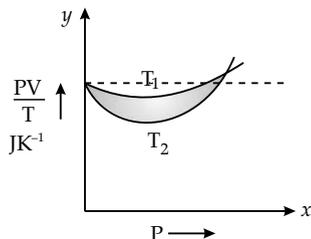
COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any 4 of the following questions on the basis of the same:

- I. A gas which obeys the ideal gas equation $PV = nRT$ at all temperature and pressure values is called an ideal gas equation. Molecules of such a gas are free from inter molecular attraction and have negligible volume. The gas particles are equally sized and motion of the particles follows Newton's laws of motion. These particles do not undergo any energy loss as they have perfect elastic collisions. Figure shows plot of $\frac{PV}{T}$ versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.



Q. 1. What does the dotted line signify in the above diagram?

- (A) Ideal gas behaviour
(B) Real gas behaviour
(C) Oxygen gas at temperature T_1
(D) Oxygen gas at temperature T_2

Ans. Option (A) is correct.

Explanation: The dotted line is a straight line parallel to P axis, so it signifies the ideal gas behaviour.

Q. 2. Which is of the following is correct ?

- (A) $T_1 < T_2$ (B) $T_1 > T_2$
(C) $T_1 = T_2$ (D) $T_1 \leq T_2$

Ans. Option (B) is correct.

Explanation: The real gas approaches the ideal behaviour and the temperature increases. Since the plot of temperature T_1 is close to the ideal behaviour, it can be concluded that $T_1 > T_2$.

Q. 3. The value of $\frac{PV}{T}$ where curves meet on y-axis is

- (A) 8.31 JK^{-1} (B) 0.26 JK^{-1}
(C) 0.52 JK^{-1} (D) 4.15 JK^{-1}

Ans. Option (B) is correct.

Explanation: The ideal gas equation is given by

$$\frac{PV}{T} = nR$$

$$M_{O_2} = 32 \text{ g}$$

$$nR = \frac{1}{32} \times 8.314 \text{ JK}^{-1}$$

$$= 0.259 \text{ JK}^{-1}$$

Q. 4. Relation between Boltzmann constant and universal gas constant is

$[k_B = \text{Boltzmann constant, } R = \text{Universal gas constant, } N = \text{Avogadro number}]$

- (A) $R = k_B/N$ (B) $N = k_B R$
(C) $k_B = \frac{N}{R}$ (D) $R = k_B N$

Ans. Option (D) is correct.

Q. 5. What will be the value of $\frac{PV}{T}$ for 1.00×10^{-3} kg

of Hydrogen gas if the oxygen gas is replaced by hydrogen gas? [Give: mass of Hydrogen = 2.02 u]

- (A) 2 JK^{-1} (B) 16.8 JK^{-1}
(C) 8.3 JK^{-1} (D) 4.11 JK^{-1}

Ans. Option (D) is correct.

Explanation: No. of moles of Hydrogen atom

$$n = \frac{1.00 \times 10^{-3} \text{kg}}{2.02 \text{u}} = 0.495$$

Since, $\frac{PV}{T} = nR = 0.495 \times 8.314 = 4.11 \text{ JK}^{-1}$

B Subjective Questions

Read the following text and answer any 4 of the following questions on the basis of the same:

There are different types of thermodynamic processes having their own characteristics.

1. **Isothermal process:** In this process the system's temperature remains constant. Heat is either supplied to or removed from the system in such a system.
2. **Adiabatic process:** In this process there is no exchange of heat with its surroundings. There The temperature of the system is constantly changing.
3. **Isobaric process:** In this process the pressure of the system remains constant.
4. **Isochoric process:** In this process the volume of the system remains constant.
5. **Reversible process:** In this process the direction of flow can be reversed at any point in the process. Throughout this process, the system maintains a virtual state of equilibrium with the surroundings.
6. **Irreversible process:** A process that cannot be reversed is known as irreversible.

7. **Cyclic process:** In this process a system goes through a sequence of states before returning to its initial state.

Q. 1. Ice starts melting at its melting point. It is similar to which type of thermodynamic process? Explain.

Ans. When ice melts at starts melting at its melting point, the temperature remains fixed till the whole ice melts. Hence, this is similar to thermodynamic isothermal process.

Q. 2. What is the shape of isothermal curves?

Ans. Shape of isothermal curves is rectangular hyperbola.

Q. 3. What is the difference between isothermal curves and adiabatic curves?

Ans. Adiabatic curves are always steeper than isothermal curves.

Q. 4. A spring is compressed its shape is deformed and when released it returns to its original shape. It is similar to which type of thermodynamic process? Explain.

Ans. Normally a spring is in state 1. When compressed it moves to state 2. When released it returns to state 1. Hence, it is similar to thermodynamic reversible process.

Q. 5. An ideal gas at 300K undergoes isobaric expansion at 2 Kpa. If volume increases from 1m³ to 5m³, find its final temperature.

Ans. In isobaric process $V \propto T$

$$\therefore \frac{V_{\text{initial}}}{V_{\text{final}}} = \frac{T_{\text{initial}}}{T_{\text{final}}}$$

$$\text{or} \quad \frac{1}{5} = \frac{300}{T_{\text{final}}}$$

$$\therefore T_{\text{final}} = 1500 \text{ K}$$



UNIT-IX

BEHAVIOUR OF PERFECT GASES AND KINETIC THEORY OF GASES

Study Time
Maximum Time: 2:30 hours
Maximum Questions: 68

CHAPTER

12

KINETIC THEORY



Syllabus

Equation of state of a perfect gas, work done in compressing a gas; Kinetic theory of gases—assumptions, concept of pressure. Kinetic interpretation of temperature, rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application of specific heat capacities of gases; concept of mean free path, Avogadro's number.



Revision Notes

FreeKItaab

➤ **Ideal Gases :**

- (a) Strictly obeys gas laws, like Boyle's law, Charles's law etc.
- (b) The size of the gas molecules is almost zero and the volume of the gas molecule is also almost zero.
- (c) There is no force of attraction or repulsion amongst the molecules.
- (d) Collisions between molecules are perfectly elastic.

➤ **Equation of state of Ideal gas:**

(a) **From Boyel's law:**

$$P_1 V_1 = P_2 V_2$$

(b) **From Charles' law:**

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

(c) **Combining Charles' and Boyel's law:**

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

or

K is a proportionality constant.

Again,

Where

∴

For 1 gram mole,

For μ mole,

Where

∴

Number of molecules in 1 mole (i.e., M gram) gas = Avogadro number (N)

So, number of moles in m gram gas = $\frac{Nm}{M}$

$$PV = KT$$

$$K = nk_B$$

n = number of molecules

K_B = Boltzmann constant

$$PV = nk_B T$$

$$PV = RT$$

$$PV = \mu RT$$

$$\mu = \frac{m}{M}$$

m = mass of gas sample

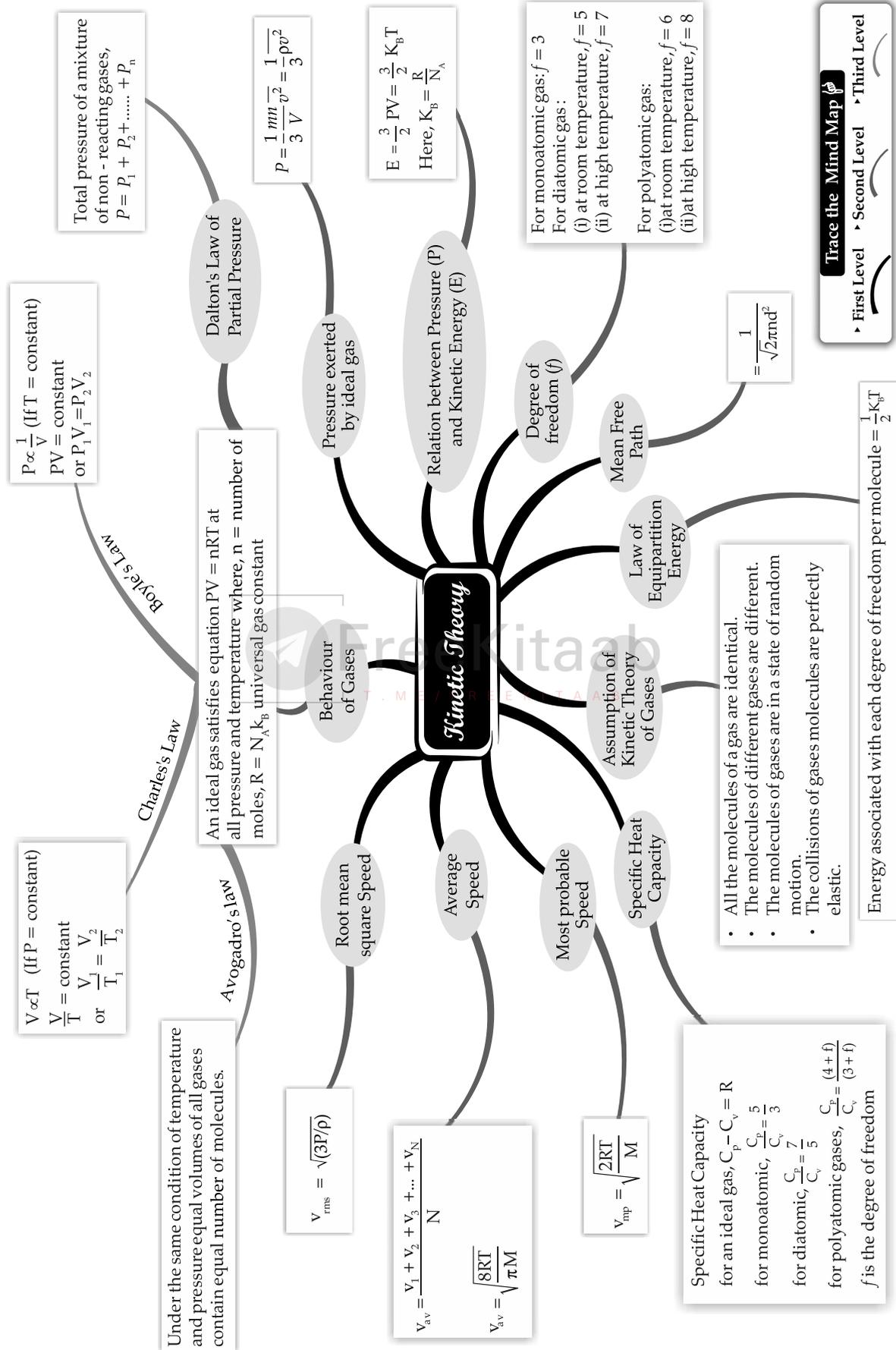
M = molecular weight

$$PV = \frac{m}{M} RT$$

Scan to know
more about
this topic



Ideal gas
equation



Trace the Mind Map

► First Level ► Second Level ► Third Level

So,
$$\frac{m}{M} = \frac{n}{N}$$

$$\therefore PV = \frac{n}{N} RT$$

Value of $R = 8.31 \text{ J mol}^{-1}\text{K}^{-1}$

Value of $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$

➤ **Assumptions of Kinetic Theory of Gases :**

- (a) A gas consists of a very large number of molecules which are perfectly elastic spheres and are identical in all respects for a given gas and are different for different gases.
- (b) The molecules of a gas are in a state of continuous, rapid and random motion.
- (c) The volume occupied by the molecules is negligible in comparison to the volume of the gas.
- (d) The molecules do not exert any force of attraction or repulsion on each other, except during collision.
- (e) The collisions of the molecules with themselves and with the walls of the vessel are perfectly elastic.
- (f) Molecular density is uniform throughout the gas.
- (g) A molecule moves along a straight line between two successive collisions.
- (h) The collisions are almost instantaneous.

➤ **Brownian Motion :** It is defined as continuous zig-zag motion of particles of macroscopic size (10^{-5} to 10^{-6} m) suspended in fluid.

Brownian motion increases :

- (a) When size of suspended object increases.
- (b) When density of fluid is decreases.
- (c) When temperature of medium increases.
- (d) When viscosity of medium decreases.

➤ **Pressure of an ideal gas:**

Pressure exerted by an ideal gas is equal to two third of its translational kinetic energy per unit volume.

$$P = \frac{2}{3}E = \frac{1}{3} \frac{mn}{V} \overline{v^2} = \frac{1}{3} \rho \overline{v^2}$$

Where, $\overline{v^2}$ = mean square velocity of the molecules

m = Mass of the whole gas

V = Volume of the gas

ρ = Density of gas

➤ **Root mean square speed**

Root mean square speed of the molecules is defined as the square root of the mean of the squares of speed of all the gas molecules.

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$$

Since,

pressure of a gas = $P = \frac{1}{3} \rho \overline{v^2}$

\therefore

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

➤ **Interpretation of temperature:**

The root mean square speed of the molecules of a gas is directly proportional to the square root of absolute temperature of gas.

\therefore

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} \propto \sqrt{T}$$

When $T = 0$, $v_{rms} = 0$

So, absolute temperature is the temperature at which the motion of the molecules of the gas becomes zero and no temperature below absolute zero is possible.

➤ **Degrees of freedom:**

Degrees of freedom: The degree of freedom for a dynamic particle is the number of directions in which that can move freely or the total number of coordinates required to describe completely the position and configuration of the particle.

A gaseous molecule has three types degrees of freedom: translational, rotational and vibrational.

- Translational degrees of freedom arise from the ability of gas molecules to move freely in space. So, the translational motion of the molecule of gas has three degrees of freedom associated with it. This is applicable for all gas molecules, whether they are monatomic, diatomic or polyatomic,
- Rotational degrees of freedom represent the number of unique ways the molecule may rotate in space about its center of mass. A monatomic gaseous molecule has no rotational degrees of freedom. Linear molecule has two rotational degrees of freedom.

However, non-linear molecules have three rotational degrees of freedom.

- Vibrational degrees of freedom arises when the bonds of the molecules behave like a spring and the molecule execute simple harmonic motion.

Atomicity		Degrees of freedom			
1		3	0	0	3
2		3	2	1	6
3	linear	3	2	4	9
	Non-linear	3	3	3	9
N	Linear	3	2	3N - 5	3N
	Non-linear	3	3	3N - 6	3N

➤ **Law of Equipartition of energy** : It states that the energy for each degree of freedom in thermal equilibrium is $12 k_B T$

➤ **Molar specific heat:**

- **For monoatomic gas:** The molecule of a monatomic gas has only three translational degrees of freedom.

The molar specific heat at constant volume, $C_V = \frac{3}{2} R$

$$C_p = C_V + R$$

$$\therefore C_p = \frac{3}{2} R + R$$

$$\therefore \gamma = \frac{C_p}{C_V} = \frac{5}{3}$$

- **For diatomic gas:** A diatomic molecule has 3 translational and 2 rotational degrees of freedom.

$$C_V = \frac{5}{2} R$$

$$C_p - C_V = R$$

$$\therefore C_p = \frac{7}{2} R$$

$$\therefore \gamma = \frac{C_p}{C_V} = \frac{9}{7}$$



Key Words

- **Most probable speed** of the molecules of a gas is that speed which is possessed by maximum fraction of total number of molecules of the gas.
- **Mean speed or average speed** is the average speed with which molecules of a gas move.
- **Root mean square speed** is defined as the square root of the mean of the squares of random velocities of individual molecules of a gas.
- **Absolute zero of temperature** may be defined as that temperature at which the root mean square velocity of gas molecules reduces to zero.
- **Pressure** exerted by gas is due to continuous bombardment of gas molecules against the walls of container.
- **Degrees of freedom** of a dynamic system is defined as the total no. of co-ordinates or independent quantities required to describe completely the position & configuration of the system.
- **Mean free path** is the average distance covered between two successive collisions by the gas molecule moving along the straight line.

Key Formulae

- Most probable speed : $c_{mp} = \sqrt{\frac{2k_B T}{m}}$
 - Average speed: $c_{av} = \sqrt{\frac{8k_B T}{\pi m}}$
 - Root mean square speed: $c_{rms} = \sqrt{\frac{3k_B T}{m}}$
- k_B = Boltzman constant, T = Temperature, m = mass
- Ratio among speeds: $c_{mp} : c_{av} : c_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$
 - Pressure exerted by ideal gas: $P = \frac{2}{3}E = \frac{1}{3} \frac{mn \bar{v}^2}{V} = \frac{1}{3} \rho \bar{v}^2$
 - Root mean square velocity: $v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$
 - Law of equipartition of energy: $E_t = \frac{1}{2} k_B T$
 - Specific Heat Capacity of :
 - (a) Monoatomic gas: $\gamma = \frac{C_p}{C_v}$ or $\gamma = \frac{5}{3} = 1.67$
 - (b) Diatomic gas: $\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$
 - (c) Triatomic gas:



Linear gas molecules $\gamma = \frac{9}{7} = 1.28$

Non-linear gas molecules $\gamma = \frac{4}{3} = 1.33$.
 - (d) Polyatomic gas: $\gamma = \left(1 + \frac{2}{n}\right)$, where n is the degree of freedom
 - Mean free path: $\lambda = \frac{1}{\sqrt{2} n \pi d^2} = \frac{k_B T}{\sqrt{2} \pi d^2 P}$



Mnemonics

Concept: Degrees of freedom.

Mnemonics: Baa Baa Black Sheep

Have you any wool?

Yes, sir, **Mom** has **3** bags full.

Dadi needs **5** bags normally rule.

Papa keeps **6** bags normal rule.

Papa Dadi each needs **2** bags more

High cold whenever, be very sure.

Interpretation: Mom has **3** bags full → Degrees of freedom of Monoatomic gas is 3.

Dadi needs **5** bags normally cool

Degrees of freedom of diatomic gas at normal → (room) temperature is 5.

Papa keeps **6** bags normal rule → Degrees of freedom of Polyatomic gas at normal (room) temperature is 6.

Papa, Dadi each needs **2** bags more → Degrees of freedom of Polyatomic gas at high temperature is $6 + 2 = 8$.

High cold whenever, be very sure → Degrees of freedom of Diatomic gas at high temperature is $5 + 2 = 7$.



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is an ideal gas ? R [DDE]

OR

What is a perfect gas ?

Ans. An ideal gas is that which obeys the gas laws *i.e.* Charles's law, Boyle's law etc. at all values of temperature and pressure. Molecules of such a gas should be free from intermolecular attraction and have negligible volume. 1

Q. 2. On which factors does the average kinetic energy of gas molecules depend : Nature of the gas, temperature, volume ? U

Ans. The average K.E. of molecules of a gas molecules depends only on the absolute temperature of the gas and is directly proportional to it. 1

Q. 3. Mention the different ways of increasing the number of molecular collisions per unit time in a gas. U

Ans. The number of collisions per unit time can be increased by

- (i) increasing the temperature of the gas.
- (ii) increasing the number of molecules, and
- (iii) decreasing the volume of the gas. 1



Commonly Made Error

- ▶ Students can not state the ways of increasing the number of molecular collisions per unit time in a gas.



Answering Tip

- ▶ The concept of molecular motion in a gas should be stressed upon.

Q. 4. The absolute temperature of a gas is increased 3 times. What will be the increase in root mean square velocity of the gas molecules ? A

Ans. $c \propto \sqrt{T}$

When temperature becomes 3 times, the r.m.s. velocity becomes $\sqrt{3}$ times. Hence, increase in r.m.s. velocity $\sqrt{3}c - c = 0.732c$. 1

Q. 5. What do you mean by mean free path of a gas molecule ? R [DDE]

Ans. The mean free path of a gas molecule is the average distance travelled by the molecule between its two successive collisions with other molecules. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. Explain absolute zero of temperature on the basis of kinetic theory of gases. U [NCT 2009]

Ans. Absolute zero of temperature may be defined as that temperature at which the root mean square velocity of the gas molecules reduces to zero. It means, molecular motion ceases at absolute zero. 2

Q. 2. Give perfect gas equations. U

Ans. Boyle's law:

$$V \propto \frac{1}{P} \quad \text{(i) } \frac{1}{2}$$

Charles's law:

$$V \propto T \quad \text{(ii) } \frac{1}{2}$$

combining eq (i) & (ii)

$$\frac{PV}{T} = \text{constant} = R \quad \frac{1}{2}$$

for n moles, $PV = nRT$ 1/2

Q. 3. On the basis of kinetic theory of gases, explain, how does a gas exert pressure ?

Ans. According to kinetic theory, the molecules of a gas are in a state of continuous random motion. They collide with one another and also with the walls of the vessel. 1

Whenever a molecule collides with the wall, it returns with a changed momentum and an equal momentum is transferred to the wall and thus creating pressure. 1

Q. 4. Two different gases have exactly the same temperature. Does this mean that their molecules move with the same r.m.s. speed ? A

Ans. When the two gases have exactly the same temperature the average kinetic energy per molecule for each gas is the same. But as the different gases may have molecules of different masses, the r.m.s. speed (c) of molecules of different gases shall be different. 2

Q. 5. The pressure of a given mass of a gas is halved at constant temperature. What will be the volume of the gas in comparison to its initial volume ? Explain on the basis of kinetic theory. A

Ans. As $P = \frac{1}{3} \frac{M}{V} c^2$

or $V = \frac{1}{3} \frac{M}{P} c^2$

i.e., $V \propto \frac{1}{P}$ 1

[$\therefore \frac{1}{3} Mc^2 = \text{a constant}$]

Therefore, if P becomes $P/2$, V becomes double. 1



Short Answer Type Questions-II

(3 marks each)

Q. 1. Define root mean square velocity of gas molecules give various relations for it. [R]

Ans. Root mean square velocity is defined as the square root of the average of the squares of the individual velocities of the gas molecules *i.e.*, 1

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}} = \sqrt{\overline{v^2}} \quad 1$$

where, $v_1, v_2, v_3, \dots, v_n$ are individual velocities

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{m}}$$

i.e., $v_{rms} \propto \sqrt{T}$ 1

Q. 2. For Brownian motion of particles of suspensions in liquid answer the following: [A]

- What should be the typical size of suspended particles?
- Bombardments of the suspended particles by molecules of the liquid are random. We should then expect equal no. of molecules hitting a suspended particle from all directions. Why the net impact is not zero?
- Can the assembly of suspended particles be considered a 'gas' of 'heavy molecules'? If so, what is the temperature of this gas if temperature of the liquid is T?

Ans. (a) 10^{-5} to 10^{-6} m. 1

(b) The average number of molecules hit in different directions so their net impact is not zero. 1

(c) Yes, since the heavy gas is in thermal equilibrium with the liquid, its temperature is equal to the temperature of liquid, *i.e.*, T. 1

Q. 3. Calculate the average kinetic energy for one molecule of gas at constant volume. [A]

Ans. Pressure exerted by one mole of gas

$$P = \frac{1}{3} \rho v_{rms}^2 = \frac{1}{3} \frac{M}{V} v_{rms}^2$$

or, $PV = \frac{1}{3} M v_{rms}^2$

But $PV = RT$ 1

$$\frac{1}{3} M v_{rms}^2 = RT$$

$$M v_{rms}^2 = 3RT$$

Now, Average KE

$$\frac{1}{3} M v_{rms}^2 = \frac{3}{2} RT$$

or $\frac{1}{2} (Nm) v^2 = \frac{3}{2} RT$ ($\because M = Nm$)

(KE)_{Avg} for one molecule

$$\frac{1}{2} M v_{rms}^2 = \frac{3}{2} \frac{R}{N} T = \frac{3}{2} k_B T \quad 1$$

\therefore Total random K.E. for one mole = $\frac{3}{2} RT$

and average K.E. per molecule = $\frac{3}{2} k_B T$ 1



Commonly Made Error

- Student could not calculate the average kinetic energy for one molecule of the gas.



Answering Tip

- Students should be familiar with the expression for the pressure exerted by the gas.



Long Answer Type Questions

(5 marks each)

Q. 1. State and explain Charles's law. [U]

Ans. It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by $1/273.15$ of its volume at 0°C for each 1°C rise or fall in temperature. 1

Let V_0 be the volume of the given mass of a gas at 0°C . According to Charles's law, its volume at 1°C is

$$V_1 = V_0 + \frac{V_0}{273.15}$$

$$= V_0 \left[1 + \frac{1}{273.15} \right] \quad 1$$

\therefore Volume of the gas at $t^\circ\text{C}$

$$V_t = V_0 \left[1 + \frac{t}{273.15} \right]$$

$$= V_0 \left(\frac{273.15 + t}{273.15} \right) \quad 1$$

If T_0 and T are temperatures on kelvin scale corresponding to 0°C and $t^\circ\text{C}$, then

$$T_0 = 273.15 + 0 = 273.15$$

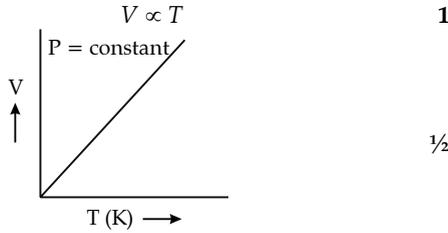
$$T = 273.15 + t$$

$$V_t = V_0 \frac{T}{T_0} \quad \frac{1}{2}$$

$$\frac{V_t}{T} = \frac{V_0}{T_0}$$

$$\frac{V}{T} = \text{constant}$$

i.e.,



Commonly Made Error

- Students can not derive the expression for Charles's law.



Answering Tip

- Step by step derivation should be stressed upon.

Q. 2. Determine the numerical values of R and k_B . □

Ans. Numerical value of R : Let us consider one mole of a gas at STP, then

$$R = \frac{P_0 V_0}{T_0} \quad 1$$

Standard pressure

$$P_0 = 0.76 \text{ m of Hg column} \\ = 0.76 \times 13.6 \times 10^3 \times 9.8 \text{ N/m}^2$$

Standard temperature = $T_0 = 273.15\text{K}$ 1

Volume of one mole of gas at

$$R = \frac{22.4 \times 10^{-3} \text{ m}^3}{273.15} \\ = \frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273.15}$$

$$= 8.31 \text{ J mole}^{-1} \text{ K}^{-1} \quad 1$$

In the C.G.S. system.

$$R = \frac{8.31}{4.2} \text{ cal mole}^{-1} \text{ }^\circ\text{C}^{-1}$$

$$= 1.98 \text{ cal mole}^{-1} \text{ }^\circ\text{C}^{-1} \quad 1$$

Numerical value of k_B :

We know that

$$k_B = \frac{R}{N}$$

$$k_B = \frac{8.31 \text{ J mole}^{-1} \text{ K}^{-1}}{6.02 \times 10^{23} \text{ mole}^{-1}}$$

$$= 1.38 \times 10^{-23} \text{ J/K.} \quad 1$$

Q. 3. Determine γ from degrees of freedom.

□ [KVS 2013]

Ans. Suppose a polyatomic gas molecule has n degree of freedom.

Total energy associated with one gram molecule of the gas, i.e.,

$$E = n \times \frac{1}{2} RT \times 1$$

$$= \frac{n}{2} RT \quad 1$$

As

$$C_V = \frac{d}{dT} (E)$$

$$= \frac{d}{dT} \left(\frac{n}{2} RT \right)$$

$$= \frac{n}{2} R \quad 1$$

$$C_p = C_V + R$$

$$C_p = \frac{n}{2} R + R$$

$$= \left(\frac{n}{2} + 1 \right) R \quad 1$$

$$\gamma = \frac{C_p}{C_V}$$

$$\gamma = \frac{\left(\frac{n}{2} + 1 \right) R}{\frac{n}{2} R} \quad 1$$

\therefore

$$= \frac{2 \left(\frac{n}{2} + 1 \right)}{n}$$

$$\gamma = 1 + \frac{2}{n} \quad 1$$



Objective Type Questions

(1 marks each)

A Multiple Choice Questions

Q. 1. The ratio of specific heats for monatomic and rigid diatomic gases are respectively

(A) $\frac{9}{7}, \frac{5}{3}$ (B) $\frac{9}{5}, \frac{7}{3}$

(C) $\frac{5}{3}, \frac{7}{5}$ (D) $\frac{3}{5}, \frac{7}{9}$

Ans. Option (C) is correct.

Explanation: The molecule of a monatomic gas has only three translational degrees of freedom.

So, $C_V = \frac{5}{2} R$

Since, $C_p - C_V = R$

$\therefore C_p = \frac{5}{2} R$

$$\frac{C_p}{C_V} = \frac{5}{3}$$

A diatomic molecule treated having a rigid dumbbell shape, has 3 translational and 2 rotational degrees of freedom.

$$\text{So, } C_V = \frac{5}{2} R$$

$$\text{Since, } C_p - C_V = R$$

$$\therefore C_p = \frac{7}{2} R$$

$$\frac{C_p}{C_V} = \frac{7}{5}$$

Q. 2. Each translational degree of freedom contributes _____, each rotational degree of freedom contributes _____ and each vibrational frequency contributes _____ to the energy of a molecule.

- (A) $\frac{1}{2} k_B T, \frac{1}{2} k_B T, \frac{1}{2} k_B T$
 (B) $k_B T, k_B T, \frac{1}{2} k_B T$
 (C) $\frac{1}{2} k_B T, \frac{1}{2} k_B T, k_B T$
 (D) $k_B T, k_B T, k_B T$

Ans. Option (C) is correct.

Q. 3. The average kinetic energy of a molecule of an ideal gas is proportional to

- (A) Absolute temperature
 (B) Pressure
 (C) Volume
 (D) Nature of gas

Ans. Option (A) is correct.

Explanation: The average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating temperature.

Q. 4. The relation between mean free path (λ) is diameter (d) of gas molecules is

- (A) $\lambda \propto d$ (B) $\lambda \propto d^2$
 (C) $\lambda \propto \frac{1}{d}$ (D) $\lambda \propto \frac{1}{d^2}$

Ans. Option (D) is correct.

$$\text{Explanation: } \lambda = \frac{1}{\sqrt{2} n \pi d^2}$$

Q. 5. Real gases satisfy the ideal gas equation only approximately,

- (A) At low pressures and high temperatures.
 (B) At low pressures and low temperatures.
 (C) At high pressures and high temperatures.
 (D) At high pressures and low temperatures.

Ans. Option (A) is correct.

B Assertion & Reason Type Questions:

Directions: These questions consist of two statements - Assertion and Reason.



NCERT Corner

Q. 1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3\AA .

Choose any one of the following four responses.

- (A) If both Assertion and Reason are true and the Reason is a correct explanation of the Assertion.
 (B) If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.
 (C) If the Assertion is true but Reason is false.
 (D) If both the Assertion and Reason are false.

Q. 1. Assertion: Non-rigid diatomic molecule has total internal energy = $\frac{7}{2} k_B T$.

Reason: Non-rigid diatomic molecule has 3 translational degrees of freedom, 2 rotational degrees of freedom and 1 vibrational mode.

Ans. Option (A) is correct.

Explanation: Non-rigid diatomic molecule has 3 translational degrees of freedom, 2 rotational degrees of freedom and 1 vibrational mode. Each translational degree of freedom contributes $\frac{1}{2}$

$k_B T$, each rotational degree of freedom contributes $\frac{1}{2} k_B T$ and each vibrational mode contributes $k_B T$ amount of energy.

$$\text{So, } U = 3 \times \frac{1}{2} k_B T + 2 \times \frac{1}{2} k_B T + k_B T = \frac{7}{2} RT$$

So, the assertion and reason both are true and the reason explains the assertion.

Q. 2. Assertion: In a diatomic gas, 60% of total internal energy is translational and rest 40% is rotational.

Reason: A diatomic gas molecule has 3 translational degrees of freedom and 3 rotational degrees of freedom.

Ans. Option (C) is correct.

Explanation: A diatomic gas molecule has 3 translational degrees of freedom and 2 rotational degrees of freedom.

$$\text{So, Translational kinetic energy} = 3 \times \frac{1}{2} k_B T = \frac{3}{2}$$

$$k_B T$$

$$\text{Rotational kinetic energy} = 2 \times \frac{1}{2} k_B T = k_B T$$

So, 60% of total internal energy is translational and rest 40% is rotational.

Hence, the assertion is true but the reason is false.

Q. 3. Assertion: A gas that satisfies $PV = \mu RT$ exactly at all pressures and temperatures is defined as ideal gas.

Reason: No real gas is truly ideal.

Ans. Option (B) is correct.

Ans. Given: diameter,

$$d = 3\text{\AA}, r = \frac{d}{2} = 1.5\text{\AA}$$

$$= 1.5 \times 10^{-8} \text{ cm}$$

$$\text{Molecular volume } V = \frac{4}{3} \pi r^3 N$$

(Here, N = Avogadro's number)

$$\begin{aligned} &= \frac{4}{3} \times \frac{22}{7} \times (1.5 \times 10^{-8})^3 \\ &\quad \times (6.023 \times 10^{23}) \\ &= 8.52 \text{ cc} \end{aligned}$$

Let V' be actual volume occupied by 1 mole of O_2 at STP = 22400cc.

$$\therefore \frac{V}{V'} = \frac{8.52}{22400} = 3.8 \times 10^{-4}$$



Commonly Made Error

- ▶ Students lack understanding of the term STP.



Answering Tip

- ▶ Students should be acquainted with the fact that STP stands for Standard temperature and pressure which is 1 atm pressure at 0°C.

Q. 2. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atm pressure, 0°C). Show that it is 22.4 litres.

Ans. Here, $n = 1$ mol, $T = 273$ K, $R = 8.31$ J/mole/K.

$$\begin{aligned} P &= 1 \text{ atm} = 0.76 \text{ m of Hg} \\ &= 0.76 \times 9.8 \times 13.6 \times 10^3 \text{ N/m}^2 \end{aligned}$$

Using the relation

$$PV = nRT,$$

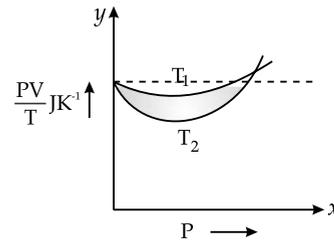
$$\text{or, } V = \frac{nRT}{P}$$

$$\begin{aligned} &= \frac{1 \times 8.31 \times 273}{0.76 \times 9.8 \times (13.6 \times 10^3)} \\ &= 22.4 \times 10^{-3} \text{ m}^3 \\ &= 22.4 \text{ lit.} \end{aligned}$$

Q. 3. Figure shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.

- What does the dotted line signify ?
- Which is true : $T_1 > T_2$ or $T_1 < T_2$?
- What is the value of $\frac{PV}{T}$ where the curves meet on the y -axis ?
- If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the Y -axis ? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ?

[Molecular mass of hydrogen (H_2) = 2.02 u, of O_2 = 32.0 u, $R = 8.31 \text{ mol}^{-1}\text{K}^{-1}$.]



- Ans. (a)** The dotted plot signifies the ideal gas behaviour as $PV/T = \text{constant}$.
- (b)** T_1 is greater than T_2 because behaviour of a real gas approaches the ideal gas behaviour when temperature is increased.

$$\begin{aligned} \text{(c) Here, } \frac{PV}{T} &= nR = \frac{8.31}{(32 \times 10^{-3})} \times 1 \times 10^{-3} \\ &= 0.26 \text{ JK}^{-1} \end{aligned}$$

($\because R = 8.31 \text{ Jmol}^{-1}\text{K}^{-1}$, $O_2 = 32 \times 10^{-3}$ kg and mass of O_2 is given as 1×10^{-3} kg)

- (d)** No, this is because PV/T depends upon the volume of gas and volume of equal masses of different gases are different. For hydrogen

$$\frac{PV}{T} = \frac{8.31 \times 10^{-3}}{2.02 \times 10^{-3}} = 4.11 \text{ JK}^{-1}$$

The value is not same.

Mass of hydrogen for same value of this term $\frac{PV}{T}$, i.e., 0.26 JK^{-1} is given by,

$$\begin{aligned} &= \frac{2.02 \times 10^{-3}}{8.31} \times 0.26 \\ &= 6.32 \times 10^{-5} \text{ kg} \end{aligned}$$

Q. 4. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.

Ans. Initially in the oxygen cylinder

$$\begin{aligned} V_1 &= 30 \text{ litres} \\ &= 30 \times 10^{-3} \text{ m}^3 \\ P_1 &= 15 \text{ atm} \\ &= 15 \times 1.01 \times 10^5 \text{ Pa.} \\ T_1 &= 27 + 273 \\ &= 300 \text{ K} \end{aligned}$$

If the cylinder contains n_1 mole of oxygen gas then

$$P_1 V_1 = n_1 R T_1$$

or

$$\begin{aligned} n_1 &= \frac{P_1 V_1}{R T_1} \\ &= \frac{15 \times 1.01 \times 10^5 \times 30 \times 10^{-3}}{8.3 \times 300} \\ &= 18.253 \end{aligned}$$

For oxygen, molecular weight $M = 32$ g

Initial mass of cylinder

$$\begin{aligned} m_1 &= n_1 M \\ &= 18 \cdot 253 \times 32 \\ &= 584.1 \text{ g} \end{aligned}$$

Finally in the oxygen cylinder, let n_2 moles of oxygen be left

$$\begin{aligned} V_2 &= 30 \times 10^{-3} \text{ m}^3 \\ P_2 &= 11 \times 1.01 \times 10^5 \text{ Pa.} \\ T_2 &= 17 + 273 = 290 \text{ K.} \end{aligned}$$

Now,

$$\begin{aligned} n_2 &= \frac{P_2 V_2}{RT_2} \\ &= \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.3 \times 290} \\ &= 13.847 \end{aligned}$$

Final mass of oxygen gas in the cylinder

$$m_2 = 13.847 \times 32 = 443.1 \text{ g}$$

Mass of the oxygen gas taken out

$$\begin{aligned} &= m_1 - m_2 = 584.1 - 443.1 \\ &= 141.0 \text{ g} \end{aligned}$$

$$\begin{aligned} V_2 &= \frac{493000 \times 1.0 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} \\ &= 5.275 \times 10^{-6} \text{ m}^3. \end{aligned}$$

Q. 6. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m³ at a temperature of 27°C and 1 atm pressure.

Ans. Here,

$$\begin{aligned} P &= 1 \text{ atm} \\ &= 1.01 \times 10^5 \text{ Pa} \\ V &= 25 \text{ m}^3 \\ k_B &= (1.38 \times 10^{-23}) \text{ J/K} \\ T &= 273 + 27 \\ &= 300 \text{ K} \end{aligned}$$

Using, $PV = nk_B T$, we get

$$\begin{aligned} n &= \frac{PV}{k_B T} \\ &= \frac{(1.01 \times 10^5)(25)}{(1.38 \times 10^{-23})(300)} \\ &= 6.1 \times 10^{26}. \end{aligned}$$

Q. 7. Estimate the average thermal energy of a helium atom at (i) room temperature (27°C), (ii) the temperature on the surface of the sun (6000 K), (iii) temperature of 10 million kelvin (the typical core temperature in the case of star).

Ans. (i)

$$\begin{aligned} K.E._{avg} &= \frac{3}{2} kT \\ &= \frac{3}{2} (1.38 \times 10^{-23}) (27 + 273) \\ &= 6.21 \times 10^{-21} \text{ J} \end{aligned}$$

(ii)

$$\begin{aligned} K.E._{avg} &= \frac{3}{2} kT \\ &= \frac{3}{2} (1.38 \times 10^{-23}) \times 6000 \\ &= 1.242 \times 10^{-19} \text{ J} \end{aligned}$$

(iii)

$$\begin{aligned} K.E._{avg} &= \frac{3}{2} kT \\ &= \frac{3}{2} (1.38 \times 10^{-23}) \times 10^7 \\ &= 2.07 \times 10^{-16} \text{ J} \end{aligned}$$



Commonly Made Error

- Students can not find the mass of oxygen taken out of the cylinder.



Answering Tip

- Students should have practice in solving numerical problems using ideal gas equation.

Q. 5. An air bubble of volume 1.0 cm³ rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35°C ? (Given, 1 atm = 1.01 × 10⁵ Pa)

Ans. When the air bubble is at 40 m depth, then

$$\begin{aligned} V_1 &= 1.0 \text{ cm}^3 \\ &= 1.0 \times 10^{-6} \text{ m}^3 \\ T_1 &= 12^\circ\text{C} \\ &= 12 + 273 = 285 \text{ K} \\ P_1 &= 1 \text{ atm} + h\rho g \\ &= 1.01 \times 10^5 + 40 \times 10^3 \times 9.8 \\ &= 4,93,000 \text{ Pa} \end{aligned}$$

When the air bubble reaches at the surface of lake, then

$$\begin{aligned} V_2 &= ? \\ T_2 &= 35^\circ\text{C} = 35 + 273 \text{ K} \\ &= 308 \text{ K} \\ P_2 &= 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} \end{aligned}$$

Now

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

or

$$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$$



Commonly Made Error

- Students can not evaluate the average thermal energy.



Answering Tip

- Students should be familiar with the relation, $K.E._{avg} = (3/2)kT$ where, Temperature (T) is in kelvin.

Q. 8. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Ans. (a) Yes, because according to Avogadro's hypothesis, equal volume of all the gases have same number of molecules under the condition of same temperature and pressure.

(b) Using $v_{rms} = \sqrt{\frac{3kT}{m}}$, we get $v_{rms} \propto \frac{1}{\sqrt{m}}$, i.e., v_{rms} will not be same in the three cases because it depends upon mass of the gas. It will be the largest for neon.

Q. 9. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at -20°C ? (Atomic mass of Ar = 39.9 u and that of He = 4.0 u.)

Ans. R.M.S. speed of argon at temperature T

$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

R.M.S. for helium at temperature -20°C or 253K is

$$v' = \sqrt{\frac{3R \times 253}{4.0}}$$

But $v = v'$ (given)

$$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4.0}}$$

or
$$\frac{T}{39.9} = \frac{253}{4.0}$$

$$T = 2523.7\text{K.}$$

Q. 10. Estimate the mean free path and collision frequency of nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17°C . Take the radius of a nitrogen molecule to be roughly 1.0 \AA . Compare the collision time with the time the molecule moves freely between two successive collisions. (Molecular mass of $\text{N}_2 = 28.0 \text{ u}$.)

Ans. From Maxwell's correction, the mean free path (λ) of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

where d = diameter of a molecule.

and
$$n = \frac{N}{V}$$

$$= \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

At 2 atm pressure, the volume of the gas of mass M is given by $V = \frac{RT}{P}$, $T = 273 + 17 = 290 \text{ K}$.

$$\therefore n = \frac{N}{V} = \frac{NP}{RT} \quad \text{(i)}$$

Now given : $N = 6.023 \times 10^{23} \text{ mole}^{-1}$
 $P = 2 \text{ atm}$
 $= 2 \times 1.013 \times 10^5 \text{ Nm}^{-2}$
 $R = 8.3 \text{ JK}^{-1} \text{ mol}^{-1}$.

From equ (i)

$$n = \frac{6.023 \times 10^{23} \times 2 \times 1.013 \times 10^5}{8.3 \times 290}$$

$$= 5.07 \times 10^{25}$$

Here, $r = 1 \text{ \AA} = 1 \times 10^{-10} \text{ m}$

So, $d = 2r = 2 \times 10^{-10} \text{ m}$

Mean free path,

$$\lambda = \frac{1}{\sqrt{2} \times 3.142 \times (2 \times 10^{-10})^2 \times 5.07 \times 10^{25}}$$

$$= 1.0 \times 10^{-7} \text{ m}$$

The r.m.s. velocity is given by

$$c = \sqrt{\frac{3RT}{M}}$$

Putting $R = 8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$, $T = 290 \text{ K}$,

$$M = 28 \times 10^{-3} \text{ kg/mol}$$

In equation (i),

$$c = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-3}}}$$

$$= 5.08 \times 10^2 \text{ ms}^{-1}$$

$$= 5.10 \times 10^2 \text{ ms}^{-1}$$

\therefore Collision frequency (ν) is given by

$$\nu = \frac{c}{\lambda} = \frac{5.1 \times 10^2}{1.0 \times 10^{-7}}$$

$$= 5.1 \times 10^9 \text{ s}^{-1}$$

Suppose τ be the time between two successive collisions,

$$\therefore \tau = \frac{\lambda}{c} = \frac{1.0 \times 10^{-7} \text{ m}}{5.1 \times 10^2 \text{ ms}^{-1}}$$

$$= 2.0 \times 10^{-10} \text{ s.} \quad \text{(ii)}$$

Also suppose t = time taken for the collision.

$$\therefore t = \frac{d}{c} = \frac{2 \times 10^{-10}}{5.10 \times 10^2}$$

$$= 4 \times 10^{-13} \text{ s} \quad \text{(iii)}$$

(ii)/(iii) gives
$$\frac{\tau}{t} = \frac{2.0 \times 10^{-10} \text{ s}}{4 \times 10^{-13} \text{ s}}$$

$$= 500$$

or $\tau = 500 t$, i.e., the time taken between successive collisions is 500 times the time taken for a collision to take place, i.e., the two molecules are in contact. This shows that the molecule in a gas moves nearly free for most of the time.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

Q. 1. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

- (A) remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas.
- (B) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.
- (C) will increase by a factor equal to $(v_{\text{rms}}^2 + (500)^2) / v_{\text{rms}}^2$ where v_{rms} was the original mean square velocity of the gas.
- (D) will be different on the top wall and bottom wall of the vessel.

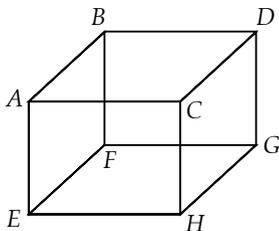
Ans. Option (B) is correct.

Explanation:
$$p = \frac{nRT}{V}$$

P remains unaffected as n , R , T and V are constant.

Q. 2. 1 mole of an ideal gas is contained in a cubical volume V . ABCDEFGH at 300 K. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time

- (A) the pressure on EFGH would be zero.
- (B) the pressure on all the faces will be equal.
- (C) the pressure of EFGH would be double the pressure on ABCD.
- (D) The pressure on EFGH would be half that of ABCD.



Ans. Option (D) is correct.

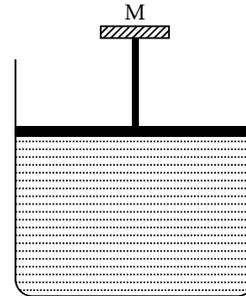
Q. 3. Boyle's law is applicable for an

- (A) adiabatic process.
- (B) isothermal process.
- (C) isobaric process.
- (D) isochoric process.

Ans. Option (B) is correct.

Explanation: Since, $V \propto \frac{1}{P}$ at constant temperature.

Q. 4. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction. If the temperature is increased,



- (A) both P and V of the gas will change.
- (B) only P will increase according to Charles's law.
- (C) V will change but not P .
- (D) P will change but not V .

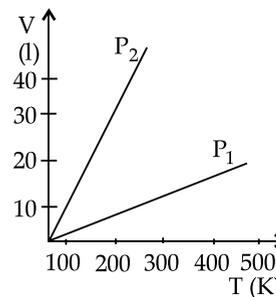
Ans. Option (C) is correct.

Explanation:
$$P = \frac{F}{A} = \frac{mg}{A} = \text{constant}$$

$\therefore V \propto T$ (at constant pressure)

Q. 5. Volume versus temperature graphs for a given mass of an ideal gas are shown in Fig. at two different values of constant pressure. What can be inferred about relation between P_1 & P_2 ?

- (A) $P_1 > P_2$
- (B) $P_1 = P_2$
- (C) $P_1 < P_2$
- (D) data is insufficient



Ans. Option (A) is correct.

Explanation: $V \propto T$

$$\frac{V}{T} = \text{constant} = \text{slope of the graph}$$

Again,
$$V = \frac{nRT}{P}$$

So, as slope increases, P decreases.

In graph, slope at P_2 is more than slope at P_1 ,
 $\therefore P_1 > P_2$



Very Short Answer Type Questions

Q. 6. Calculate the number of atoms in 39.4 g gold. Molar mass of gold is 197g mol⁻¹.

Ans. ∴ Molar mass of gold is 197 g mol⁻¹, the number of atoms = 6.023 × 10²³

$$\therefore \text{No. of atoms in 39.4g} = \frac{6.023 \times 10^{23} \times 39.4}{197} = 1.2 \times 10^{23}$$

Q. 7. The molecules of a given mass of a gas have root mean square speeds of 100 ms⁻¹ at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?

Ans.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{P_2 T_1}{P_1 T_2} = \frac{2 \times 300}{400} = \frac{3}{2}$$

$$P_1 = \frac{1}{3} \frac{M}{V_1} \overline{C_1^2}$$

$$P_2 = \frac{1}{3} \frac{M}{V_2} \overline{C_2^2}$$

$$\therefore \frac{P_1}{P_2} = \frac{V_1 \overline{C_1^2}}{V_2 \overline{C_2^2}}$$

or

$$\overline{C_2^2} = \frac{V_2 P_2 \overline{C_1^2}}{V_1 P_1}$$

$$\text{or rms speed} = \sqrt{\overline{C_2^2}} = \sqrt{\frac{2}{3}} \sqrt{2} \sqrt{100^2}$$

$$\therefore \text{rms speed} = \frac{200}{\sqrt{3}} \text{ ms}^{-1}$$

Q. 8. A gas mixture consists of 2.0 moles of oxygen and 4.0 moles of neon at temperature T. Neglecting all vibrational modes, calculate the total internal

energy of the system. (Oxygen has two rotational modes.)

Ans. O₂ has 5 degrees of freedom. Therefore, energy per

$$\text{mole} = \frac{5}{2} RT$$

$$\therefore \text{For 2 moles of O}_2, \text{ energy} = 5RT$$

Neon has 3 degrees of freedom

$$\therefore \text{Energy per mole} = \frac{3}{2} RT$$

$$\therefore \text{For 4 mole of neon, energy} = 4 \times \frac{3}{2} RT = 6RT$$

$$\therefore \text{Total energy} = 11RT.$$

Q. 9. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 Å and 2 Å. The gases may be considered under identical conditions of temperature, pressure and volume.

$$\text{Ans.} \quad \lambda \propto \frac{1}{d^2}$$

$$d_1 = 1 \text{ \AA}, d_2 = 2 \text{ \AA}$$

$$\lambda_1 : \lambda_2 = 4 : 1$$



Commonly Made Error

Students can not evaluate the ratio of mean free path for two gases.



Answering Tip

Students should be acquainted with the concept of mean free path of the gas molecules.



Short Answer Type Questions

Q. 10. The container shown in Fig. has two chambers, separated by a partition, of volumes is V₁ = 2.0 litre and V₂ = 3.0 litre. The chambers contain μ₁ = 4.0 and μ₂ = 5.0 moles of a gas at pressures P₁ = 1.00 atm and P₂ = 2.00 atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.

Ans. V₁ = 2.0 litre, V₂ = 3.0 litre

$$\mu_1 = 4.0 \text{ moles}, \mu_2 = 5.0 \text{ moles}$$

$$P_1 = 1.00 \text{ atm}, P_2 = 2.00 \text{ atm}$$

$$P_1 V_1 = \mu_1 RT_1, P_2 V_2 = \mu_2 RT_2$$

V ₁	V ₂
μ ₁	μ ₂
P ₁	P ₂

$$\mu = \mu_1 + \mu_2 \quad V = V_1 + V_2$$

$$\text{For 1 mole,} \quad PV = \frac{2}{3} E$$

$$\text{For } \mu_1 \text{ moles,} \quad P_1 V_1 = \frac{2}{3} \mu_1 E_1$$

$$\text{For } \mu_2 \text{ moles,} \quad P_2 V_2 = \frac{2}{3} \mu_2 E_2$$

$$\text{Total energy is } (\mu_1 E_1 + \mu_2 E_2) = \frac{3}{2} (P_1 V_1 + P_2 V_2)$$

When partition is removed

$$PV = \frac{2}{3} E_{\text{total}} = \frac{2}{3} \mu E_{\text{per mole}}$$

$$P(V_1 + V_2) = \frac{2}{3} \times \frac{3}{2} (P_1V_1 + P_2V_2)$$

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

$$= \left(\frac{1.00 \times 2.0 + 2.00 \times 3.0}{2.0 + 3.0} \right) \text{atm}$$

$$= \frac{8.0}{5.0} = 1.60 \text{ atm.}$$



Commonly Made Error

- Students can not evaluate the pressure of the mixture of two gases when the partition is removed.



Answering Tip

- Students should be acquainted with relation between pressure and energy.

Q. 11. When air is pumped into a cycle tyre the volume and pressure of the air in the tyre both are increased. What about Boyle's law in this case?

Ans. When air is pumped, more molecules are pumped in. Boyle's law is stated for situation where number of molecules remain constant. So, Boyle's law is not applicable here.

Q. 12. A balloon has 5.0 g mole of helium at 7°C. Calculate (a) the number of atoms of helium in the balloon, (b) the total internal energy of the system.

Ans. $\mu = 5.0T$
 $T = 280K$

$$\text{No. of atoms} = \mu N_A = 5.0 \times 6.02 \times 10^{23}$$

$$= 30 \times 10^{23}$$

Average kinetic energy per molecule = $\frac{3}{2}kT$

$$\therefore \text{Total internal energy} = \frac{3}{2}kT \times N$$

$$= \frac{3}{2} \times 30 \times 10^{23} \times 1.38 \times 10^{-23} \times 280$$

$$= 1.74 \times 10^4 \text{ J}$$

Q. 13. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP.

Ans. Volume occupied by 1 gram mole of gas at NTP = 22400cc

$$\therefore \text{Number of molecules in 1cc of hydrogen}$$

$$= \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$$

As each diatomic molecule has 5 degrees of freedom, hydrogen being diatomic also has 5 degrees of freedom.

$$\therefore \text{Total no. of degrees of freedom} = 5 \times 2.688 \times 10^{19}$$

$$= 1.344 \times 10^{20}$$



Commonly Made Error

- Students can not evaluate the degrees of freedom for the molecules of hydrogen.



Answering Tip

- Students should be familiar with the concept of degrees of freedom for monatomic, diatomic and triatomic molecules.



Long Answer Type Questions

Q. 14. Explain why

- there is no atmosphere on moon.
- there is fall in temperature with altitude.

Ans. (a) Acceleration due to gravity in moon = $\frac{1}{6}$ of the

$$\text{acceleration due to gravity of earth} \approx \frac{1.63\text{m}}{\text{s}^2}$$

$$\text{So, escape velocity in moon} = \sqrt{(2gR)}$$

$$= \sqrt{(2 \times 1.63 \times 1.7 \times 10^6)} \text{ [considering radius of moon} = 1.6 \times 10^6 \text{ m]}$$

$$= 2.36 \times 10^6 \text{ m/s}$$

$$= 2.36 \times 10^3 \text{ km/s}$$

Considering the initial temperature 300K and mass of hydrogen atom = $m = 1.67 \times 10^{-27} \text{ kg}$

$$V_{\text{rms}} \text{ of Hydrogen} = \sqrt{(3k_B T/m)}$$

$$= \sqrt{\left(\frac{3 \times 1.38 \times 10^{-23} \times 300}{1.67 \times 10^{-27}} \right)}$$

$$= 2.727 \text{ m/s}$$

Since, v_{rms} is greater than escape velocity and gravitational force is low, the gas will escape.

The distance of moon from sun is almost equal to that of earth and sometimes less also. So, it receives same amount of energy of sun as earth receives. The temperature rises, v_{rms} rises and more and more gas escapes. Thus over a long time the moon has lost most of its atmosphere.

- As the molecules move higher their potential energy increases and hence kinetic energy decreases and hence temperature reduces.

At greater height more volume is available and gas expands and hence some cooling takes place.



Commonly Made Error

- ▶ Students can not reason out for the absence of atmosphere on the surface of moon.



Answering Tip

- ▶ Students should have an understanding about the escape velocity and r.m.s. velocity of molecules on the surface of moon.



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any 4 of the following questions on the basis of the same:

I. Atomic Hypothesis in Ancient India and Greece

Though John Dalton is credited with the introduction of atomic view point in modern science, scholars in ancient India and Greece conjectured long before the existence of atoms and molecules.

In the Vaisheshika school of thought in India founded by Kanada (Sixth century B.C.) the atomic picture was developed in considerable detail. Atoms were thought to be eternal, indivisible, infinitesimal and ultimate parts of matter. It was argued that if matter could be subdivided without an end, there would be no difference between a mustard seed and the Meru mountain.

The four kinds of atoms (Paramanu — Sanskrit word for the smallest particle) postulated were Bhoomi (Earth), Ap (water), Tejas (fire) and Vayu (air) that have characteristic mass and other attributes, were propounded.

Akasa (space) was thought to have no atomic structure and was continuous and inert.

Atoms combine to form different molecules (e.g., two atoms combine to form a diatomic molecule dvyanuka, three atoms form a tryanuka or a triatomic molecule), their properties depending upon the nature and ratio of the constituent atoms. The size of the atoms was also estimated, by conjecture or by methods that are not known to us. The estimates vary.

In Lalitavistara, a famous biography of the Buddha written mainly in the second century B.C., the estimate is close to the modern estimate of atomic size, of the order of 10^{-10} m.

In ancient Greece, Democritus (Fourth century B.C.) is best known for his atomic hypothesis. The word 'atom' means 'indivisible' in Greek. According to him, atoms differ from each other physically, in shape, size and other properties and this resulted in the different properties of the substances formed by their combination. The atoms of water were smooth and round and unable to 'hook' on to each other, which is why liquid /water flows easily. The atoms of earth were rough and jagged, so they held together to form hard substances. The atoms of fire were

thorny which is why it caused painful burns. These fascinating ideas, despite their ingenuity, could not evolve much further, perhaps because they were intuitive conjectures and speculations not tested and modified by quantitative experiments—the hallmark of modern science.

Q. 1. In Greek, "atom" means

- (A) Smallest (B) Indivisible
(C) Invisible (D) Unbreakable

Ans. Option (B) is correct.

Explanation: The word 'atom' means 'indivisible' in Greek.

Q. 2. In which biography the size of the atoms was estimated close to the modern estimate

- (A) biography of the Buddha
(B) biography of Jesus Christ
(C) biography of Muhammad
(D) biography of Tirthankara

Ans. Option (A) is correct.

Explanation: In Lalitavistara, a famous biography of the Buddha written mainly in the second century B.C., the estimate is close to the modern estimate of atomic size, of the order of 10^{-10} m.

Q. 3. The kinds of atoms which were propounded in ancient India are

- (A) Paramanu , Bhoomi , Ap, Tejas and Vayu
(B) Ap, Tejas and Vayu
(C) Bhoomi , Ap, Tejas and Vayu
(D) Bhoomi , Tejas and Vayu

Ans. Option (C) is correct.

Explanation: The four kinds of atoms (Paramanu—Sanskrit word for the smallest particle) postulated were Bhoomi (Earth), Ap (water), Tejas (fire) and Vayu (air) that have characteristic mass and other attributes, were propounded.

Q. 4. Which of the followings was thought to have no atomic structure and was continuous and inert?

- (A) Bhoomi (B) Vayu
(C) Tejas (D) Akash

Ans. Option (D) is correct.

Explanation: Akash (space) was thought to have no atomic structure and was continuous and inert.

Q. 5. In ancient Greece, who is best known for his atomic hypothesis?

- (A) Democritus (B) Plate
(C) Pythagoras (D) Aristotle

Ans. Option (A) is correct.

Explanation: In ancient Greece, Democritus (Fourth century B.C.) is best known for his atomic hypothesis.

B Subjective Questions

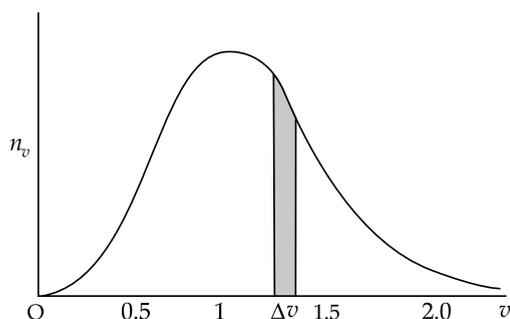
Maxwell Distribution Function

In a given mass of gas, the velocities of all molecules are not the same, even when bulk parameters like pressure, volume and temperature are fixed.

Collisions change the direction and the speed of molecules. However in a state of equilibrium, the distribution of speeds is constant or fixed.

Distributions are very important and useful when dealing with systems containing large number of objects. As an example consider the ages of different persons in a city. It is not feasible to deal with the age of each individual. We can divide the people into groups: children up to age 20 years, adults between ages of 20 and 60, old people above 60. If we want more detailed information we can choose smaller intervals, 0–1, 1–2, ..., 99–100 of age groups. When the size of the interval becomes smaller, say half year, the number of persons in the interval will also reduce, roughly half the original number in the one year interval. The number of persons $dN(x)$ in the age interval x and $x + dx$ is proportional to dx or $dN(x) = n_x dx$. We have used n_x to denote the number of persons at the value of x .

In a similar way the molecular speed distribution gives the number of molecules between the speeds v and $v + dv$. $dN(v) = 4p N a^3 e^{-bv^2} v^2 dv = n_v dv$. This is called Maxwell distribution.



Maxwell distribution of molecular speeds

The plot of n_v against v is shown in the figure. The fraction of the molecules with speeds v and $v + dv$

is equal to the area of the strip shown. The average of any quantity like v^2 is defined by the integral

$$\begin{aligned} \langle v^2 \rangle &= \frac{1}{N} \int v^2 dN(v) \\ &= \sqrt{\frac{3k_B T}{m}} \end{aligned}$$

which agrees with the result derived from more elementary considerations.

Q. 1. Which parameter of molecules of a given mass of gas is not constant even when parameters like pressure, volume, temperature are fixed? Why?

Ans. In a given mass of gas, the velocities of all molecules are not the same, even when bulk parameters like pressure, volume and temperature are fixed. Collisions change the direction and the speed of molecules.

Q. 2. When the speed of the molecules of a given mass of gas becomes constant?

Ans. In a given mass of gas, when bulk parameters like pressure, volume and temperature are fixed and the state of equilibrium is achieved, the distribution of speeds becomes constant or fixed.

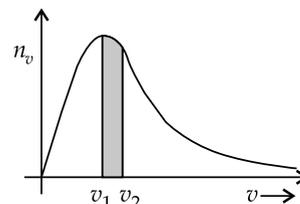
Q. 3. Write down the expression of molecular speed distribution for the number of molecules between the speeds v and $v + dv$.

Ans. The molecular speed distribution for the number of molecules between the speeds v and $v + dv$ is

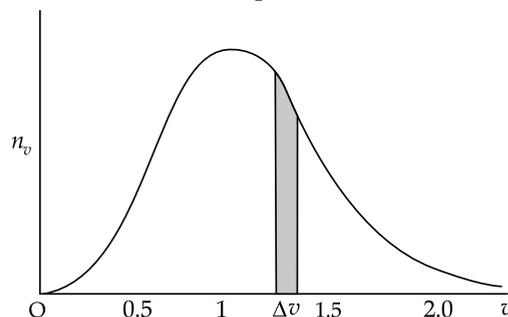
$$\begin{aligned} dN(v) &= 4p N a^3 e^{-bv^2} v^2 dv \\ &= n_v dv. \end{aligned}$$

Q. 4. Plot n_v against v showing the fraction of the molecules with speeds between v_1 and v_2 by shaded area.

Ans.



Q. 5. Study the given graph and determine the number of molecules at zero speed.



Ans. Since, in the graph n_v and v both starts from zero, there will be no molecules at zero speed.

UNIT-X OSCILLATIONS AND WAVES

Study Time
 Maximum Time: 3 : 00 hours
 Maximum Questions: 79

CHAPTER

13

OSCILLATIONS



Syllabus

Periodic motion - time period, frequency, displacement as a function of time, periodic function and their application; Simple harmonic motion (S.H.M.) and its equation, phase, oscillations of a loaded spring- restoring force and force constant, energy in S.H.M., Kinetic and potential energies, simple pendulum derivation of expression for its time period.



Revision Notes

- **Harmonic Oscillations :** Those oscillations which can be expressed in terms of single harmonic function. i.e., (sine function or cosine function).

$$y = a \sin \omega t \text{ or } y = a \cos \omega t.$$

- **Non-Harmonic Oscillations :** Those oscillations which cannot be expressed in terms of single harmonic function i.e.,

$$y = a \sin \omega t + b \sin 2 \omega t.$$

- **Periodic Functions :** Those functions which are used to represent periodic motion i.e.,

$$f(t) = f(t + T) = f(t + 2T).$$

sine & cosine functions are periodic functions.

- **Phase :** Phase of vibrating particle at any instant is a physical quantity which completely expresses the position and direction of motion of particle at that instant with respect to its mean position.

- **Some Facts :**

- (a) In oscillatory motion, the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalised equation of motion of the vibrating particle.
- (b) When the displacement of the particle executing a vibratory motion is represented by $y = a \sin (\omega t + \phi)$, then $(\omega t + \phi)$ is called phase of the vibrating particle.
- (c) ϕ is called the initial phase of the vibrating particle.

- **Simple Harmonic Motion :** It is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean (i.e., equilibrium) position under a restoring force, which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean (i.e., equilibrium) position at that instant, i.e.,

$$F = -ky$$

where k is known as force constant. Here negative sign represent that the restoring force (F) is always directed towards the mean position.

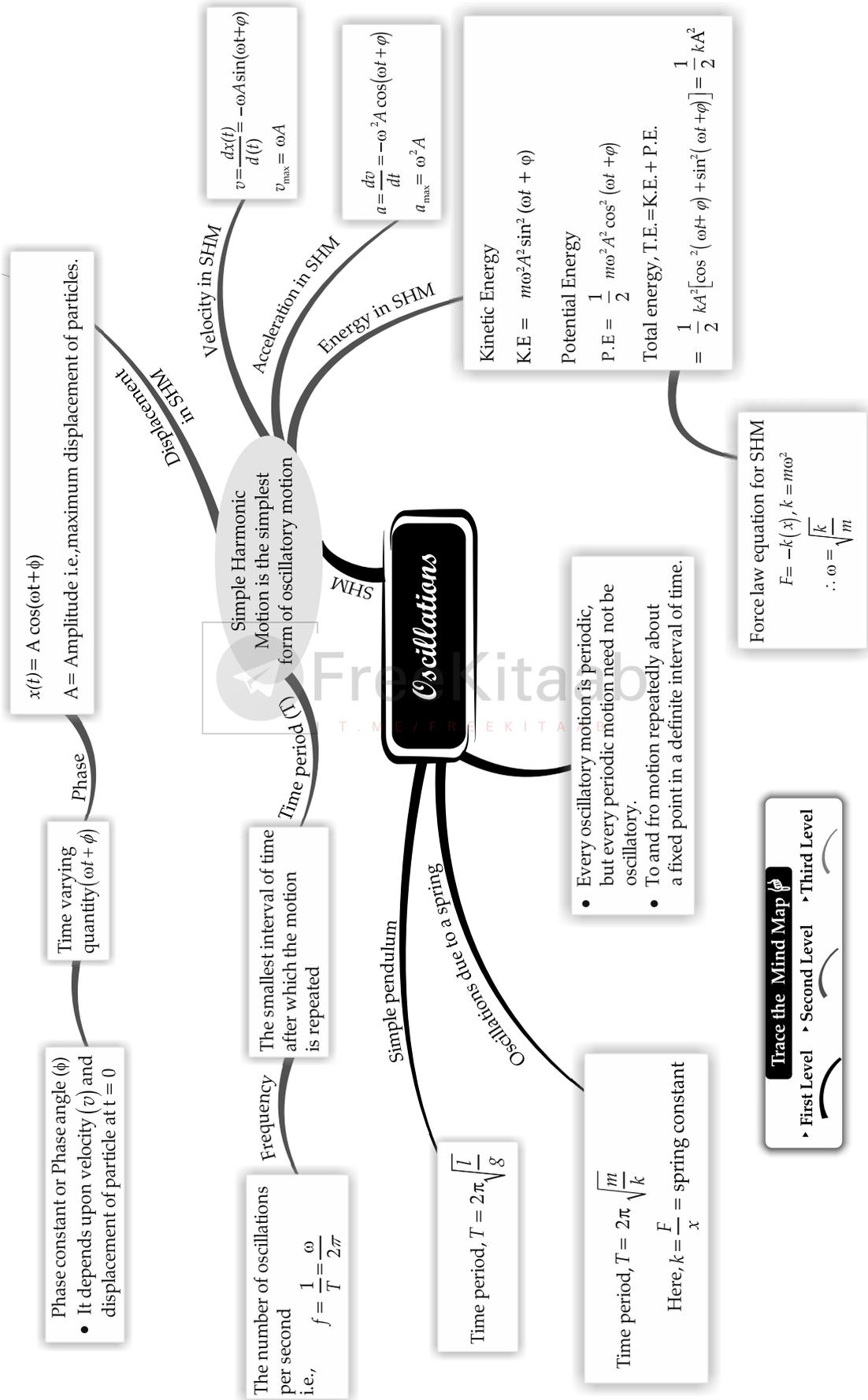
- **Geometrical interpretation of S.H.M. :** S.H.M. is defined as the projection of a uniform circular motion on any diameter of a circle of reference.

- (a) S.H.M. may be linear and angular S.H.M.
- (b) The linear S.H.M. is always along a straight line about a fixed point on a line, whereas the angular S.H.M. is always along an arc of a circle about a fixed point on the arc.
- (c) The linear S.H.M. is controlled by force law, where $F = -ky$, where k is the restoring force constant, i.e., force per unit displacement.

Scan to know more about this topic



Simple harmonic motion



- (d) The angular S.H.M. is controlled by torque law, where $\tau = -C\theta$, where C is the restoring torque constant, i.e., restoring torque per unit twist.
- **Characteristics of S.H.M. :**
- (a) **Displacement :** The displacement of a particle executing linear S.H.M. at an instant is defined as the distance of the particle from the mean position at that instant.
- (b) **Velocity :** is defined as the time rate of change of the displacement of the particle at the given instant.
- (c) **Amplitude :** The maximum displacement on either side of mean position.
- (d) **Acceleration :** It is defined as the time rate of change of the velocity of the particle at the given instant.
- (e) **Time Period :** It is defined as the time taken by the particle executing S.H.M. to complete one vibration.
- **Restoring Force & Force Constant :** Force constant is the force required to give unit displacement to the body.

$$F = -ky \qquad \text{Here, } k \text{ is force constant.}$$

- **Simple Pendulum :** It is most common example of S.H.M. An ideal simple pendulum consists of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

$$\text{Time period, } = 2\pi\sqrt{\frac{l}{g}}$$

- **Energy of S.H.M :** A particle executing S.H.M. possesses two types of energy :
- (a) **Potential Energy :** This energy is on account of the displacement of the particle from its mean position.
- (b) **Kinetic Energy :** This energy is on account of the velocity of the particle.

Key Words

- **Periodic motion :** The motion which is identically repeated after a fixed interval of time.
- **Oscillatory motion or vibratory motion :**
The motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position), in a definite interval of time.
- Some terms related to periodic motion :**
- Time period :** It is the least interval of the time after which the periodic motion of a body repeats itself.
- Frequency :** It is defined as the no. of periodic motions executed by the body per second.
- Angular frequency :** It is equal to the product of frequency of the body with factor 2π . i.e., $\omega = 2\pi\nu$.
- Displacement :** It is the change in position under consideration with time in a periodic motion.

Key Formulae

- **Periodic Motion.**

(a) **Frequency,** $v = \frac{1}{T}$

(b) **Angular Frequency,** $\omega = v \times 2\pi$
or $\omega = \frac{2\pi}{T}$

➤ $\text{Phase} = (\omega t + \phi) = \left(\frac{2\pi t}{T} + \phi\right) = (2\pi\nu t + \phi)$

- **Simple Harmonic Motion :**

- (a) **Differential Equation,**

(i) **Linear S.H.M.** $= \frac{d^2y}{dt^2} + \omega^2 y = 0$, where $\omega^2 = k/m$, here, m is the mass of the body

(ii) **Angular S.H.M.** $= \frac{d^2\theta}{dt^2} + \omega^2 \theta = 0$, where $\omega^2 = C/I$, here, I = moment of inertia

- (b) **General equation—**

(i) **Linear S.H.M.** $y = y_0 \sin(\omega t + \phi)$
(ii) **Angular S.H.M.** $\theta = \theta_0 \sin(\omega t + \phi_0)$

(c) Displacement,
or

$$y = A \sin \omega t$$

(d) Velocity,

$$y = A \cos \omega t$$

$$v = \omega \sqrt{A^2 - y^2}$$

(e) Acceleration,

$$a = \frac{dv}{dt} = -\omega^2 A \sin \omega t$$

(f) Time Period,

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \quad \text{or} \quad 2\pi \sqrt{\frac{l}{C}}$$

➤ Time period :

For simple pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Potential energy of SHM:

$$U = \frac{1}{2} m \omega^2 y^2$$

Kinetic energy of SHM:

$$K = \frac{1}{2} m \omega^2 (a^2 - y^2)$$

Total energy:

$$E = \frac{1}{2} m \omega^2 a^2$$



Mnemonics

Concept: Time period of a simple pendulum.

Mnemonics: Tanu Paul speaks lot of Asian languages.

Interpretation:

T - Time period

P - proportional to

S - square root of

L - length

O - on

A - acceleration due to gravity

Time period

$$\propto \sqrt{\frac{\text{Length}}{\text{Acceleration due to gravity}}}$$



Very Short Answer Type Questions

(1 mark each)

Q. 1. What is an epoch ? Name the unit in which it is measured. [R]

Ans. The initial phase of the oscillating particle is called epoch. It is expressed in radians. 1

Q. 2. What is seconds pendulum ? [R] [A] [I]

Ans. A pendulum, whose time period is 2 second is called a seconds pendulum. 1

Q. 3. What is the basic condition for the motion of a particle to be S.H.M. ? [R]

Ans. The necessary and sufficient condition for motion to be simple harmonic is that the restoring force must be linear, i.e., $F = -ky$ or torque, $\tau = -C\theta$. 1



Commonly Made Error

- ▶ Student could not state the basic condition for the motion of a particle to be S.H.M.



Answering Tip

- ▶ The brief understanding of Simple Harmonic Motion should be there.

Q. 4. Can a motion be periodic but not oscillatory ? If yes, give an example. If not, explain why ? [A]

Ans. Yes, uniform circular motion is the example of it.1

Q. 5. How will the period of a simple pendulum change when its length is doubled ? [U] (DDE)

Ans. The time period becomes $\sqrt{2}$ times the original value since $T \propto \sqrt{l}$. 1



Short Answer Type Questions-I

(2 marks each)

Q. 1. When a pendulum clock gains time, what adjustments should be made ? [U] [A] [I]

Ans. When a pendulum clock gains time, it means it has gone fast, i.e., it makes more vibrations per day than required. 1

This shows that the time period of oscillations has decreased. Therefore, to correct it, the length of pendulum should be properly increased. 1

Q. 2. What would happen to the motion of an oscillating system if the sign of the force term in equation $F = -kx$ is changed ? [U]

Ans. When the sign in the force equation is changed from ' $-ve$ ' to ' $+ve$ ', the force and hence acceleration will not be opposite to displacement. Due to which the particle will not oscillate but will

be accelerated in the direction of displacement. As a result of it, the motion will become a linearly accelerated motion. 2



Commonly Made Error

- ▶ Students can not explain the consequences to the motion in case the sign of the force is changed.



Answering Tip

- ▶ Students should have an understanding about forces supporting motion and forces opposing motion.

Q. 3. Show that in S.H.M., the acceleration is directly proportional to its displacement at the given instant. □ (KVS 2008; NCT 2008)

Ans. In S.H.M., the displacement of the particle at an instant is given by :

$$y = r \sin \omega t.$$

Velocity, $v = \frac{dy}{dt} = r\omega \cos \omega t$

Acceleration, $a = \frac{dv}{dt} = -\omega^2 r \sin \omega t$

$$= -\omega^2 y. \quad 1$$

From the above equation, we note that $a \propto y$, i.e., acceleration in S.H.M. at an instant is directly proportional to the displacement of the particle from the mean position at that instant and opposite in direction. 1

Q. 4. The periodic time of a body executing S.H.M. is 2 sec. After how much interval from $t = 0$, will its displacement be half of its amplitude ? □

Ans. Here, $T = 2s$; $t = ?$, $y = a/2$

Now, $y = a \sin \omega t = a \sin \pi t$ ($\because T = 2s$) 1/2

$$\therefore \frac{a}{2} = a \sin \frac{2\pi}{T} t = a \sin \pi t \quad 1/2$$

or $\sin \pi t = \frac{1}{2}$

$$\therefore \sin \pi t = \sin \frac{\pi}{6} \quad 1/2$$

or $\pi t = \frac{\pi}{6}$ or, $t = \frac{1}{6} s$ 1/2

Q. 5. Two exactly identical pendulums are oscillating with amplitude 2 cm and 6 cm. Calculate the ratio of their energies of oscillations. □

Ans. Total energy of the bob of simple pendulum is given by $E = m\omega^2 r^2$, i.e., $E \propto r^2$, 1

So $\frac{E_1}{E_2} = \frac{r_1^2}{r_2^2} = \left(\frac{2}{6}\right)^2 = \frac{1}{9}$ 1



Short Answer Type Questions-II

(3 marks each)

Q. 1. The periodic time of a mass suspended by a spring (force constant k) is T . If the spring is cut in three equal pieces, what will be the force constant of each part ? If the same mass be suspended from one piece, what will be the periodic time ? □

Ans. Consider the spring be made of combination of three springs in series each of spring constant k . The effective spring constant K is given by :

$$\frac{1}{K} = \frac{1}{k} + \frac{1}{k} + \frac{1}{k} = \frac{3}{k}$$

or $K = \frac{k}{3}$

or $k = 3K$ 1

\therefore Time period of vibration of a body attached to the end of this spring,

$$\begin{aligned} T &= 2\pi \sqrt{\frac{m}{K}} \\ &= 2\pi \sqrt{\frac{m}{(k/3)}} \\ &= 2\pi \sqrt{\frac{3m}{k}} \quad \dots(i) \quad 1 \end{aligned}$$

When the spring is cut into three pieces, each having spring constant = k .

Time period of vibration of a body attached to the end of this spring,

$$T_1 = 2\pi \sqrt{\frac{m}{k}} \quad \dots(ii)$$

From the equation (i) and (ii),

$$\frac{T_1}{T} = \frac{1}{\sqrt{3}}$$

$$\therefore T_1 = \frac{T}{\sqrt{3}} \quad 1$$

Q. 2. A simple harmonic motion is represented by $x(t) = 10 \sin (20t + 0.5)$ □

Find its amplitude, frequency and initial phase.

Ans. Comparing the given equation with standard equation of S.H.M.

$$x(t) = a \sin (\omega t + \phi_0) \quad 1$$

(i) Amplitude, $a = 10 m$ 1/2

(ii) Angular frequency, $\omega = 20 \text{ rad s}^{-1}$ 1/2

(iii) Frequency is given by $v = \frac{\omega}{2\pi} = 3.18 \text{ Hz}$ 1/2

(iv) Initial phase, $\phi_0 = 0.5 \text{ rad.}$ 1/2



Commonly Made Error

- Students can not evaluate amplitude, frequency and initial phase from a given simple harmonic motion.



Answering Tip

- Students should be familiar with the standard equation of S.H.M.



Objective Type Questions

(1 marks each)

A Multiple Choice Questions

Q. 1. A particle is executing simple harmonic motion. When its displacement is x , its total energy is

- (A) Proportional to x
- (B) Proportional to $\frac{1}{x}$
- (C) Proportional to x^2
- (D) Independent of x

Ans. Option (D) is correct.

Explanation: Total energy = $\frac{1}{2} m\omega^2 A^2$

So, it is independent of x .

Q. 2. Which of the following statement is incorrect?

- (A) Every periodic motion is oscillatory, but every oscillatory motion need not be periodic.
- (B) Simple harmonic motion is the simplest form of oscillatory motion.
- (C) There is no significant difference between oscillations and vibrations. It seems that when the frequency is small, we call it oscillation, while when the frequency is high, we call it vibration.
- (D) Oscillating bodies eventually come to rest at their equilibrium positions.

Ans. Option (A) is correct.

Explanation: Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. Example: Circular motion is a periodic motion, but it is not oscillatory.

Q. 3. A particle moves such that its acceleration a is given by $a = -bx$, where x is the displacement from equilibrium position and b is a constant.

The period of oscillation is

- (A) $\frac{2\pi}{\sqrt{b}}$
- (B) $\frac{\sqrt{b}}{2\pi}$
- (C) $\frac{2\pi}{\sqrt{b}}$
- (D) $2\pi b$

Ans. Option (A) is correct.

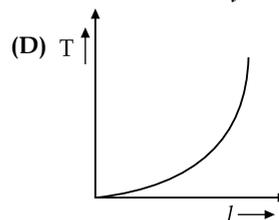
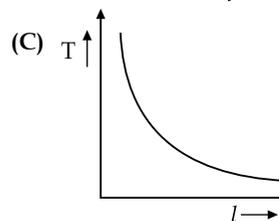
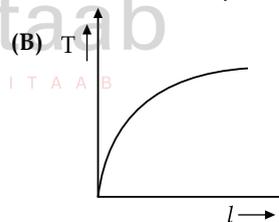
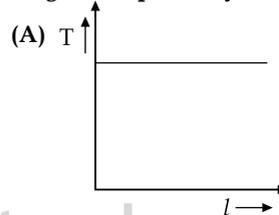
Explanation: Time period = $2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$

$$= 2\pi \sqrt{\left(\frac{x}{bx}\right)}$$

$$= 2\pi \sqrt{\left(\frac{1}{b}\right)}$$

$$= \frac{2\pi}{\sqrt{b}}$$

Q. 4. In case of a simple pendulum, time period versus length is depicted by



Ans. Option (B) is correct.

Explanation: For SHM, $T^2 \propto l$

It is similar to equation of parabola, $y^2 = 4ax$

Q. 5. A particle executes S.H.M. with a period of 8 second and amplitude of 4 cm. Its maximum speed in cm/sec is

- (A) π
- (B) 2π
- (C) 3π
- (D) $\frac{\pi}{2}$

Ans. Option (a) is correct.

Explanation: $v_{\max} = A\omega$

or $v_{\max} = A \frac{2\pi}{T}$

or $v_{\max} = 4 \times \frac{2\pi}{8}$

$\therefore v_{\max} = \pi$

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A
- (C) A is true but R is false
- (D) A is false and R is true

Q. 1. Assertion (A): Any motion that repeats itself at regular intervals of time is called periodic motion.

Reason (R): The smallest interval of time after which the motion is repeated is called its period.

Ans. Option (B) is correct.

Q. 2. Assertion (A): One of the simplest periodic functions is $f(t) = A \cos \omega t$.

Reason (R): If the argument of $A \cos \omega t$ is increased by an integral multiple of 2π radians, the value of the function remains the same.

Ans. Option (A) is correct.

Explanation: The function that repeats itself at regular intervals of time is called periodic function.

$A \cos 0 = A \cos 2\pi = A \cos 4\pi \dots\dots\dots$

Hence, the value of $A \cos \omega t$ repeats as the argument is increased by an integral multiple of 2π radians. Hence, $A \cos \omega t$ is a periodic function. So, the assertion and reason both are true and the reason explains the assertion.

Q. 3. Assertion (A): The total mechanical energy of a harmonic oscillator is independent of time.

Reason (R): The potential energy of a particle executing simple harmonic motion is periodic.

Ans. Option (b) is correct.



NCERT Corner

Q. 1. Which of the following example represent periodic motion :

- (i) A swimmer completing one (return) trip from one bank of a river to other and back,
- (ii) A freely suspended bar magnet displaced from its N-S direction and released,
- (iii) A hydrogen molecule rotating about its centre of mass.
- (iv) An arrow released from a bow.

Ans. (i) It is not a periodic motion. Though the motion of a swimmer is to and fro but will not have a definite period.

(ii) It is a periodic motion because a freely suspended magnet if once displaced from N-S direction and let it go, it oscillates about its position. Hence it is S.H.M. also.

(iii) It is a periodic motion.

(iv) It is not a periodic motion.

Q. 2. Which of the following examples represent (nearly) simple harmonic motion, and which represent periodic but not S.H.M. :

- (i) The motion of earth about its axis.
- (ii) Motion of an oscillating mercury column in a U-tube.
- (iii) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lowermost position.
- (iv) General vibration of a polyatomic molecule about its equilibrium position.

Ans. (i) It is periodic but not S.H.M. because it is not to and fro motion about a fixed point.

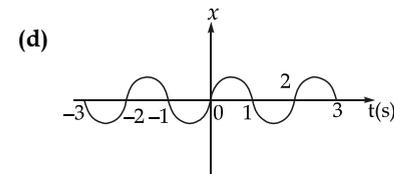
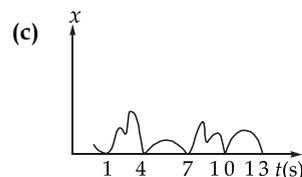
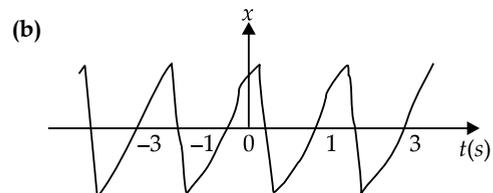
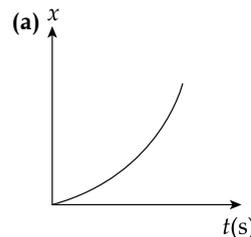
(ii) It is S.H.M.

(iii) It is S.H.M.

(iv) It is a periodic motion but not S.H.M. A polyatomic gas molecule has a no. of natural frequencies and its general motion is the resultant of S.H.M. of no.

of different frequencies. The resultant motion is periodic but not S.H.M.

Q. 3. Given below are four $x-t$ plots for linear motion of a particle. Which of the plots represent periodic motion ? What is the period of motion (in case of periodic motion) ?



Velocity, $v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$

or $\omega = -A\omega \sin(\pi \times 0 + \phi)$

$$1 = -A \sin \phi \text{ or } A \sin \phi = -1 \quad \text{(ii)}$$

Squaring the eqⁿ. (s) (i) & (ii), then adding -

$$A^2(\cos^2 \phi + \sin^2 \phi) = 1 + 1 = 2$$

$$A^2 = 2 \text{ or } A = \sqrt{2} \text{ cm}$$

Now, Dividing eqⁿ. (ii) by eqⁿ. (i).

$$\tan \phi = -1$$

$$\phi = -\frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

If we choose sine function instead of cosine function, then,

$$x(t) = B \sin(\omega t + \alpha) \quad \text{(iii)}$$

$$x = 1 \text{ cm}$$

or $1 = B \sin(0 + \alpha)$

$$B \sin \alpha = 1 \quad \text{(iv)}$$

Velocity,

$$v(t) = \frac{dx(t)}{dt} = B\omega \cos(\omega t + \alpha) \quad \text{[using (iii)]}$$

At $t = 0$, $v(t) = \omega \text{ cm/s}$

$$\omega = +B\omega \cos(0 + \alpha)$$

$$B \cos \alpha = +1 \quad \text{(v)}$$

Squaring (iv) & (v), then adding -

$$B^2 \sin^2 \alpha + B^2 \cos^2 \alpha = 1 + 1$$

or, $B^2(\sin^2 \alpha + \cos^2 \alpha) = 2$

$$\therefore B = \pm\sqrt{2} \text{ cm}$$

Hence, amplitude of motion = $\sqrt{2} \text{ cm}$.

Now, Dividing equation (iv) by (v),

$$\frac{B \sin \alpha}{B \cos \alpha} = \frac{1}{1} \text{ or, } \tan \alpha = 1 \text{ or, } \alpha = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}$$

Q. 8. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s, what is the weight of the body?

Ans. Given : mass, $m = 50 \text{ kg}$,
maximum extension, $y = 20 - 0 = 20 \text{ cm}$
 $= 0.2 \text{ m}$

Time, $t = 0.6 \text{ s}$.

Force, $F = mg = 50 \times 9.8 \text{ N}$

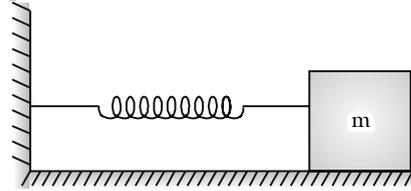
$$\therefore k = \frac{F}{y} = \frac{50 \times 9.8}{0.2} = 2450 \text{ N/m.}$$

Now, $T = 2\pi\sqrt{\frac{m}{k}}$ or $m = \frac{T^2 k}{4\pi^2}$

$$m = \frac{(0.6)^2 \times 2450}{4 \times (3.14)^2} = 22.36 \text{ kg}$$

so, weight of the body = mg
 $= 22.36 \times 9.8$
 $W = 219.1 \text{ N.}$

Q. 9. A spring having a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.



Ans. (i) Using, $v = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$, we get

$$v = \frac{1}{2\pi} \sqrt{\frac{1200}{3}}$$

$$= 3.18 \text{ s}^{-1}$$

(ii) Maximum acceleration = $r\omega^2$
 $= r(2\pi v)^2$
 $= 4\pi^2 v^2 r$
 $= 4\pi^2 (3.18)^2 \times 0.02$
 $= 7.98 \text{ ms}^{-2}$

(iii) Maximum speed = $r\omega$
 $= r2\pi v$
 $= 2\pi \times 3.18 \times 0.02$
 $= 0.4 \text{ ms}^{-1}$.

Q. 10. In the above question, let us take the position of mass when the spring is in natural length, i.e. unstretched as $x = 0$, and the direction from left to right as the positive direction of x -axis, Given x as a function of time t for the oscillating mass if at the moment we start the stop watch ($t = 0$), the mass is :

- at the mean position,
- at the maximum stretched position, and
- at the maximum compressed position.

In what way do these functions for S.H.M. differ from each other, in frequency, in amplitude or the initial phase ?

Ans. (a) When $t = 0$, the mass is at mean position.

Using, $x = r \sin(\omega t + \phi)$,

$$\omega = 2\pi f$$

$$= \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}}$$

$$= 20 \text{ rad s}^{-1}$$

We get $x = r \sin(20 \times t + 0)$
 $= 2 \sin 20 t$

(b) at $t = 0$, mass is at extreme stretched position.

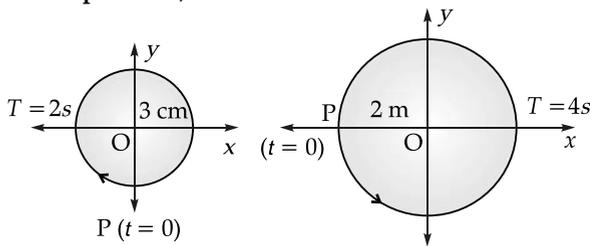
Then, $x = 2 \sin\left(20 \times t + \frac{\pi}{2}\right)$
 $= 2 \cos 20t$

(c) at $t = 0$, mass is at extreme compressed position

$$\begin{aligned} \text{Then } x &= 2\sin\left(20 \times t + \frac{3\pi}{2}\right) \\ &= -2\cos 20t. \end{aligned}$$

These functions differ in initial phase.

Q. 11. Figure below corresponds to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e., clockwise or anti clockwise) are indicated on each figure. Obtain the corresponding simple harmonic motion of the x -projection of the radius vector of the revolving particle P, in each case.



Ans. (a) Here, at $t = 0$, OP makes an angle $\frac{\pi}{2}$ with

x -axis. As motion is clockwise, so $\phi = \frac{+\pi}{2}$ radian.

So, the x -projection of OP at time t will give us the equation of S.H.M. given by.

$$\begin{aligned} x &= A\cos\left(\frac{2\pi t}{T} + \phi\right) \\ &= 3\cos\left(\frac{2\pi t}{2} + \frac{\pi}{2}\right) \\ &\quad (\because A = 3 \text{ cm}, T = 2 \text{ s}) \end{aligned}$$

$$\begin{aligned} \text{or } x &= 3\cos\left(\pi + \frac{\pi}{2}\right) \\ &= -3\sin \pi t \quad (x \text{ is in cm}) \end{aligned}$$

$\therefore x = -3\sin \pi t$ which is the equation of S.H.M.

(b) Here $T = 4 \text{ s}, A = 2 \text{ m}$

At $t = 0$, OP makes an angle π with the positive direction of x -axis, i.e., $\phi = +\pi$.

Hence, the x -projection of OP at time t will give us the equation of S.H.M.

$$\begin{aligned} \text{as : } x &= A\cos\left(\frac{2\pi}{T}t + \phi\right) \\ &= 2\cos\left(\frac{2\pi}{4}t + \pi\right) \\ &= -2\cos\left(\frac{\pi}{2}t\right) \end{aligned}$$

$$\text{or } x = -2\cos\left(\frac{\pi}{2}t\right) \text{ (m).}$$

which is the equation of S.H.M.

Q. 12. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anti-clockwise in every case:

(x is in cm and t is in s).

(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right)$

(b) $x = \cos\left(\frac{\pi}{6} - t\right)$

(c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

(d) $x = 2\cos \pi t$

Ans. (a)

$$x = -2\sin\left(3t + \frac{\pi}{3}\right) = 2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right) = 2\cos\left(3t + \frac{5\pi}{6}\right)$$

Comparing with standard SHM equation,

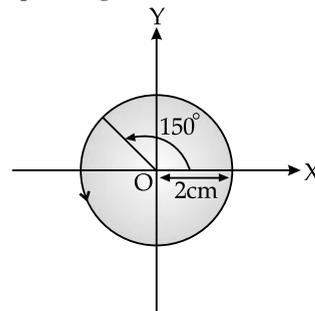
$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$$

\therefore Amplitude, $A = 2 \text{ cm}$

Phase angle, $\phi = \frac{5\pi}{6} = 150^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 3 \text{ rad/s}$

Corresponding reference circle is –



(b) $x = \cos\left(\frac{\pi}{6} - t\right)$

$$x = \cos\left(t - \frac{\pi}{6}\right) \quad [\because \cos(-\theta) = \cos\theta]$$

Comparing with standard SHM equation,

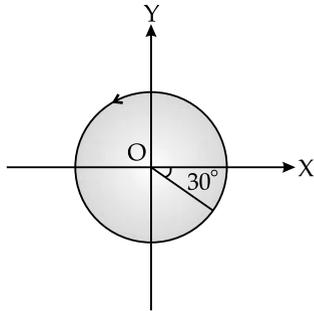
$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right),$$

\therefore Amplitude, $A = 1$

Phase angle, $\phi = \frac{-\pi}{6} = -30^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 1 \text{ rad/s}$.

Corresponding reference circle is –



(c) $x = 3 \sin\left(2\pi t + \frac{\pi}{4}\right)$

$$x = -3 \cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] \left[\because \cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta\right]$$

$$\text{or } x = -3 \cos\left(2\pi t + \frac{3\pi}{4}\right)$$

Comparing with standard SHM equation,

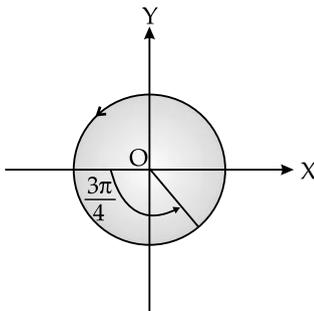
$$x = A \cos\left(\frac{2\pi}{T}t + \phi\right)$$

Amplitude, $A = 3 \text{ cm}$

Phase angle, $\phi = \frac{3\pi}{4} = 135^\circ$

Angular velocity, $\omega = \frac{2\pi}{T} = 2\pi \text{ rad/s}$

Corresponding reference circle is



(d) $x = 2 \cos \pi t$

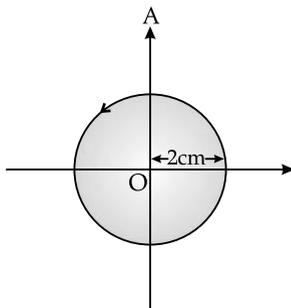
Comparing with standard SHM equation

∴ Amplitude, $A = 2 \text{ cm}$

Phase angle, $\phi = 0$

Angular velocity, $\omega = \pi \text{ rad/s}$

Corresponding reference circle is –



Commonly Made Error

- Students can not plot the reference circle for the simple harmonic motions.



Answering Tip

- The reference line is OX for positive amplitude and OX' for negative amplitude. From the reference line the initial phase angle is plotted.

Q. 13. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force applied at the free end stretches the spring.

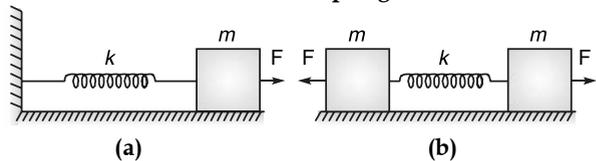


Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in figure (b) is stretched by the same force F .

- What is the maximum extension of the spring in the two cases ?
- If the mass in figure (a) and the two masses in figure (b) are released free, what is the period of oscillations in each case ?

Ans. (i) (a) Using $F = ky$,

The maximum extension,

$$y = \frac{F}{k}$$

- (b) Here, the springs have no rigid support, so, each mass behaves as a support for the spring against force applied on the other end.

∴ Maximum extension,

$$y = \frac{F}{k}$$

- (ii) (a) Here, time period,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- (b) In figure (b), we have a two body system of spring constant k and reduced mass,

$$m' = \frac{m \times m}{m + m} = \frac{m}{2}$$

Inertia factor = $m/2$

Spring factor = k

$$\therefore T = 2\pi\sqrt{\frac{m}{2k}}$$

Q. 14. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If

the piston moves with S.H.M. with an angular frequency of 200 rad/min. What is its maximum speed ?

Ans. Given, $a = \frac{1}{2} m$, $\omega = 200$ rad/min.

$$v_{\max} = a\omega = \frac{1}{2} \times 200 = 100 \text{ m/min.}$$

Q. 15. The acceleration due to gravity on the surface of Moon is 1.7 ms^{-2} . What is the time period of a simple pendulum on the surface of Moon if its time period on the surface of Earth is 3.5 s? (g on the surface of Earth is 9.8 ms^{-2})

Ans. Acceleration due to gravity on the surface of Moon, $g_m = 1.7 \text{ ms}^{-2}$
Acceleration due to gravity on the surface of Earth, $g_e = 9.8 \text{ ms}^{-2}$
Time period of a simple pendulum on Earth $T_e = 3.5 \text{ s}$

$$T_e = 2\pi\sqrt{\frac{l}{g_e}}$$

Where,

l is the length of the pendulum

$$\therefore l = \frac{T_e^2}{4 \times (3.14)^2 \times g_e} = \frac{(3.5)^2}{4 \times (3.14)^2 \times 9.8 \text{ m}}$$

The length of pendulum remains constant.

On Moon's surface, time period, $T_m = 2\pi\sqrt{\frac{l}{g_m}}$

$$T_m = 2\pi\sqrt{\frac{(3.5)^2 \times 9.8}{4 \times (3.14)^2 \times 1.7}} = 8.4 \text{ s}$$

Hence, the period of the simple pendulum on the surface of Moon is 8.4s.

Q. 16. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v . If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period ?

Ans. As the pendulum is oscillating in the radial direction, it is acted upon by an additional acceleration equal to the centripetal acceleration of the car on the circular track of radius R , i.e., v^2/R where v = uniform speed of the car. So, there are two accelerations acting on the pendulum in mutually perpendicular directions. i.e.,

(i) acceleration due to gravity (g)

(ii) a_c = centripetal acceleration (v^2/R)

When a be the resultant acceleration of the pendulum, then

$$a = \sqrt{g^2 + a_c^2}$$

$$a = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

$$a = \sqrt{g^2 + \frac{v^4}{R^2}}$$

When T be the time period of oscillation of the pendulum, then

$$T = 2\pi\sqrt{\frac{l}{a}}$$

$$= 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$$

$$\therefore T = 2\pi\sqrt{\frac{l}{\left(g^2 + \frac{v^4}{R^2}\right)^{\frac{1}{2}}}}$$

Q. 17. A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_l . The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$

Where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

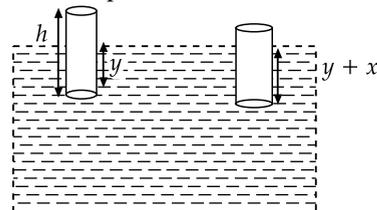
Ans. Let height of cylinder be y inside the liquid (Initially in equilibrium).

Then, weight of the cylinder = upthrust due to liquid displaced

$$\therefore Ah\rho g = Ay\rho_l g$$

Mass of the cylinder = $m = Ah\rho$

When cork of cylinder is slightly depressed by x and released, a restoring force is equal to additional upthrust, acts on it.



$$\text{Upthrust} = F = \rho_l (y + x) Ag$$

$$\text{Restoring force} = -(F - mg)$$

$$= -[(\rho_l \times (y + x) Ag) - \rho_l Ayg]$$

$$= -\rho_l Agx$$

$$= -kx \quad (\text{where } k = \rho_l Ag)$$

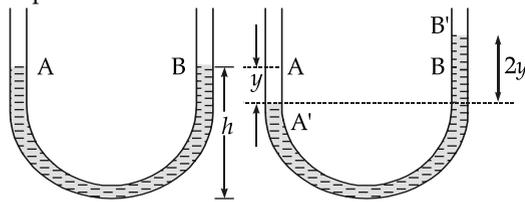
So, the cork exhibits a SHM.

$$\text{Time period} = T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

or
$$T = 2\pi\sqrt{\left(\frac{\rho Ah}{\rho_1 Ag}\right)}$$

Q. 18. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the liquid column of mercury in the U-tube executes simple harmonic motion. [DDE][KVS 2008]

Ans. The suction pump creates the pressure difference. Hence mercury rises in one limb of the U-tube. When it is removed a net force acts on the mercury column due to the difference in levels of mercury in the two limbs and therefore the mercury column executes S.H.M. which can be explained as follows :



The mercury contained in a vertical U-tube upto the level A and B in its two limbs.

Suppose ρ = density of the mercury.

L = Total length of the mercury column in both the limbs.

A = internal cross-sectional area of U-tube.

m = mass of mercury in U-tube = $LA\rho$.

Suppose the mercury be depressed in left limb to A' by a small distance y , then it rises by the same amount in the right limb to position B' .

\therefore Difference in levels in the two limbs
= $A'B' = 2y$.

\therefore Volume of mercury contained in the column of length $2y$ = $A \times 2y$

\therefore $m = A \times 2y \times \rho$.

When W = weight of liquid contained in the column of length $2y$.

Then $W = mg = A \times 2y \times \rho \times g$ $\frac{1}{2}$

This weight produced the restoring force (F) which tends to bring back the mercury to its equilibrium position.

$$F = -2Ay\rho g = -(2A\rho g)y$$

When a = acceleration produced in the liquid column, Then

$$a = \frac{F}{m} = -\frac{(2A\rho g)y}{LA\rho}$$

or
$$a = -\frac{2\rho y g}{L\rho} = -\frac{2\rho g}{2h\rho}y \quad \dots(i)$$

($\because L = 2h$)

where h = height of mercury in each limb. Now from (i) it is clear that $a \propto y$ and $-ve$ sign shows that it acts opposite to y , so the motion of mercury in U-tube is simple harmonic in nature having time period (T) given by

$$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{2h\rho}{2\rho g}}$$

$$= 2\pi\sqrt{\frac{h\rho}{\rho g}}$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

Q. 1. The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of

the particle is

- (A) simple harmonic with period $2\pi/\omega$.
- (B) simple harmonic with period π/ω .
- (C) periodic but not simple harmonic.
- (D) non - periodic.

Ans. Option (B) is correct.

Explanation:

$$\begin{aligned} y &= 3\cos\left(\frac{\pi}{4} - 2\omega t\right) = 3\cos\left[-\left(2\omega t - \frac{\pi}{4}\right)\right] \\ &= 3\cos\left(2\omega t - \frac{\pi}{4}\right) \quad \{\because \cos(-\theta) = \cos\theta\} \end{aligned}$$

This shows simple harmonic motion with time

$$\text{period } T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

Q. 2. The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is

- (A) non - periodic.
- (B) periodic but not simple harmonic.
- (C) simple harmonic with period $2\pi/\omega$.
- (D) simple harmonic with period π/ω .

Ans. Option (B) is correct.

Explanation: as $\frac{d^2y}{dt^2}$ is not $\propto (-y)$

\therefore It is not SHM but it is a periodic motion with period $\frac{2\pi}{\omega}$

Q. 3. The relation between acceleration and displacement of four particles are given below

- (A) $a_x = +2x$. (B) $a_x = +2x^2$.
(C) $a_x = -2x^2$. (D) $a_x = -2x$.

Which one of the particles is executing simple harmonic motion?

Ans. Option (D) is correct.

Explanation: because for SHM, $a_x \propto (-x)$

Q. 4. Motion of an oscillating liquid column in a U-tube is

- (A) periodic but not simple harmonic.
(B) non-periodic.
(C) simple harmonic and time period is independent of the density of the liquid.
(D) simple harmonic and time-period is directly proportional to the density of the liquid.

Ans. Option (C) is correct.



Commonly Made Error

- Students can not judge the motion of an oscillating liquid column in a U-tube.



Answering Tip

- Students should be familiar with the type of motion executed by different systems.

Q. 5. A particle is acted simultaneously by mutually perpendicular simple harmonic motions $x = a \cos \omega t$ and $y = a \sin \omega t$. The trajectory of motion of the particle will be

- (A) an ellipse.
(B) a parabola.
(C) a circle.
(D) a straight line.

Ans. Option (C) is correct.

Explanation:

$$x = a \cos \omega t \quad (i)$$

$$y = a \sin \omega t \quad (ii)$$

squaring and adding (i) & (ii),

we get

$$\begin{aligned} x^2 + y^2 &= a^2 \cos^2 \omega t + a^2 \sin^2 \omega t \\ &= a^2 \end{aligned}$$

The trajectory of motion will be a circle.

Q. 6. The displacement of a particle varies with according to the relation $y = a \sin \omega t + b \cos \omega t$.

- (A) The motion is oscillatory but not S.H.M.
(B) The motion is S.H.M. with amplitude $a + b$.
(C) The motion is S.H.M. with amplitude $a^2 + b^2$.
(D) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$

Ans. Option (D) is correct.

Explanation: Let

$$a = A \cos \phi \quad (i)$$

$$b = A \sin \phi \quad (ii)$$

Squaring and adding (i) & (ii)

$$a^2 + b^2 = A^2$$

$$y = a \sin \omega t + b \cos \omega t \quad (iii)$$

or $y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$

$$= A \sin(\omega t + \phi)$$

This is an equation of SHM with amplitude

$$A = \sqrt{a^2 + b^2}$$

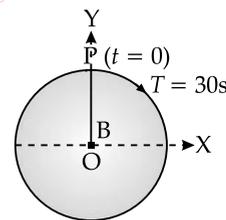
Q. 7. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the x -projection of the radius vector of the rotating particle P is

(A) $x(t) = B \sin\left(\frac{2\pi t}{30}\right)$.

(B) $x(t) = B \sin\left(\frac{\pi t}{15}\right)$.

(C) $x(t) = B \sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$.

(D) $x(t) = B \sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$.



Ans. Option (A) is correct.

Explanation: Projection of OP on x -axis at time t is

$$x(t) = B \cos\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right) = B \sin\left(\frac{2\pi}{T}t\right)$$

$$x(t) = B \sin\left(\frac{2\pi}{30}t\right) \quad \{\because T = 30s\}$$

Q. 8. The equation of motion of a particle is $x = a \cos(\alpha t)^2$.

The motion is

- (A) Periodic but not oscillatory.
(B) Periodic and oscillatory.
(C) Oscillatory but not periodic.
(D) Neither periodic nor oscillatory.

Ans. Option (C) is correct.

Q. 9. A particle executing S.H.M. has a maximum speed of 30 cm/s and a maximum acceleration of 60 cm/s². The period of oscillation



Answering Tip

- Motion of simple pendulum is simple harmonic if the angle by which the bob is displaced from the mean position is small.

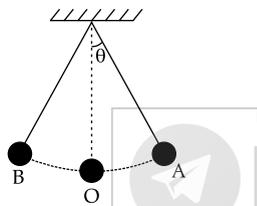
Q. 15. What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator? [NCERT Exemplar Q. No. 23]

Ans. $\frac{a_{\max}}{v_{\max}} = \frac{\omega^2 A}{\omega A} = \omega$

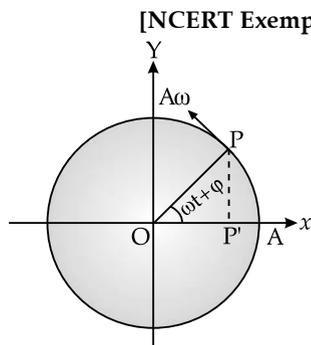
Q. 16. What is the ratio between the distance travelled by the oscillator in one time period and amplitude? [NCERT Exemplar Q. No. 24]

Ans. In SHM,
The distance travelled by particle in one complete oscillation = $AO + OB + BO + OA = 4AO$
Amplitude = AO

\therefore ratio = $\frac{4AO}{AO} = 4$



Q. 17. In Fig., what will be the sign of the velocity of the point P' , which is the projection of the velocity of the reference particle P . P is moving in a circle of radius R in anti clockwise direction? [NCERT Exemplar Q. No. 25]



Ans. Projection of P on x -axis

$x = A \cos(\omega t + \phi)$

$v_x = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$

\therefore sign of velocity is negative

Q. 18. Show that for a particle executing S.H.M. velocity and displacement have a phase difference of $\pi/2$. [NCERT Exemplar Q. No. 26]

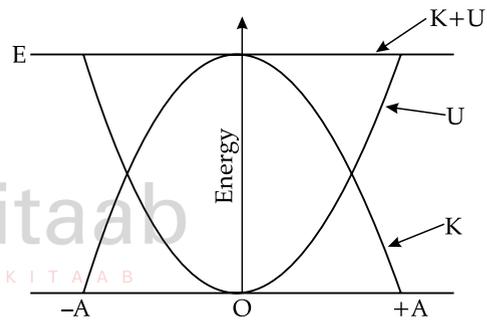
Ans. Displacement, $y = A \sin \omega t$
{particle is executing SHM}
 $v = \frac{dy}{dt} = A\omega \cos \omega t = A\omega \sin\left(\omega t + \frac{\pi}{2}\right)$

\therefore phase difference between y and v is

$\Delta\phi = \left\{ \left(\omega t + \frac{\pi}{2} \right) - \omega t \right\} = \frac{\pi}{2}$

Q. 19. Draw a graph to show the variation of P.E., K.E. and total energy of a simple harmonic oscillator with displacement. [NCERT Exemplar Q. No. 27]

Ans.



$E = K + U = \text{constant}$

Q. 20. The length of a second's pendulum on the surface of Earth is 1m. What will be the length of a second's pendulum on the moon? [NCERT Exemplar Q. No. 28]

Ans. For Earth, $T = 2\pi \sqrt{\frac{l_{\text{earth}}}{g_{\text{earth}}}}$

For Moon, $T' = 2\pi \sqrt{\frac{l_{\text{moon}}}{g_{\text{moon}}}}$

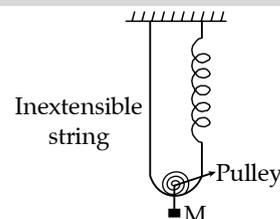
Since $T = T'$

So, $\frac{l_{\text{moon}}}{g_{\text{moon}}} = \frac{l_{\text{earth}}}{g_{\text{earth}}}$ or, $l_{\text{moon}} = l_{\text{earth}} \left(\frac{g_{\text{moon}}}{g_{\text{earth}}} \right) = 1 \times \frac{1}{6}$

So, $l_{\text{moon}} = \frac{1}{6} \text{ m}$

Short Answer Type Questions

Q. 21. Find the time period of mass M when displaced from its equilibrium position and then released for the system shown in Fig. [NCERT Exemplar Q. No. 29]



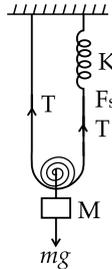
Ans. $Mg = T + T$ {system is in equilibrium}
 $\therefore Mg = 2T$

The spring elongated by $2l$. Where l is the distance moved by hanging mass.

$T = F_s$ (In the spring)

$T = 2kl$

$\therefore Mg = 2(2kl) = 2k(2l)$



displacing mass through y distance downwards
 Restoring force

$$F = Mg - 2k(2l + 2y)$$

$$= Mg - (2k)(2l) - 4ky$$

or $F = Mg - Mg - 4ky = -4ky$

$$M \frac{d^2y}{dt^2} = -4ky$$

$$\frac{d^2y}{dt^2} = -\frac{4k}{M}y \quad \left\{ \text{comparing with } \frac{d^2y}{dt^2} = -\omega^2y \right\}$$

$$\therefore \omega = \sqrt{\frac{4k}{M}} \quad \therefore T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{M}{4k}}$$

Q. 22. Show that the motion of a particle represented by $y = \sin \omega t - \cos \omega t$ is simple harmonic with a period of $2\pi/\omega$. [NCERT Exemplar Q. No. 30]

Ans. $y = \sin \omega t - \cos \omega t$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin \omega t - \frac{1}{\sqrt{2}} \cos \omega t \right)$$

$$= \sqrt{2} \left(\cos \frac{\pi}{4} \sin \omega t - \sin \frac{\pi}{4} \cos \omega t \right)$$

$$\therefore y = \sqrt{2} \sin \left(\omega t - \frac{\pi}{4} \right)$$

$\therefore (\sin \omega t - \cos \omega t)$ represents SHM.

$$y = \sqrt{2} \sin \left(\omega t - \frac{\pi}{4} \right) = \sqrt{2} \sin \left(\omega t - \frac{\pi}{4} + 2\pi \right)$$

$$= \sqrt{2} \sin \left(\omega \left(t + \frac{2\pi}{\omega} \right) - \frac{\pi}{4} \right)$$

$$\therefore \text{Time period} = \frac{2\pi}{\omega}$$

Q. 23. Find the displacement of a simple harmonic oscillator at which its P.E. is half of the maximum energy of the oscillator. [NCERT Exemplar Q. 31]

Ans. PE of oscillator, $U = \frac{1}{2} m \omega^2 y^2$

{ y =displacement}

Maximum-energy of oscillator, $E = \frac{1}{2} m \omega^2 A^2$

$$U = \frac{1}{2} E$$

$$\text{or } \frac{1}{2} m \omega^2 y^2 = \frac{1}{4} m \omega^2 A^2$$

$$\text{or } y^2 = \frac{A^2}{2}$$

$$\text{or } y = \pm \frac{A}{\sqrt{2}}$$

Q. 24. A mass of 2 kg is attached to the spring of spring constant 50 Nm^{-1} . The block is pulled to a distance of 5 cm from its equilibrium position at $x = 0$ on a horizontal frictionless surface from rest at $t = 0$. Write the expression for its displacement at anytime t . [NCERT Exemplar Q. No. 33]

Ans. Given: $m = 2 \text{ kg}$, $k = 50 \text{ Nm}^{-1}$, $A = 5 \text{ cm}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = \sqrt{25} = 5 \text{ rad/s}$$

At $t = 0$, $x = 0$

\therefore displacement at any time t

$$x = A \sin \omega t \quad \text{or } x = 5 \sin \omega t$$

Q. 25. Consider a pair of identical pendulums, which oscillate with equal amplitude independently such that when one pendulum is at its extreme position making an angle of 2° to the right with the vertical. The other pendulum makes an angle of 1° to the left of the vertical. What is the phase difference between the pendulums?

[NCERT Exemplar Q. No. 34]

Ans. Supposing θ_0 is angular amplitude of each of the pendulums.

$$\theta_1 = \theta_0 \sin(\omega t + \delta_1)$$

$$\theta_2 = \theta_0 \sin(\omega t + \delta_2) \quad \left\{ \text{SHM of pendulums} \right\}$$

One pendulum is making 2° angle with vertical, $\theta_0 = 2^\circ$.

$$\therefore 2^\circ = 2^\circ \sin(\omega t + \delta_1)$$

$$\sin(\omega t + \delta_1) = 1$$

$$\omega t + \delta_1 = 90^\circ$$

For other pendulum, $\theta_2 = -1^\circ$

$$\therefore -1^\circ = 2^\circ \sin(\omega t + \delta_2)$$

$$\text{or } \sin(\omega t + \delta_2) = -\frac{1}{2}$$

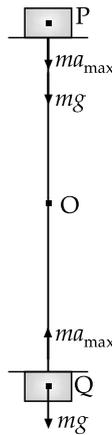
$$\omega t + \delta_2 = -30^\circ$$

$$\therefore (\omega t + \delta_1) - (\omega t + \delta_2) = 90^\circ - (-30^\circ)$$

$$\text{Phase difference} = \delta_1 - \delta_2 = 120^\circ$$

👁️ Long Answer Type Questions

Q. 26. A person normally weighing 50 kg stands on a massless platform which oscillates up and down harmonically at a frequency of 2.0 s⁻¹ and an amplitude 5.0 cm. A weighting machine on the platform gives the persons weight against time.



- (a) Will there be any change in weight of the body, during the oscillation?
 (b) If answer to part (a) is yes, what will be the maximum and minimum reading in the machine and at which position?

[NCERT Exemplar Q. No. 35]

Ans. (a) Yes, change of weight will be there.

(b) Given mass = $m = 50$ kg, $v = 2$ s⁻¹,
 $A = 5$ cm = 0.05 m

Let the platform vibrate between two extreme positions P and Q about mean position O.

a_{\max} = maximum acceleration towards mean position O

$$a_{\max} = \omega^2 A = (2\pi v)^2 A$$

$$a_{\max} = 4\pi^2 v^2 A = 4 \times (3.14)^2 \times 2^2 \times 0.05 = 7.9 \text{ ms}^{-2}$$

At P, restoring force (ma_{\max}) and weight (mg) are directed towards mean position. So, net weight at P given by

$$W_1 = mg + ma_{\max} = m(g + a_{\max}) = 50(9.8 + 7.9) = 885 \text{ N}$$

Similarly, net weight at Q

$$W_2 = mg - ma_{\max} = m(g - a_{\max}) = 50(9.8 - 7.9) = 95 \text{ N}$$

Weight is maximum at top position and minimum at lowermost position.



Commonly Made Error

- ▶ Students can not judge the change in weight during oscillations.



Answering Tip

- ▶ Students should keep in mind that weight is a force acting in the downward direction.

Q. 27. A body of mass m is attached to one end of a massless spring which is suspended vertically from a fixed point. The mass is held in hand so that the spring is neither stretched nor compressed.

Suddenly the support of the hand is removed. The lowest position attained by the mass during oscillation is 4 cm below the point, where it was held in hand.

- (a) What is the amplitude of oscillation?
 (b) Find the frequency of oscillation?

[NCERT Exemplar Q. No. 36]

Ans. (a)

$$\text{Amplitude} = \frac{\text{distance between 2 extreme positions}}{2} = \frac{4 \text{ cm}}{2} = 2 \text{ cm}$$

(b) Frequency = $\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

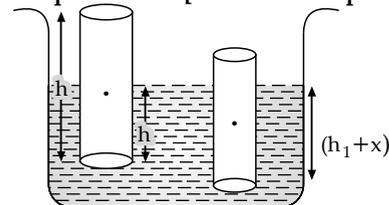
At mean position, $mg = k(2)$ or, $\frac{k}{m} = \frac{g}{2}$

$$\therefore \nu = \frac{1}{2\pi} \sqrt{\frac{g}{2}} = \frac{1}{2 \times 3.14} \sqrt{\frac{9.8}{2}} = \frac{1}{6.28} \sqrt{4.9} = 3.5 \text{ s}^{-1}$$

Q. 28. A cylindrical log of wood of height H and area of cross-section A floats in water. It is pressed and then released. Show that the log would execute S.H.M. with a time period.

$$T = 2\pi \sqrt{\frac{m}{A\rho g}}$$

Where m is mass of the body and ρ is density of the liquid. [NCERT Exemplar Q. No. 37]



- Ans.** Try yourself. refer Q.17. of NCERT Corner.
Q. 29. A tunnel is dug through the centre of the Earth. Show that a body of mass ' m ' when dropped from one end of the tunnel will execute simple harmonic motion. [NCERT Exemplar Q. No. 39]
Ans. Acceleration due to gravity inside Earth

$$g' = \frac{GM}{R^3} x = \frac{g}{R} x$$

Here, x = distance of the point from centre of Earth ($x < R$)

If, block of mass m is placed along the diameter inside the Earth.

So, force on block

$$F = -\frac{mg}{R} x = -kx$$

$\therefore k = \frac{mg}{R}$ (motion will be SHM)

$\therefore T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{mg/R}} = 2\pi \sqrt{\frac{R}{g}}$



Commonly Made Error

- ▶ Students can not show that the motion of a mass 'm' inside Earth will be simple harmonic.



Answering Tip

- ▶ Students should be familiar with the acceleration due to gravity inside Earth.



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any 4 of the following questions on the basis of the same:

I. Galileo's Isochronous Pendulum:

Galileo observed a lamp swinging from ceiling at Pisa cathedral ceiling. He was the first scientist to observe how long it took any object suspended from a rope or chain (a pendulum) to swing back and forth. There were no wrist watches at that time, so Galileo used his own pulse as a time measurement. Galileo observed that no matter how big the swings were, as in when the lamp was first swung, to how small the swings were as the lamp returned to a standstill, the time it took for each swing to complete was exactly the same. So, he concluded that the oscillations are isochronous. Anyhow, it was proved by advanced experimentation that the isochronism of simple pendulum is correct within 1% under the 30° of amplitude. It is confirmed that the period changes less than 1% until the amplitude is 30°. On the other hand, the period increased by more than 10% when the amplitude became larger than 80°. At the end of his life he devised a scheme for using a pendulum to regulate a mechanical clock. However, the first reliable pendulum clock was only demonstrated by Huygens 15 years after Galileo's death.

Q. 1. Who first observed the time period of a pendulum?

- (A) Aristotle (B) Pythagoras
(C) Galileo (D) Democritus

Ans. Option (C) is correct.

Explanation: Galileo observed a lamp swinging from ceiling at Pisa cathedral ceiling. He was the first scientist to observe how long it took any object suspended from a rope or chain (a pendulum) to swing back and forth.

Q. 2. How the time period of a pendulum was measured in the beginning of 16th century?

- (A) By a stop watch
(B) By using pulse beats
(C) By using vibration of a stringed instrument
(D) By using a sand clock

Ans. Option (B) is correct.

Explanation: Since, no watches were available at that time, Galileo used his own pulse as a time measurement.

Q. 3. Galileo observed, that the time period of the pendulum

- (A) Does not vary with the size of swing
(B) Increases when the size of swing increases
(C) Decreases when the size of swing increases
(D) Unpredictable

Ans. Option (A) is correct.

Explanation: Galileo observed that no matter how big the swings were, as in when the lamp was first swung, to how small the swings were as the lamp returned to a standstill, the time it took for each swing to complete was exactly the same.

Q. 4. First reliable pendulum clock was only demonstrated by

- (A) Vincenzo Viviani (B) Huygens
(C) Maffeo Barberini (D) Newton

Ans. Option (B) is correct.

Explanation: The first reliable pendulum clock was only demonstrated by Huygens 15 years after Galileo's death.

Q. 5. Isochronism of simple pendulum is within acceptable limit as long as the amplitude is under

- (A) 90° (B) 60°
(C) 30° (D) 80°

Ans. Option (C) is correct.

Explanation: Anyhow, it was proved by advanced experimentation that the isochronism of simple pendulum is correct within 1% under the 30° of amplitude. It is confirmed that the period changes less than 1% until the amplitude is 30°. On the other hand, the period increased by more than 10% when the amplitude became larger than 80°.

B Subjective Questions

I. Read the following text and answer any 4 of the following questions on the basis of the same:

Ordinary clock pendulums expand and contract with changes in temperature. The period of the pendulum depends on its length. So, it varies with changes in ambient temperature.

British clockmaker John Harrison developed gridiron pendulum which is temperature-compensated. It consists of alternating parallel rods of two metals with different thermal coefficients, such as brass and iron. Later instead of brass rod zinc rods were used. Design of gridiron pendulum using brass is more complex because brass does not expand as much as zinc. So, for appropriate compensation of length, five iron and four brass rods are required. The exact degree of compensation is adjusted by a section of the central rod which is partly brass and partly iron.

The rods are connected by a frame in such a way that their different thermal expansions (or contractions) are compensated by each other, so that the overall length of the pendulum and thus its period, stays constant with temperature.

Its simplest form consists of five rods. A central iron rod runs up from the bob to a point immediately below the suspension.

At that point a cross-piece extends from the central rod and connects to two zinc rods, one on each side of the central rod. The zinc rods reach just above the bob and connect to two other iron rods which run back up to the top near the suspension and are connected with each other there.

The upward expansion of the zinc rods counteracts the combined downward expansion of the iron. The rod lengths are calculated so that the effective length of the zinc rods multiplied by zinc's thermal expansion coefficient equals the effective length of the iron rods multiplied by iron's expansion coefficient, thereby keeping the pendulum the same length.



Q. 1. Why pendulum clocks run slow in summer?

Ans. Time period of pendulum $= T = 2\pi\sqrt{\frac{L}{g}}$

As temperature increases in summer, the length of pendulum increases and the value of time period increases. Hence, the clock runs slow.

Q. 2. What is the length of a simple pendulum, which ticks seconds ?

Ans. The time period of a simple pendulum, which ticks seconds, is 2 s.

$$\text{Time period of pendulum} = T = 2\pi\sqrt{\frac{L}{g}}$$

$$\text{or} \quad 2 = 2\pi\sqrt{\frac{L}{9.8}}$$

$$\text{or} \quad L = \frac{9.8}{(3.14)^2}$$

$$\therefore L \approx 1\text{m}$$

Q. 3. A pendulum clock consists of an iron rod. It keeps correct time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = $\frac{1.2 \times 10^{-6}}{^\circ\text{C}}$.

Ans. Change in length due to change in temperature $= \Delta L = L\alpha\Delta\theta$

$$\text{or} \quad \frac{\Delta L}{L} = \alpha\Delta\theta$$

$$\text{or} \quad \frac{\Delta L}{L} = 1.2 \times 10^{-6} \times (40^\circ - 20^\circ)$$

$$\therefore \frac{\Delta L}{L} = 2.4 \times 10^{-5}$$

$$\text{Time period of pendulum} = T = 2\pi\sqrt{\frac{L}{g}}$$

$$\text{or} \log T = \log 2\pi + \frac{1}{2} \log L - \frac{1}{2} \log g$$

Differentiating,

$$\frac{dT}{T} = 0 + \frac{1}{2} \frac{dL}{L} - 0$$

$$\text{or} \quad \frac{dT}{T} = \frac{1}{2} \times 2.4 \times 10^{-5}$$

$$\text{Time loss in 24 hours} = \frac{1}{2} \times 2.4 \times 10^{-5} \times 24 \times 60$$

$$\times 60 = 1.0368 \text{ s}$$

Q. 4. What is the basic principle of temperature compensated pendulum?

Ans. Temperature compensated pendulum consists of alternating parallel rods of two metals with different thermal coefficients. The rods are connected by a frame in such a way that their different thermal expansions (or contractions) are compensated by each other, so that the overall length of the pendulum and thus its period, stays constant with temperature.

Q. 5. Initially grid iron pendulum was constructed using iron and brass. Later brass rods were replaced by zinc. Why?

Ans. Design of gridiron pendulum using brass is more complex because brass does not expand as much as zinc. So, for appropriate compensation of length, five iron and four brass rods are required. The exact degree of compensation is adjusted by a section of the central rod which is partly brass and partly iron. To avoid this complexity of design brass was replaced by zinc.

CHAPTER 14 WAVES

Study Time
Maximum Time: 3 : 00 hours
Maximum Questions: 86


Syllabus

Wave motion : Transverse and longitudinal waves, speed of wave motion. Displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats.

Topic-1 Waves & Wave Motion

TOPIC - 1
 Waves & Wave Motion
 P. 293

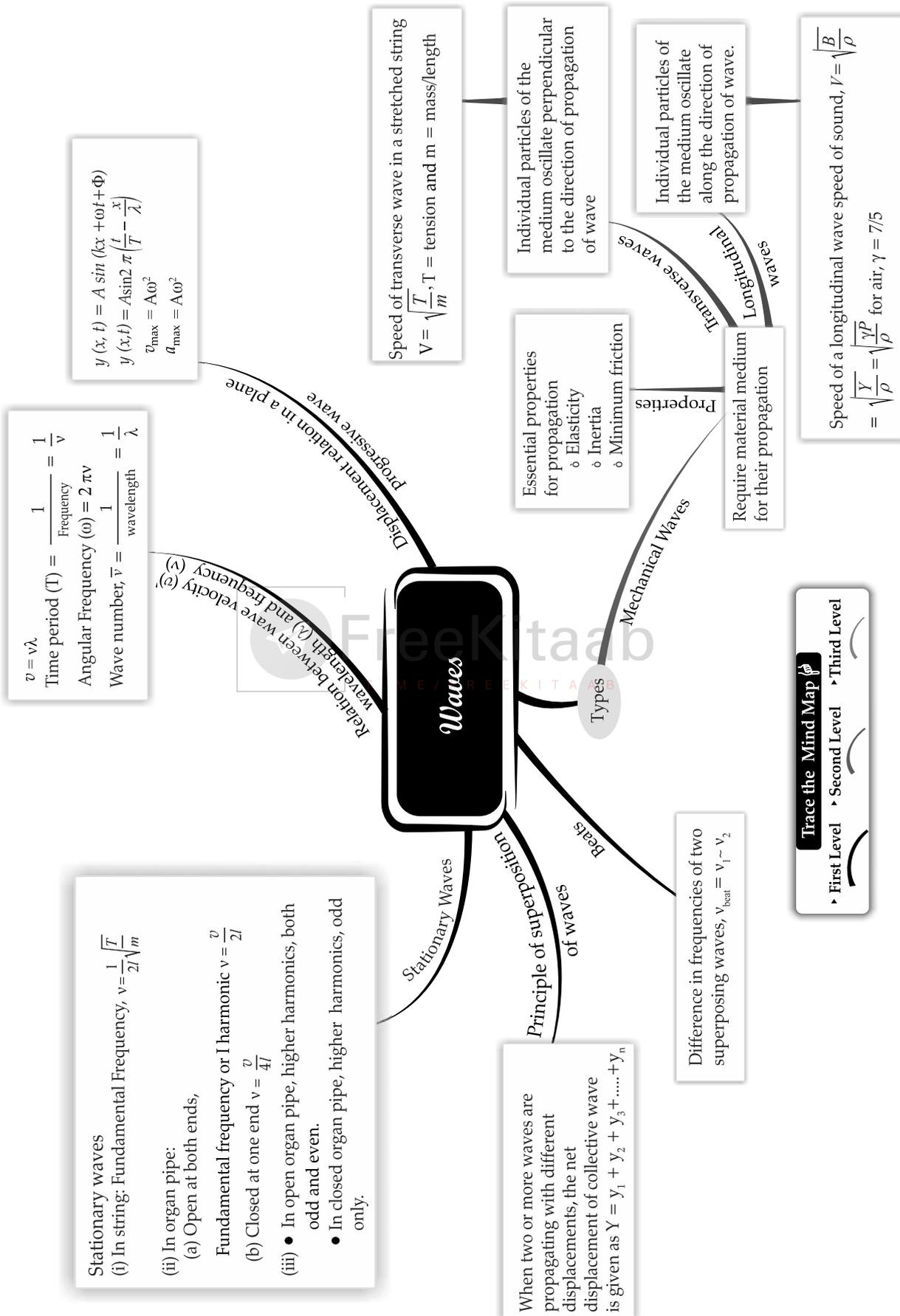
TOPIC - 2
 Superposition Principle
 P. 298

Revision Notes

- **Wave motion** is a kind of disturbance which travels through a medium on account of repeated periodic vibrations of the particles of the medium about their mean position.
- **The medium** for wave propagation should have three properties :
 - (a) elasticity
 - (b) inertia
 - (c) minimum frictional resistance.
- **Kinds of waves :**
 - (i) **Mechanical waves** or elastic waves: The waves which require a material medium for propagation. Example: sound waves, waves on the surface of water, waves on strings.
 - (ii) **Electromagnetic waves:** The waves which require no medium for propagation. Example: light waves, radio waves.
 - (iii) **Longitudinal waves :** In which particles vibrate in the direction of propagation of waves. Longitudinal waves travel through a medium in the form of compressions and rarefactions involving changes in pressure and volume and can travel in all modes and cannot be polarised. The medium required must possess elasticity of volume. Example: Sound waves in air are longitudinal.
 - (iv) **Transverse waves :** In which particles vibrate in a direction perpendicular to the direction of propagation of waves. Transverse waves travel through a medium in the form of crests and troughs involving changes in shape can travel in solid and liquid and can be polarised. The medium required must possess elasticity of shape. Example: Vibrations in strings
 - (v) **Matter waves:** These waves are associated with constituents of matter - electrons, protons, neutrons, atoms and molecules. Those arise in quantum mechanical description of nature.
 - (vi) **Capillary waves and gravity waves:** The waves on the surface of water are of two kinds: capillary waves and gravity waves.

Ripple wave: These are ripples of fairly short wavelength—not more than a few centimetre—and the restoring force that produces them is the surface tension of water.





Trace the Mind Map

- First Level
- Second Level
- Third Level

Gravity waves: These are have wavelengths typically ranging from several metres to several hundred meters. The restoring force that produces these waves is the pull of gravity, which tends to keep the water surface at its lowest level.

➤ **Laplace correction :** According to Laplace, the changes in pressure & volume of a gas, when sound waves propagate through it are not isothermal but it is adiabatic. since,

(a) Velocity of sound in gas is quite large.

(b) A gas is bad conductor of heat.

$$\therefore \text{Velocity of sound, } v = \sqrt{\frac{B_a}{\rho}}$$

B_a = Bulk modulus = γP , ρ = density of gas

Key Words

➤ **Some parameters related to wave motion**

(a) **Displacement** of a particle is the distance covered by the particle from the mean position.

(b) **Amplitude** is the maximum displacement of the particle from the equilibrium position.

(c) **One wavelength** is the distance travelled by the wave, during the time the particle completes one vibration about its mean position. We may also define, one wavelength = smallest distance between two particles vibrating in the same phase = distance between the centres of two consecutive crests/troughs/compressions/rarefactions.

(d) **Angular wave number or propagation constant :** It is 2π times the no. of waves that can be accommodated per unit length. It is denoted by K .

(e) **Frequency :** It is the no. of complete wavelengths traversed by the wave in one second.

(f) **Time period :** It is equal to time taken by wave to travel a distance equal to wavelength.

(g) **Particle velocity** = velocity of particle executing SHM = $\frac{dx}{dt}$. Its value changes with time.

(h) **Wave velocity** is the velocity with which disturbance travels in the medium.

$$v = n\lambda = \frac{\lambda}{T} = \text{constant for a wave motion.}$$

➤ **Phase or phase angle** is the physical quantity which tells us by what amount the two waves or the two particles lag or lead in terms of angle or time or distance.

Key Formulae

➤ **A longitudinal wave** can be represented by

$$x = a \sin(\omega t \pm kx)$$

A transverse wave can be represented by

$$y = a \sin(\omega t \pm kz); y = a \sin(\omega t \pm kx)$$

$$z = a \sin(\omega t \pm kx); z = a \sin(\omega t \pm ky)$$

$$x = a \sin(\omega t \pm ky); x = a \sin(\omega t \pm kz)$$

➤ **Relation between particle velocity & wave velocity.**

$$u(x, t) = -v \frac{d}{dx} [y(x, t)]$$

Where

u = particle velocity, v = wave velocity.

➤ **Particle Acceleration :**

$$a(x, t) = -(2\pi v)^2 y = -\omega^2 y.$$

➤ **Plane progressive wave :**

(a) **Standard Equations :**
$$y = r \sin \left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right]$$

or

$$y = r \cos \left[\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right]$$

where y = displacement, r = amplitude, T = time period, λ = wavelength, x = starting distance of wave from origin.

- (b) Angular frequency : $\omega = \frac{2\pi}{T}$
- (c) Propagation constant : $k = \frac{2\pi}{\lambda}$
- (d) Velocity of wave : $v = v\lambda = \frac{\lambda}{T}$
- (e) Velocity of particle : $u = \frac{dy}{dt}, u_{max} = r\omega$
- (f) Acceleration of particle : $a = \frac{d^2y}{dt^2}, a_{max} = -\omega^2r$
- (g) Phase:
For wave $y(x, t) = a \sin(kx - \omega t + \phi)$
Phase = $(kx - \omega t + \phi)$



Mnemonics

Concept: Transverse and longitudinal wave.

Mnemonics: Teacher Punished lazy dog.

Interpretation:

T - Transverse wave

P - perpendicular to direction of propagation

L - Longitudinal wave

D - direction of propagation

If constituents of the medium oscillate perpendicular to the direction of wave propagation, we call the wave a transverse wave. If constituents of the medium oscillate along the direction of wave propagation, we call the wave a longitudinal wave.



Very Short Answer Type Questions

(1 mark each)

Q. 1. Is it possible to have longitudinal waves on a string?

Ans. No, because string is not stretchable. It can neither be compressed nor rarefied. 1

Ans. Yes, transverse waves are possible in a steel rod, because steel has elasticity of shape. 1

Q. 3. What type of mechanical waves do you expect to exist in (a) vacuum, (b) air, (c) inside the water, (d) rock (e) on the surface of water?

Ans. (a) No wave, (b) longitudinal waves, (c) longitudinal waves, (d) transverse or longitudinal or both (separately), (e) combined longitudinal and transverse (ripples). 1

Q. 4. What is the phase difference between the waves $y = a \cos(\omega t + kx)$ and $y = a \sin(\omega t + kx)$?

Ans. Phase difference = $\frac{\pi}{2} = 90^\circ$. 1

Q. 5. Why are longitudinal waves called pressure waves?

Ans. This is because propagation of longitudinal waves through a medium involves changes in pressure and volume of air, when compressions and rarefactions are formed. 1



Commonly Made Error

- Students can not predict the possibility of existence of longitudinal waves on a string.



Answering Tip

- The distinction between longitudinal and transverse waves should be understood clearly.

Q. 2. Is it possible to have transverse wave in a steel rod?



Short Answer Type Questions-I

(2 marks each)

Q. 1. Explain why waves on strings are always transverse?

Ans. A string is non-stretchable, i.e., compressions and rarefactions cannot be produced in strings. Therefore, longitudinal waves in strings are

not possible. Strings do have elasticity of shape. Therefore, waves on strings are transverse. 2

Q. 2. Liquids and gases cannot propagate transverse waves. Why?

Ans. Liquids and gases cannot sustain shearing stress. Therefore, transverse waves in the form of crests and troughs (involving change of shape) are not possible in fluids. Rather, the fluid possesses volume elasticity. Therefore, compressions and rarefactions (involving changes in volume) can be propagated through fluids. 2

Q. 3. What is periodic wave function ? U

Ans. A wave function (x, t) which satisfies the periodicity conditions of position and time is called a periodic wave function, i.e.,

(i) $y(x + m\lambda, t) = y(x, t)$

(ii) $y(x, t + nT) = y(x, t)$

where λ = wavelength of wave, T = period of the wave, n and m are integers. 2

Q. 4. Write down the expression for speed of transverse waves in solids and in a stretched string. U

Ans. In a solid :

Speed of transverse wave, $v = \sqrt{\frac{\eta}{\rho}}$

where, η is modulus of rigidity and ρ is density of material of solid.

In a stretched string : 1

Here, $v = \sqrt{\frac{T}{m}}$

where T is tension in the string and m is mass per unit length of the string. 1

Q. 5. What is phase in wave equation U

$y = r \sin \left(2\pi \frac{t}{T} + 2\pi \frac{x}{\lambda} \right)$

Ans. Rewriting $y = r \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$ 1

Here, $2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$ is phase of the wave. 1

Short Answer Type Questions-II

(3 marks each)

Q. 1. Given below are some examples of wave motion. State in each case, whether the wave motion is transverse, longitudinal or a combination both.

(i) Motion of a kink in a long coil spring produced by displacing one end of the string sideways.

(ii) Waves produced in a cylinder containing a liquid by moving its piston back and forth.

(iii) Waves produced by a motor boat sailing in water.

(iv) Light waves travelling from sun to earth.

(v) Ultrasonic waves in air produced by a vibrating quartz crystal. A

Ans. (i) When the spring is pulled sideways, the kink moves at 90° to the length of the spring. Waves are transverse. 1/2

(ii) Waves in this case are longitudinal, because molecules of the liquid will move along the direction of motion of the piston. 1/2

(iii) The water surface is cut laterally and pushed backwards by the propeller of motor boat. Therefore, the waves are a mixture of longitudinal and transverse waves. 1/2

(iv) Light waves (from sun to earth) are electromagnetic waves which are transverse in nature. 1/2

(v) Ultrasonic waves in air are basically sound waves of frequency greater than the audible frequencies. They are, therefore, longitudinal waves. 1

Q. 2. Give a relation for velocity of a wave. U

Ans. Velocity, $v = \frac{\text{Displacement}}{\text{Time}}$

$= \frac{\text{Wavelength}}{\text{Time for one cycle}}$ 1

$= \frac{\lambda}{T}$ (∵ $\frac{1}{T} = f$) 1

$= \lambda f$ 1

Q. 3. Obtain an equation of a progressive wave. U

Ans. For a simple harmonic motion, displacement

$y = r \sin (\omega t + \phi)$

where r is amplitude, ω is angular velocity and ϕ is phase difference 1

But $\omega = \frac{2\pi}{T} = 2\pi f$ 1

and $\phi = \frac{2\pi}{\lambda} x$

where λ is wavelength and x is path difference.

∴ $y = r \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$ 1

Long Answer Type Questions

(5 marks each)

Q. 1. Discuss some of the important characteristics of wave motion. U

Ans. The important characteristics of wave motion :

(1) Wave motion is a sort of disturbance which travels through a medium. 1

(2) A material medium is essential for the propagation of mechanical waves. The medium must possess three properties, viz., elasticity, inertia and minimum friction amongst the particles of the medium. 1

- (3) The velocity of the particles during their vibration is different at different positions. 1
- (4) Energy is propagated along with the wave motion without any net transport of the medium. 1
- (5) There is a continuous phase difference amongst successive particles of the medium. 1
- (6) Waves are partly reflected and partly refracted from the interface of two media. 1



Commonly Made Error

- ▶ Students can not state the characteristics of wave motion.



Answering Tip

- ▶ Students should be familiar with the properties of wave and its motion in detail.

Topic-2 Superposition Principle



Revision Notes

➤ **Principle of Superposition of Waves :**

- (a) According to this principle, overlapping waves add algebraically to produce a resultant wave or a net wave.

or

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of displacements due to each wave at that time.

i.e., $y = y_1 + y_2 + \dots + y_n$

Important phenomenon related to Superposition of waves:

- (i) **Stationary waves :** When two identical transverse or longitudinal progressive waves travel in a bounded medium with same speed but in opposite directions, then by their superposition a new type of wave is produced which appears stationary in the medium. This is known as stationary or standing wave.
- (ii) **Beats:** When two sound waves of nearly equal frequencies superimpose, the intensity of the resultant wave increases and decreases alternately with time. Then phenomenon is known as beats.
- (iii) **Interference of waves:** When two waves of same frequency travel in a medium simultaneously in the same direction, then due to their superposition, the resultant intensity at any point of the medium becomes different from the sum of the intensities of the two waves. At some points the intensity of resultant wave is very large while at some other points it is very low or zero. This phenomenon is known as interference.

➤ **Laws of Vibrations of Stretched Strings.**

Fundamental Frequency of vibration of stretched string

$$v = \frac{1}{2L} \sqrt{\frac{T}{m}} = \frac{1}{LD} \sqrt{\frac{T}{\pi\rho}}$$

- (a) **Law of Length :**

$$v \propto \frac{1}{L}$$

Fundamental frequency is inversely proportional to length.

- (b) **Law of Tension :** Fundamental frequency is directly proportional to square root of tension. i.e.,

$$v \propto \sqrt{T}$$

- (c) **Law of Mass :** Fundamental frequency is inversely proportional to the square root of mass :

$$v \propto \frac{1}{\sqrt{m}}$$

- (d) **Law of Diameter :**

$$v \propto \frac{1}{D}$$

- (e) **Law of Density :**

$$v \propto \frac{1}{\sqrt{\rho}}$$

➤ **Vibration of air column in closed organ pipe:**

The tube which is closed at one end and open at other end is called closed organ pipe.

When air is blown at open end, a longitudinal wave travels towards the closed end and reflects back towards the open end. These two waves superimpose and form stationary longitudinal wave.

$$\lambda = \frac{4L}{2m-1}$$

$$v = \frac{v}{4L} \times (2m-1)$$

where $m = 1, 2, 3, 4, \dots$

➤ **Vibration of air column in closed organ pipe:**

The tube which is open at both ends is called closed organ pipe.

When air is blown at one open end, a longitudinal wave travels towards the other end and reflects back towards the first open end. These two waves superimpose and form stationary longitudinal wave.

$$\lambda = \frac{2l}{m}$$

$$v = \frac{v}{2L} \times m$$

where $m = 1, 2, 3, 4, \dots$

Key Words

- **Stationary Waves** : If two waves of same type having same amplitude, same frequency and same wavelength, travelling with same speed in opposite directions along a straight line superimpose each other, they give rise to a new kind of waves known as stationary wave.
- **Beats** is the phenomenon of regular variation in the intensity of sound when two sources of nearly equal frequencies are sounded together.
 - (i) **Beat period** is the time interval between two successive beats. **A A B**
 - (ii) **Beat frequency** is the number of beats produced per second.

Scan to know more about this topic



Stationary waves

Key Formulae

➤ **Equation of stationary wave :**

When incident wave: $y_1 = A \sin \frac{2\pi}{\lambda} (vt - x)$ and

Reflected wave: $y_2 = A \sin \frac{2\pi}{\lambda} (vt - x)$

Then the standing wave: $y = 2A \sin \frac{2\pi}{\lambda} vt \cos \frac{2\pi}{\lambda} x$

➤ **Normal modes of vibration of strings :**

(a) **Frequency of fundamental tone:** $v_0 = \frac{1}{2l} \sqrt{\frac{T}{m}}$

(b) **1st overtone or 2nd harmonic :** $v_1 = 2v_0$

(c) **2nd overtone or 3rd harmonic :** $v_2 = 3v_0$

➤ **Organ Pipes :**

(a) **Closed organ pipe :**

(i) **Fundamental note** $v_0 = \frac{v}{4L}$

(ii) **1st overtone or 3rd harmonic** $v_1 = 3v_0$

(iii) **2nd overtone or 5th harmonic** $v_2 = 5v_0$

(b) Open organ pipe :

- (i) Fundamental note $v_1 = \frac{v}{2L}$
- (ii) 1st overtone or 2nd harmonic $v_1 = 2v_0$
- (iii) 2nd overtone or 3rd harmonic $v_2 = 3v_0$
- Beats : Beat frequency $n_1 - n_2$ or $n_2 - n_1$



Very Short Answer Type Questions

(1 mark each)

- Q. 1. What are harmonics ? [R]
 Ans. Harmonics are the notes of frequencies which are integral multiple of the fundamental frequency. 1
- Q. 2. Is superposition principle applicable to electromagnetic waves ? [U]
 Ans. Yes, it is applicable to electromagnetic waves. 1
- Q. 3. Where will a person hear maximum sound, at node or antinode ? [U] [A]
 Ans. Perception of sound is due to pressure variations which is maximum at nodes. 1
- Q. 4. What is the distance between a node and an adjoining antinode in a stationary wave ? [U]
 Ans. It is one quarter of the wavelength (i.e., $\lambda/4$). 1
- Q. 5. An open organ pipe is closed at one end. How will the frequency of fundamental note of the pipe change ? [A]

Ans. The fundamental frequency shall reduce to half the original value. 1



Commonly Made Error

- ▶ Students can not state the change in frequency of fundamental note when one end of open organ pipe is closed.



Answering Tip

- ▶ Students should have an understanding of frequency of sound wave in open and closed organ pipes.



Short Answer Type Questions-I

(2 marks each)

- Q. 1. What are fundamental note and overtones ? [R]
 Ans. When a source is sounded, it generally vibrates in more than one mode and therefore, emits tones of different frequencies. The tone of lowest frequency is called the fundamental note and the tones of higher frequencies are called overtones. 2
- Q. 2. A bugle has no valves. How then can we sound different notes on it ? [U]
 Ans. We know that the frequency of a note produced by an organ pipe (bugle) depends upon the pressure with which air is forced into it. Therefore, sound of different notes can be produced by forcing air through the mouth piece at different pressures in the bugle. That is why bugle player expands his mouth unequally, while playing upon the bugle. 2
- Q. 3. Set up a relation between speed of sound in a gas and root mean square velocity of the molecules of that gas. [U]

Ans. Speed of sound in a gas is $v = \sqrt{\frac{\gamma P}{\rho}}$..(i) ½

According to kinetic theory of gases, root mean square velocity (c) of molecules of gas is obtained from the relation

$$P = \frac{1}{3} \rho c^2, c = \sqrt{\frac{3P}{\rho}} \quad \text{..(ii) } \frac{1}{2}$$

Dividing (i) by (ii),

$$\frac{v}{c} = \sqrt{\frac{\gamma}{3}}$$

or $v = c \sqrt{\frac{\gamma}{3}}$ 1

This is the required relation.

- Q. 4. A vessel is placed below a water tap. We can estimate the height of the water level reached in the vessel from a distance simply by listening to the sound. Why ? [A]
 Ans. The frequency of sound produced in an air column is inversely proportional to the length of the air column. As level of water in the vessel increases, length of air column above it decreases. Hence the frequency of sound produced goes on increasing. The sound becomes shriller. 2
- Q. 5. Can beats be observed in two light sources of nearly equal frequencies ? [A]

Ans. No, the emission of light from atom is a random and rapid phenomena. The phase at a point due to two independent light sources will change rapidly and randomly. Therefore, instead of beats, we shall get uniform intensity. 1
 However, if light sources are laser beams of nearly equal frequencies, we may be able to observe the phenomenon of beats in light. 1



Short Answer Type Questions-II

(3 marks each)

Q. 1. What is standing wave ? Give its characteristics. R

Ans. Standing or stationary wave results when two progressive waves of same wavelength and amplitude travelling with same speed in opposite directions superimpose. ½

Characteristics :

1. Energy is not transferred from particle to particle. ½
2. Amplitude of vibration of particles is maximum at antinode and minimum (say zero) at nodes. ½
3. Time period of all particles is same except at nodes. ½
4. Wavelength and frequency of standing wave is same as that of the component wave. ½
5. Pressure variation is minimum at antinode and maximum at nodes. ½

Q. 2. Explain the factors on which the pitch of a tuning fork depends. U

Ans. (i) Length of the prongs (l) : The pitch (ν) is inversely proportional to the square of the length (l) of the prongs, i.e., $\nu \propto 1/l^2$. ½

(ii) Thickness of the prongs (b) : The pitch (ν) is directly proportional to the thickness (b) of the prong, i.e., $\nu \propto b$. ½

(iii) Elasticity of the material (Y) : The pitch is directly proportional to the square root of the elasticity (Y) of the material, i.e., $\nu \propto \sqrt{Y}$. 1

(iv) Density of the material (ρ) : The pitch (ν) is inversely proportional to the square root of the density of the material, i.e., $\nu \propto \frac{1}{\sqrt{\rho}}$. 1

Q. 3. A standing wave is represented by $y = 2A \sin kx \cos \omega t$. If one of the component waves is $y_1 = A \sin (kx - \omega t)$. What is the equation of the second component wave ? A

Ans. When a standing wave $y = 2A \sin kx \cos \omega t$ is produced then if one component wave is $A \sin(kx - \omega t)$ then the other component wave is $A \sin(kx + \omega t)$. 1



Commonly Made Error

- Students can not find the equation of the second component wave.



Answering Tip

- Students should be familiarized with superposition principle.



Objective Type Questions

(1 mark each)

A Multiple Choice Questions

Q. 1. Following are the equations of transverse waves:

- (i) $Z = A \cos (kx - \omega t)$
- (ii) $Z = A \cos (kx + \omega t)$
- (iii) $Z = A \cos(ky - \omega t)$

Which may superimpose to form stationary wave?

- (A) (i) and (ii) (B) (i) and (iii)
(C) (ii) and (iii) (D) (i) and (iii)

Ans. Option (A) is correct.

Q. 2. In a stationary wave, the distance between two consecutive nodes is

- (A) $\frac{\lambda}{4}$ (B) $\frac{\lambda}{2}$
(C) λ (D) 2λ

Ans. Option (B) is correct.

Q. 3. Fundamental tone of same frequency is produced by an open organ pipe and a closed organ pipe. The ratio of their lengths is

- (A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 1 : 4

Ans. Option (C) is correct.

Explanation: Frequency of fundamental tone produced by open organ pipe = $\nu_0 = \frac{v}{2L_{\text{open}}}$

Frequency of fundamental tone produced by closed organ pipe = $\nu_0 = \frac{v}{4L_{\text{closed}}}$

$$\frac{v}{2L_{\text{open}}} = \frac{v}{4L_{\text{closed}}}$$

or,

$$L_{\text{open}} = 2L_{\text{closed}}$$

∴

$$L_{\text{open}} : L_{\text{closed}} = 2 : 1$$

Q. 4. Waves produced in a cylinder containing a liquid by moving its piston back and forth are

- (A) Transverse and longitudinal
(B) Longitudinal
(C) Transverse and longitudinal
(D) Longitudinal

Ans. Option (B) is correct.

Q. 5. Identify the incorrect statement.

- (A) Echo is an example of reflection by a rigid boundary.

- (B) Modification of Newton's formula for speed of light is referred to as the Laplace correction.
- (C) In a longitudinal wave, the constituents of the medium oscillate forward and backward in the direction of propagation of the wave.
- (D) Matter waves are associated with constituents of matter : electrons, protons, neutrons, atoms and molecules

Ans. Option (B) is correct.

Explanation: Modification of Newton's formula for speed of sound is referred to as the Laplace correction.

$$v = \sqrt{\left(\frac{P}{\rho}\right)}$$

This relation was first given by Newton and is known as Newton's formula.

$$v = \sqrt{\left(\frac{\gamma P}{\rho}\right)}$$

This modification of Newton's formula is referred to as the Laplace correction.

B Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

- (A) Both A and R are true and R is the correct explanation of A
- (B) Both A and R are true but R is NOT the correct explanation of A

- (C) A is true but R is false
- (D) A is false and R is true

Q.1. Assertion (A): Transverse wave cannot travel through gaseous medium.

Reason (R): Gas cannot sustain shearing stress.

Ans. Option (A) is correct.

Explanation: In transverse waves, the particle motion is normal to the direction of propagation of the wave. Therefore, as the wave propagates, each element of the medium undergoes a shearing strain. Transverse waves can, therefore, be propagated only in those media, which can sustain shearing stress, such as solids and not in fluids.

So, the assertion and reason both are true and the reason explains the assertion.

Q.2. Assertion (A): Beats are audible when two vibrating tuning forks of frequency 1024Hz and 2048 Hz are brought closer.

Reason (R): Beats are clearly audible if the difference of frequencies of the two waves is more than 1000Hz.

Ans. Option (D) is correct.

Explanation: Beats are not audible when two vibrating tuning forks of frequency 1024Hz and 2048 Hz are brought closer since to hear the beats clearly the difference of frequencies of the two waves should be less than 10Hz. So, assertion and reason both are false.

Q.3. Assertion (A): Light wave can travel through vacuum but sound wave cannot.

Reason (R): Light wave can be polarised but sound wave cannot be polarised.

Ans. Option (B) is correct.



NCERT Corner

Q.1. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end ?

Ans. Using $v = \sqrt{\frac{T}{m}}$

Putting, $m = \frac{2.5}{20} = 0.125 \text{ kgm}^{-1}, T = 200 \text{ N},$

$$v = \sqrt{\frac{200}{0.125}}$$

$$= 40 \text{ ms}^{-1}$$

Time, $t = \frac{l}{v} = \frac{20}{40} = 0.5 \text{ s}$

Q.2. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 ms⁻¹ ? (g = 9.8 ms⁻²)

Ans. Let total time $t = t_1 + t_2$, where t_1 is the time taken by the stone to reach the surface of water and t_2 is the time taken by sound to reach the top.

Calculation of t_1 :

Using $s = ut_1 + \frac{1}{2}at_1^2$

Putting, $u = 0, a = g = 9.8 \text{ ms}^{-2}$
and $s = 300 \text{ m},$

$$300 = \frac{1}{2} \times 9.8 \times t_1^2$$

or $t_1 = 7.8255 \text{ s}$

Calculation of t_2 :

Using $v = \frac{s}{t_2},$

$$t_2 = \frac{s}{v} = \frac{300}{340} = 0.88 \text{ s}$$

$$t = t_1 + t_2$$

$$= 7.82 + 0.88 = 8.70 \text{ s}$$

Q. 3. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343 ms⁻¹

Ans. Using $v = \sqrt{\frac{T}{m}}$
we have
 $T = v^2 m$
where, m is mass per unit length
Now $m = \frac{M}{l} = \frac{2.1}{12}$
 $= 0.175 \text{ kg m}^{-1}$
and $v = 343 \text{ ms}^{-1}$
we get $T = (343)^2 \times 0.175 = 20588.6$
 $= 2.06 \times 10^4 \text{ N}$



Commonly Made Error

► Students can not evaluate 'm' correctly in the formula of speed of transverse wave in stretched string.



Answering Tip

► m is the mass per unit length.

Q. 4. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the

speed of sound in air :

- (a) is independent of pressure.
- (b) increases with temperature.
- (c) increases with humidity.

Ans. (a) The formula for velocity of sound in air is given by.

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \dots(1)$$

where γ = constant for a given gas, i.e., air

ρ = density of the gas

From gas equation,

$$PV = RT$$

$$\text{or} \quad P = \frac{RT}{V} \quad \dots(2)$$

∴ From equation (1) and (2),

$$v = \sqrt{\frac{\gamma RT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}} \quad \dots(3)$$

where $\rho V = M$ = molecular weight of air or gas.

For a given gas, M = constant

R is also constant.

When T = constant, then from equation (3), we conclude that v is independent of the pressure of air (gas) if temperature remains constant.

(b) Effect of temperature :

$$\text{From eqn. (3)} \quad \sqrt{\frac{\gamma RT}{M}}$$

Since, γ , R and M are constants, so

$$v \propto \sqrt{T}$$

It means velocity of sound in a gas is directly proportional to the square root of its temperature, hence we conclude that the velocity of sound in air increases with increase in temperature.

(c) Effect of humidity : The presence of water vapours in air changes the density of air, thus the velocity of sound changes with humidity of air.

Let ρ_m = density of moist air
 ρ_d = density of dry air
 v_m = velocity of sound in moist air
 v_d = velocity of sound in dry air

$$\text{From eqn. (1)} \quad v = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{In moist air,} \quad v_m = \sqrt{\frac{\gamma P}{\rho_m}} \quad \dots(4)$$

$$\text{In dry air,} \quad v_d = \sqrt{\frac{\gamma P}{\rho_d}} \quad \dots(5)$$

Dividing equation (4) by equation (5),

$$\frac{v_m}{v_d} = \sqrt{\frac{\rho_d}{\rho_m}} \quad \dots(6)$$

Also we know that density of moist air is less than the density of dry air. It means dry air is heavier than moist air. so

$$\rho_m < \rho_d$$

$$\text{or} \quad \frac{\rho_d}{\rho_m} > 1 \quad \dots(iv)$$

∴ from equation (6)

$$\frac{v_m}{v_d} > 1$$

$$\therefore v_m > v_d$$

It means speed of sound increases with humidity.

Q. 5. You have learnt, that a travelling wave in one dimension is represented by a function $y = f(x, t)$, where x and t must appear in the combination $(x - vt)$ or $(x + vt)$, i.e., $y = f(x \pm vt)$. Is the converse true? That is, does every function of $(x - vt)$ or $(x + vt)$ represent a travelling wave? Examine, if the following function for y can possibly represent a travelling wave :

(a) $(x - vt)^2$ (b) $\log \left[\frac{x + vt}{x_0} \right]$

(c) $\frac{1}{x + vt}$

Ans. No, the converse is not true. The basic requirement for a wave function to represent a travelling wave is that for all values of x and t , wave function must have a finite value.

Out of the given functions for y only (c) satisfies this condition. The other functions cannot represent a travelling wave as the necessary condition is not satisfied by these functions.



Commonly Made Error

- Students cannot answer this question properly due to lack of knowledge about the basic requirement for a wave function to represent a travelling wave.



Answering Tip

- A wave function must have finite values for all values of x and t to represent a travelling wave.

Q. 6. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is 340 ms^{-1} and in water 1486 ms^{-1} .

Ans. (a) The reflected sound :

Using $v = \frac{\lambda}{T} = \lambda \nu$

$$\lambda = \frac{v}{\nu} = \frac{340}{10^6} = 3.4 \times 10^{-4} \text{ m}$$

(b) The transmitted sound :

Here, $\lambda' = \frac{1486}{10^6} = 1.486 \times 10^{-3} \text{ m}$



Commonly Made Error

- Students cannot answer this question correctly due to confusion in finding wavelength of sound in air and water.



Answering Tip

- Students must keep in mind that the frequency of wave is independent of medium.

Q. 7. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wave length of sound in the tissue in which the speed of sound is 1.7 kms^{-1} ? The operating frequency of the scanner is 4.2 MHz.

Ans. Using $\lambda = \frac{v}{\nu}$,

or $\lambda = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 0.405 \times 10^{-3} \text{ m}$.

Q. 8. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$

where x and y are in cm and t in sec. The positive direction of x is from left to right.

- Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?
- What are its amplitude and frequency?
- What is the initial phase at the origin?
- What is the least distance between two successive crests in the wave?

Ans. Here $y(x, t) = 3 \sin(36t + 0.018x + \pi/4)$
 $= 3 \sin[0.018(2000t + x) + \pi/4]$

Comparing with standard equation

$$y(x, t) = r \sin\left[\frac{2\pi}{\lambda}(vt + x) + \phi\right]$$

It is observed that the given equation represents a travelling waveform right to left.

(a) Here, $v = 2000 \text{ cms}^{-1}$

(b) $r = 3 \text{ cm}$

Also, $\frac{2\pi}{\lambda} = 0.018$

or $\lambda = \frac{2\pi}{0.018} \text{ cm}$

Using $v = \lambda \nu$,
 $\nu = \frac{v}{\lambda} = \frac{2000}{2\pi} \times 0.018 = 5.73 \text{ s}^{-1}$

(c) Initial phase at origin, $\phi = \frac{\pi}{4}$ radian

(d) Distance between successive crests
 $= \lambda = \frac{2\pi}{0.018} = 349 \text{ cm} = 3.49 \text{ m}$

Q. 9. For the wave described in the above question plot the displacement (y) versus (t) graphs for $x = 0, 2$ and 4 cm . What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling waves differ from one point to another : amplitude, frequency or phase?

Ans. $y(x, t) = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right)$

When $x = 0$,

$$y(0, t) = 3.0 \sin\left(36t + \frac{\pi}{4}\right)$$

Initial displacement $= y(0, 0) = 3.0 \sin \frac{\pi}{4} = \frac{3}{\sqrt{2}} = 2.121 \text{ cm}$

When $x = 2$,

$$y(2, t) = 3.0 \sin\left(36t + 0.036 + \frac{\pi}{4}\right)$$

Initial displacement $= y(2, 0) = 3.0 \sin\left(0.036 + \frac{\pi}{4}\right) = 2.122 \text{ cm}$

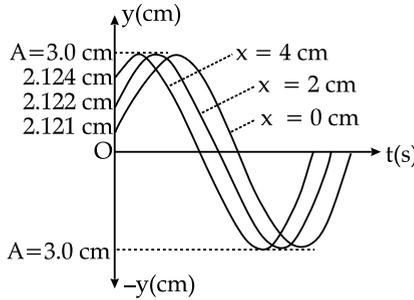
When $x = 4$,

$$y(2, t) = 3.0 \sin \left(36t + 0.072 + \frac{\pi}{4} \right)$$

$$\begin{aligned} \text{Initial displacement} &= y(4, 0) = 3.0 \sin \left(0.072 + \frac{\pi}{4} \right) \\ &= 2.124 \text{ cm} \end{aligned}$$

All the three graphs are sine graphs. All are having same amplitude (3 cm) and same frequency

$$\left(\frac{2\pi}{36} = \frac{\pi \text{ rad}}{18 \text{ s}} \right). \text{ Graphs differ in initial phase.}$$



Q. 10. For the travelling harmonic wave

$$y(x, t) = 2.0 \cos 2\pi [10t - 0.0080x + 0.35]$$

where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of (a) 4 m, (b) 0.5 m, (c) $\lambda/2$ (d) $3\lambda/4$. [KVS 2013]

Ans. Here

$$\begin{aligned} y &= 2 \cos 2\pi(10t - 0.0080x + 0.35) \\ &= 2 \cos [2\pi(10t - 0.0080x) + 2\pi(0.35)] \\ &= 2 \cos \left[2\pi \times 0.0080 \left(\frac{10}{0.0080}t - x \right) + 2\pi \times 0.35 \right] \end{aligned}$$

Standard equation for a travelling wave is

$$y = r \cos \left[\frac{2\pi}{\lambda}(vt - x) + \phi \right]$$

$$\text{Here } \phi = \frac{2\pi}{\lambda}x = 2\pi \times 0.008x$$

$$\left[\therefore \frac{1}{\lambda} = 0.008 \text{ cm} \right]$$

(a) when $x = 4 \text{ m} = 400 \text{ cm}$,
 $\phi = 2\pi \times 0.008 \times 400 = 6.4\pi \text{ rad}$

(b) when $x = 0.5 \text{ m} = 50 \text{ cm}$,
 $\phi = 2\pi \times 0.008 \times 50 = 0.8\pi \text{ rad}$

(c) when $x = \frac{\lambda}{2}$, $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ rad}$

(d) when $x = \frac{3}{4}\lambda$, $\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2} \text{ rad}$

Q. 11. The transverse displacement of a string (clamped at its both ends) is given by

$$y(x, t) = 0.06 \sin \left(\frac{2\pi}{3}x \right) \cos(120\pi t)$$

Where x and y are in m and t in s. The length of the string is 1.5 m and its mass is $3.0 \times 10^{-2} \text{ kg}$.

Answer the following:

(a) Does the function represent a travelling wave or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

Ans. (a) Given: $y(x, t) = 0.06 \sin \left(\frac{2\pi}{3}x \right) \cos(120\pi t)$

It is same as the equation for stationary wave-
 $y(x, t) = 2a \sin kx \cos \omega t$

Therefore, this represents stationary wave.

(b) Standing wave produced by two waves-
 $y(x, t) = 2a \sin kx \cos \omega t$

Due to superposition of these waves-

$$y_1(x, t) = a \sin(\omega t - kx)$$

$$y_2(x, t) = a \sin(\omega t + kx)$$

Equation given-

$$y(x, t) = 0.06 \sin \left(\frac{2\pi x}{3} \right) \cos(120\pi t)$$

So, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{3} \therefore \lambda = 3 \text{ m}$

$$\omega = 120\pi \text{ rad/s}$$

$$\omega = 2\pi\nu \text{ or } \nu = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi}$$

$$\therefore \nu = 60 \text{ Hz}$$

(c) Speed of wave, $v = \nu \lambda$

$$= 60 \times 3$$

$$= 180 \text{ m/s}$$

$$2a = 0.06 \text{ or } a = 0.03 \text{ m}$$

$$\therefore y_1 = 0.03 \sin \left(120\pi t - \frac{2\pi}{3}x \right)$$

$$y_2 = 0.03 \sin \left(120\pi t + \frac{2\pi}{3}x \right)$$

Wave's speed = 180 m/s

Mass per unit length

$$\mu = \frac{m}{l}$$

$$= \frac{3.0}{1.5} \times 10^{-2} \text{ Kg m}^{-1}$$

$$= 2 \times 10^{-2} \text{ Kg m}^{-1}$$

Therefore, $T = v^2\mu$

$$T = (180)^2 \times 2 \times 10^{-2}$$

$$\text{Tension, } T = 648 \text{ N}$$

Q. 12. For the wave on a string described in above question do all the points on the string oscillate with the same (i) (a) frequency, (b) phase, (c) amplitude? Explain your answers. (ii) What is the amplitude of a point 0.375 m away from one end?

Ans (i) Transverse Displacement is-

$$y(x, t) = 0.06 \sin \frac{2\pi}{3}x \cos 120\pi t$$

(a) Yes, $\cos 120\pi t$ represents its frequency, as this function does not depend on x , so oscillation's frequency of all points on the string is same.

- (b) Yes, phase of all points on string is same, except at nodes.
 (c) As amplitude depends on x , it will not be same at all points on the string.
 (ii) Now, amplitude at a point-0.375 m away from one end-

$$a = 0.06 \sin \frac{2\pi}{3} \times 0.375$$

$$= 0.06 \sin 0.7854$$

$$= 0.06 \times 0.707$$

$$a = 0.042 \text{ m}$$

Q. 13. Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent

- (i) a travelling wave,
 (ii) a stationary wave, or
 (iii) none of all ?

- (a) $y = 2\cos(3x) \sin(10t)$
 (b) $y = 2\sqrt{x-vt}$
 (c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$
 (d) $y = \cos x \sin t + \cos 2x \sin 2t$

Ans. (a) It represents a stationary wave in harmonic functions of x and t are contained separately in the equation.

- (b) It cannot represent any type of wave.
 (c) It represents a progressive travelling harmonic wave.
 (d) This equation is sum of two functions each representing a stationary wave. Therefore, it represents superposition of two stationary waves.

Q. 14. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear density is 4.0×10^{-2} kg m^{-1} . What is

- (a) the speed of a transverse wave on the string, and
 (b) the tension in the string ?

Ans. Using $l = \frac{M}{m}$,

$$= \frac{3.5 \times 10^{-2}}{4 \times 10^{-2}}$$

$$= 0.875 \text{ m}$$

(a) In fundamental mode,

$$l = \frac{\lambda}{2}$$

or $\lambda = 2l = 2 \times 0.875 = 1.75 \text{ m}$
 Velocity = $f\lambda = 45 \times 1.75$
 $= 78.75 \text{ ms}^{-1}$

(b) $v = \sqrt{T/m}$
 or, $T = v^2 m$
 $= (78.75)^2 \times 4 \times 10^{-2}$
 $\approx 248 \text{ N}$.

Q. 15. A 1 metre long tube open at one end, with a movable piston at other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Ans. For the fundamental tone in a closed organ pipe,

$$\frac{\lambda_0}{4} = L = 25.5 \text{ cm}$$

$$\therefore \lambda_0 = 102 \text{ cm} = 1.02 \text{ m}$$

$$\therefore \text{Frequency} = v = 340 \text{ Hz}$$

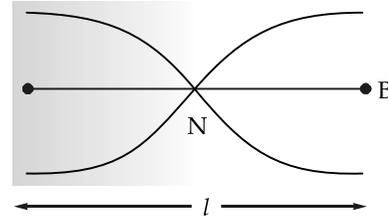
$$\therefore \text{Velocity of sound} = v\lambda_0$$

$$= 340 \times 1.02$$

$$= 346.8 \text{ ms}^{-1}$$

Q. 16. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod are given to be 2.53 kHz. What is the speed of sound in steel ?

Ans. When a rod clamped in the middle it has anti-nodes (A) at its ends and node (N) at the point of clamping. In fundamental mode, so the length of the rod is



$$l = \frac{\lambda}{2} \text{ or } \lambda = 2l,$$

where l = length of rod
 and λ = wavelength of the wave

Given : $l = 100 \text{ cm}$, $v = 2.53 \text{ kHz} = 2.53 \times 10^3 \text{ Hz}$

$$\therefore \lambda = 2 \times 100 = 200 \text{ cm}$$

When v be the speed of sound in steel, then

$$v = v\lambda = 2.53 \times 10^3 \times 200$$

$$= 506 \times 10^3 \text{ cms}^{-1}$$

$$v = 5.06 \times 10^3 \text{ ms}^{-1}$$



Commonly Made Error

- Students can not relate the length of rod to the wavelength of the wave produced.



Answering Tip

- Students should have an understanding about the formation of nodes and antinodes.

Q. 17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 ms^{-1}).

Ans Given: Pipe's length, $l = 20 \text{ cm}$
 $= 0.2 \text{ m}$

frequency, $v = 430 \text{ Hz}$

speed of sound = 340 m/s

For closed pipe, fundamental frequency,

$$v_1 = \frac{v}{4l}$$

$$= \frac{340}{4 \times 0.2}$$

$$= 425 \text{ Hz}$$

Hence, fundamental frequency will resonate with the source.

For open pipe, fundamental frequency,

$$v_2 = \frac{v}{2l}$$

$$= \frac{340}{2 \times 0.2}$$

$$= 850 \text{ Hz}$$

Then, there will be no resonance.



Commonly Made Error

- ▶ Students can not judge the condition for resonance in both the cases.



Answering Tip

- ▶ Resonance is marked by matching the frequency of the source to the natural frequency of vibration of sound in pipe.

Q. 18. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

Ans. Let v_A, v_B be frequency of A and B respectively

$$v_A - v_B = \pm 6$$

or

$$v_B = v_A \pm 6 = 324 \pm 6$$

$$v_B = 330 \text{ or } 318 \text{ Hz}$$

When tension is reduced, frequency also decreases-

$$v \propto \sqrt{T}$$

∴ No. of Beats also decreases
Hence, $v_B = 324 - 6 = 318 \text{ Hz}$

Q. 19. Explain why (or how):

- (a) in a sound wave, a displacement node is a pressure antinode and vice versa.
- (b) bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes".
- (c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes.
- (d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and
- (e) the shape of a pulse gets distorted during propagation in a dispersive medium.

- Ans.**
- (a) Displacement node is a point of zero displacement, so pressure is maximum at this point where as on anti-nodes the reverse happens, hence it is a pressure antinode.
 - (b) Bats produce and detect ultrasonic waves and emit sound of frequencies between 20-100 KHz for this purpose, hence they developed 'Echo-location' system.
 - (c) Because nature of waveforms and presence of overtones are different in the given two cases.
 - (d) Gases cannot sustain shearing stress as in case of transverse wave propagation, the medium must have property of sustaining shearing. So gases cannot support transverse wave propagation.
 - (e) This is the property of dispersive medium that waves of different wave length travel with different speeds in different directions or with different velocities. Thus, when sound pulse (a combination of waves of different wavelengths) travels through it, gets distorted.



NCERT Exemplar (Only Important Questions)

Multiple Choice Questions

Q. 1. Water waves produced by a motor boat sailing in water are

- (A) neither longitudinal nor transverse.
- (B) both longitudinal and transverse.
- (C) only longitudinal.
- (D) only transverse.

Ans. Option (B) is correct.

Q. 2. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s. Wavelength of sound waves in the second medium is

- (A) λ
- (B) $\frac{\lambda}{2}$
- (C) 2λ
- (D) 4λ

Ans. Option (C) is correct.

Explanation: Wavelength in I medium,

$$\lambda = \frac{v}{\nu} \quad (i)$$

Wavelength in II medium,

$$\lambda' = \frac{2v}{\nu} \quad (ii)$$

∴ $\nu = \nu'$ {∴ frequency remains unchanged}

(ii) ÷ (i), we get

$$\frac{\lambda'}{\lambda} = 2$$

∴ $\lambda' = 2\lambda$

Q. 3. Speed of sound wave in air

- (A) is independent of temperature.
- (B) increases with pressure.
- (C) increases with increase in humidity.
- (D) decreases with increase in humidity.

Ans. Option (C) is correct.

Q. 4. Change in temperature of the medium changes

- (A) frequency of sound waves.
- (B) amplitude of sound waves.
- (C) wavelength of sound waves.
- (D) loudness of sound waves.

Ans. Option (C) is correct.

Q. 5. With propagation of longitudinal waves through a medium, the quantity transmitted is

- (A) matter.
- (B) energy.
- (C) energy and matter.
- (D) energy, matter and momentum.

Ans. Option (B) is correct.

Q. 6. Equation of a plane progressive wave is given by $y = 0.6 \sin 2\pi \left(t - \frac{x}{2} \right)$. On reflection from a

denser medium its amplitude becomes $\frac{2}{3}$ of the amplitude of the incident wave. The equation of the reflected wave is

- (A) $y = 0.6 \sin 2\pi \left(t + \frac{x}{2} \right)$
- (B) $y = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$
- (C) $y = 0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$
- (D) $y = -0.4 \sin 2\pi \left(t - \frac{x}{2} \right)$.

[NCERT Exemplar Q. No. 8]

Ans. Option (B) is correct.

Explanation: Amplitude of reflected wave

$$= \frac{2}{3} \times 0.6$$

$$= 0.4$$

There will be phase change of π because of reflection from denser to rarer medium

$$y = 0.4 \sin 2\pi \left(t + \frac{x}{2} + \pi \right) = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$$



Commonly Made Error

- Students can not find the equation for the reflected wave.



Answering Tip

- Students should have an understanding that when the wave is reflected at the denser medium, there is a phase change of 180° .

Q. 7. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in
[NCERT Exemplar Q. No. 9]

- (A) one second
- (B) 0.5 second
- (C) 2 seconds
- (D) data given is insufficient.

Ans. Option (B) is correct.

Explanation:

$$\mu = \frac{M}{L} = \frac{2.5}{20 \text{ m}} = 0.125 \text{ kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200 \text{ N}}{0.125 \text{ kg/m}}}$$

$$= 40 \text{ m/s}$$

$$t = \frac{L}{v} = \frac{20 \text{ m}}{40 \text{ m/s}} = 0.5 \text{ s}$$

Very Short Answer Type Questions

Q. 8. A sonometer wire is vibrating in resonance with a tuning fork. Keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be in resonance with the wire?

[NCERT Exemplar Q. No. 18]

Ans. When a wire of length L vibrates its resonant frequency in n th mode after stretching it by

$$\text{tension } T, \text{ then frequency, } v = \frac{n}{2L} \sqrt{\frac{T}{m}}$$

[Here, m = mass per unit length of stretched wire]

Let us consider two given cases:

$$v_1 = \frac{n_1}{2L_1} \sqrt{\frac{T_1}{m_1}} \quad \& \quad v_2 = \frac{n_2}{2L_2} \sqrt{\frac{T_2}{m_2}}$$

from question,

$$T_1 = T_2 = T, \quad m_1 = m_2 = m \text{ (as wires are same)}$$

$$L_2 = 2L_1$$

$$\frac{v_1}{v_2} = \left(\frac{n_1}{2L_1} \sqrt{\frac{T}{m}} \right) \div \left(\frac{n_2}{2(2L_1)} \sqrt{\frac{T}{m}} \right) = \frac{2n_1}{n_2}$$

As tuning fork is same, $\therefore v_1 = v_2$

$$\frac{2n_1}{n_2} = 1 \quad \text{or} \quad n_2 = 2n_1$$

Hence, when length of wire double the number of harmonics double for same resonant frequency.

Q. 9. A tuning fork A, marked 512 Hz, produces 5 beats per second, when sounded with another unmarked tuning fork B. If B is loaded with wax the number of beats is again 5 per second. What is the frequency of the tuning fork B when not loaded?
[NCERT Exemplar Q. No. 20]

Ans. Frequency of tuning fork A,

$$v_A = 512 \text{ Hz}$$

Now, frequency of tuning fork B may be-

$$v_B = v_A \pm 5$$

$$= 512 \pm 5$$

$$= 517 \text{ or } 507 \text{ Hz}$$

As on loading B, frequency decreases

If $v_B = 517 \text{ Hz}$, it might be reduced to 507 Hz, to give again a beat of 5 Hz.

If $v_B = 507 \text{ Hz}$, reduction will increase the frequency of beat. So, $v_B = 517 \text{ Hz}$.

Q. 10. The displacement of an elastic wave is given by the function

$$y = 3 \sin \omega t + 4 \cos \omega t.$$

Where y is in cm and t is in second. Calculate the resultant amplitude.

[NCERT Exemplar Q. No. 21]

Ans. $y = 3 \sin \omega t + 4 \cos \omega t$ (i)

Let, $3 = a \cos \phi$ (ii)

$4 = a \sin \phi$ (iii)

$\therefore y = a \cos \phi \sin \omega t + a \sin \phi \cos \omega t$ (iv)

$y = a \sin(\omega t + \phi)$

from eq. (s) (i) & (iii):

$\tan \phi = \frac{4}{3}$ or $\phi = \tan^{-1} \frac{4}{3}$

Squaring (ii) & (iii), then adding-

$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$

$a^2 (\cos^2 \phi + \sin^2 \phi) = 9 + 16$

$a^2 = 25$

or $a = 5$ i.e., amplitude, $a = 5$ cm

$\therefore y_1 = 5 \sin(\omega t + \phi), \therefore \phi = \tan^{-1} \frac{4}{3}$

So, the resultant amplitude, $a = 5$ cm.

Q. 11. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, by what factor will the frequency change?

[NCERT Exemplar Q. No. 22]

Ans. Wire is stretched, so frequency of stretched wire is,

$$v = \frac{n}{2L} \sqrt{\frac{T}{m}}$$

Mass per unit length,

$$m = \frac{\text{Mass}}{\text{Length}} = \frac{\pi r^2 l \rho}{l}$$

or $m = \pi r^2 \rho$

$$v = \frac{n}{2l} \sqrt{\frac{T}{\pi r^2 \rho}}$$

$$v \propto \sqrt{\frac{1}{r^2}} \propto \frac{1}{r}$$

$$v \propto \frac{1}{r} \propto n$$

So, frequency of sitar is reduced by $\frac{1}{3}$ of its

previous value.

Q. 12. At what temperatures (in °C) will the speed of sound in air be 3 times its value at 0° C?

[NCERT Exemplar Q. No. 23]

Ans. $v \propto \sqrt{T}$

or $\frac{v_T}{v_0} = \sqrt{\frac{T}{T_0}}$

From given condition, $v_T = 3v_0$

$\therefore \frac{3v_0}{v_0} = \sqrt{\frac{T}{273 + 0}}$

or $\sqrt{T} = 3\sqrt{273}$

$T = 9 \times 273 = 2457 \text{ K}$

$T = (2457 - 273)^\circ\text{C}$

$T = 2184^\circ\text{C}$



Commonly Made Error

► Students can not evaluate the speed of sound at a particular temperature.



Answering Tip

► Speed of sound varies as the square root of absolute temperature.

Q. 13. When two waves of almost equal frequencies n_1 and n_2 reach at a point simultaneously, what is the time interval between successive maxima?

[NCERT Exemplar Q. No. 24]

Ans. Assume, $n_1 > n_2$

Frequency of Beat, $v_b = n_1 - n_2$

\therefore Time Period, $T_b = \frac{1}{v_b}$

$$= \frac{1}{n_1 - n_2} \text{ s}$$

Short Answer Type Questions

Q. 14. A steel wire has a length of 12 m and a mass of 2.10 kg. What will be the speed of a transverse wave on this wire when a tension of 2.06×10^4 N is applied? [NCERT Exemplar Q. No. 25]

Ans. Given: $l = 12$ m, mass = 2.10 kg

Tension, $T = 2.06 \times 10^4$ N,

$$m = \frac{M}{l} = \frac{2.10}{12}$$

$\therefore v = \sqrt{\frac{T}{m}} = \sqrt{\frac{2.06 \times 10^4 \times 12}{2.10}}$

$$= \sqrt{11.77 \times 10^4}$$

$$v = 3.43 \times 10^2 \text{ m/s}$$

$$v = 343 \text{ m/s}$$

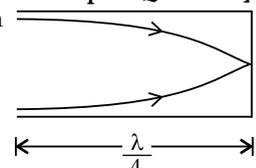
Q. 15. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a source of 1237.5 Hz? (sound velocity in air = 330 m s^{-1}) [NCERT Exemplar Q. No. 26]

Ans. Given: $l = 20$ cm = 0.2 m

$v = 1237.5 \text{ Hz}$

$v = 330 \text{ m/s}$

$l = \frac{\lambda}{4}$ or $\lambda = 4l$



$$v_{\text{fundamental}} = \frac{v}{4l}$$

$$v_{\text{fundamental}} = \frac{330}{4 \times 0.2} = 412.5 \text{ Hz}$$

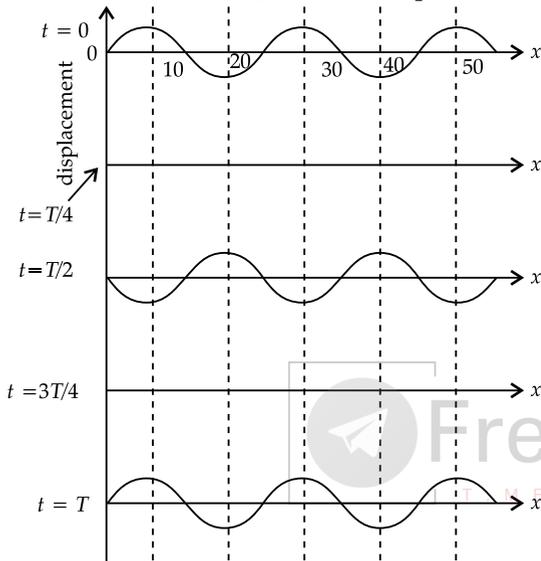
$$v_{\text{given}} = 1237.5 \text{ Hz}$$

$$\frac{v_{\text{given}}}{v_{\text{fundamental}}} = \frac{1237.5}{412.5} = \frac{3}{1}$$

So, 3rd harmonic node of pipe is excited by 1237.5 Hz frequency.

Q. 16. The wave pattern on a stretched string is shown in Fig. Interpret what kind of wave this is and find its wavelength.

[NCERT Exemplar Q. No. 28]



Ans. Nature of two waves is decided by observing the displacement and position of different points. The graph shows stationary wave. Points on positions $x = 10, 20, 30, 40$, are nodes.

The wave is stationary wave.

$$\text{Distance between successive nodes} = \frac{\lambda}{2}$$

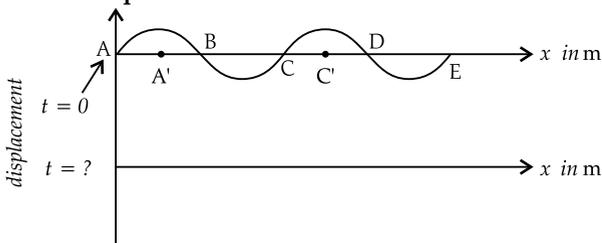
$$\lambda = 2 \times (\text{node-to-node distance})$$

$$= 2 \times (20 - 10)$$

$$= 2 \times 10$$

$$\lambda = 20 \text{ cm}$$

Q. 17. The pattern of standing waves formed on a stretched string at two instants of time are shown in Fig. The velocity of two waves superimposing to form stationary waves is 360 ms^{-1} and their frequencies are 256 Hz .



- (a) Calculate the time at which the second curve is plotted.
- (b) Mark nodes and anti-nodes on the curve.
- (c) Calculate the distance between A' and C'.

[NCERT Exemplar Q. No. 29]

Ans. Given: frequency of wave, $v = 256 \text{ Hz}$

$$\text{Time period, } T = \frac{1}{v} = \frac{1}{256}$$

$$= 3.9 \times 10^{-3} \text{ s}$$

- (a) Time taken to pass through mean position

$$t = \frac{T}{4}$$

$$= \frac{3.9 \times 10^{-3}}{4} \text{ s}$$

$$= 9.8 \times 10^{-4} \text{ s}$$

- (b) Nodes- A, B, C, D, E (zero displacement)
Anti-nodes- A', C' (maximum displacement)
- (c) At A', C', there are consecutive anti-nodes, so distance between A' and C',

$$\lambda = \frac{v}{f} = \frac{360}{256}$$

$$\lambda = 1.41 \text{ m}$$

Q. 18. Show that when a string fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1:2:3:4.

[NCERT Exemplar Q. No. 31]

Ans. Length for each loop = $\frac{\lambda}{2}$

Now,

$$L = \frac{n\lambda}{2}$$

$$\lambda = \frac{2L}{n} \tag{1}$$

$$\text{But } v = v\lambda \text{ or } \lambda = \frac{v}{v}$$

Putting in eqn. (1)

$$\frac{v}{v} = \frac{2L}{n}$$

$$v = \frac{n}{2L} v$$

$$v = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad \left[\because v = \sqrt{\frac{T}{\mu}} \right]$$

$$\therefore v \propto n$$

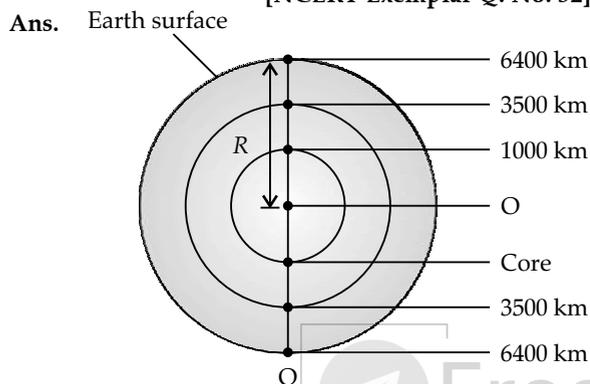
Therefore, $v_1 : v_2 : v_3 : v_4 = n_1 : n_2 : n_3 : n_4$

$$v_1 : v_2 : v_3 : v_4 = 1 : 2 : 3 : 4$$

Long Answer Type Questions

- Q. 19. The earth has a radius of 6400 km. The inner core of 1000 km radius is solid. Outside it, there is a region from 1000 km to a radius of 3500 km which is in molten state. Then again from 3500 km to 6400 km the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that the P wave has a speed of 8 km s^{-1} in solid parts and of 5 km s^{-1} in liquid parts of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter?

[NCERT Exemplar Q. No. 32]



$$\text{Time required} = \frac{\text{Distance}}{\text{speed}}$$

$$\text{Time} = \left[\frac{1000 - 0}{8} + \frac{3500 - 1000}{5} + \frac{6400 - 3500}{8} \right] \times 2$$

$$\text{Time} = \left[\frac{1000}{8} + \frac{2500}{5} + \frac{2900}{8} \right] \times 2$$

$$\text{Time required} = 1975 \text{ sec}$$

$$\text{Time required} = 32 \text{ min } 55 \text{ sec.}$$



Commonly Made Error

- Students fail to consider the factor of '2' in calculating time period.



Answering Tip

- Students should keep in mind that the earthquake has occurred close to the surface of the earth. So, they have to calculate the time required by the wave to travel a distance equal to the diameter.

- Q. 20. Given below are some functions of x and t to represent the displacement of an elastic wave.

(a) $y = 5 \cos(4x) \sin(20t)$

(b) $y = 4 \sin(5x - t/2) + 3 \cos(5x - t/2)$

(c) $y = 10 \cos[(252 - 250)\pi t] \cos[(252 + 250)\pi t]$

(d) $y = 100 \cos(100\pi t + 0.5x)$

State which of these represent

(a) a travelling wave along $-x$ direction

(b) a stationary wave

(c) beats

(d) a travelling wave along $+x$ direction.

Given reasons for your answers.

[NCERT Exemplar Q. No. 34]

Ans. (a) Equation $y = 100 \cos(100\pi t + 0.5x)$ represents a travelling wave along $-x$ -direction.

(b) Equation $y = 5 \cos(4x) \sin(20t)$ represents stationary wave, as this is combination of progressive waves (contains sin, cos, terms)

(c) Equation

$$y = 10 \cos[(252 - 250)\pi t] \cos[(252 + 250)\pi t]$$

represents beats formation because this equation involve sum & difference of two nearby frequencies i.e., 252, 250.

(d) Equation $y = 4 \sin\left(5x - \frac{t}{2}\right) + 3 \cos\left(5x - \frac{t}{2}\right)$

represents a travelling wave along $+x$ direction because it involves negative sign with t .



COMPETENCY BASED QUESTIONS

These questions have been specially developed as per the latest typologies prescribed by CBSE in accordance with NEP 2020

A Objective Questions

Read the following text and answer any 4 of the following questions on the basis of the same:

I. Longitudinal and transverse wave:

Longitudinal Waves are defined as waves those are capable of displacing the medium in a direction either in the direction of the waves or opposite. Longitudinal mechanical waves are known as compressional waves. This is because these

mechanical waves produce a lot of compression and rarefaction while travelling through medium. These waves are also called as pressure waves as then is an increase and decrease in pressure while travelling.

Sound Waves like vibrations, P-Waves created through earthquakes, etc., are some kinds of longitudinal waves.

A transverse wave is defined as the wave that moves in the perpendicular direction of the vibration. One of the most important examples of

transverse waves includes the waves created by the drum's beating. The membrane of the drum moves perpendicular to the surface. Another example of a transverse wave is light. Transverse waves travels through crests and troughs.

Transverse waves are mostly present in solids those have profound elasticity. In some cases, when there is a deformation in the material, the wave is called a shear wave.

Q. 1. Which wave is also known as shear wave?

- (A) Longitudinal wave (B) Transverse wave
(C) Both (A) and (B) (D) None of these

Ans. Option (B) is correct.

Explanation: Transverse waves are mostly present in solids those have profound elasticity. In some cases, when there is a deformation in the material, the wave is called a shear wave.

Q. 2. Which wave is also known as compressional wave?

- (A) Longitudinal wave
(B) Transverse wave
(C) Both (A) and (B)
(D) None of the above

Ans. Option (A) is correct.

Explanation: Longitudinal mechanical waves are known as compressional waves. This is because these mechanical waves produce a lot of compression and rarefaction while travelling through medium.

Q. 3. Which wave is also known as pressure wave?

- (A) Longitudinal wave (B) Transverse wave
(C) Both (A) and (B) (D) None of these

Ans. Option (A) is correct.

Explanation: These waves are also called as pressure waves as there is an increase and decrease in pressure while traveling,

Q. 4. Beating of drums produces

- (A) Longitudinal wave (B) Transverse wave
(C) Both (A) and (B) (D) None of these

Ans. Option (B) is correct.

Explanation: One of the most important examples of transverse waves includes the waves created by the drum's beating. The membrane of the drum moves perpendicular to the surface.

Q. 5. Which wave produces compressions and rarefactions in the medium?

- (A) Longitudinal wave (B) Transverse wave
(C) Both (A) and (B) (D) None of these

Ans. Option (A) is correct.

Explanation: Longitudinal mechanical waves produce a lot of compression and rarefaction while travelling through medium. Hence, they are also known as compressional waves.

B Subjective Questions

Tuning a piano wire:

When we hear two frequencies which are very close to each other but not exactly equal then distinct waxing and waning of the intensity of the sound is heard. These are known as beats. The beat frequency is equal to the absolute value of the difference in frequency of the two waves. To tune a piano, a musician takes help of beat frequency. He plucks the string and tap a tuning fork at the same time. If the two sound sources— the piano string and the tuning fork, do not produce identical frequency then the musician hears detectable beats.

Musician will then adjust the tension of the piano string and repeat the process until the beats are no longer be heard. Tension of the wire changes the frequency of the wire.

$$f \propto \sqrt{T}$$

As the piano string becomes more in tune with the tuning fork, the beat frequency reduces and approaches 0 Hz.

When beats are no longer heard, the piano string is tuned to the tuning fork; i.e., they play the same frequency. The process allows the musician to match the strings' frequencies to the frequencies of a standardized set of tuning forks.

Q. 1. A musician while tuning a piano with a standard tuning fork of frequency 256 Hz, hears beats of 5 Hz. What may be the frequency of the piano?

Ans. The beat frequency is equal to the absolute value of the difference in frequency of the two waves. So, piano wire frequency is $(256 + 5) \text{ Hz} = 261 \text{ Hz}$ or $(256 - 5) \text{ Hz} = 251 \text{ Hz}$.

Q. 2. A musician while tuning a piano with a standard tuning fork of frequency 256 Hz, hears beats of 5 Hz. Which parameter of the wire he should change to achieve the tuning?

Ans. Since, tension of the wire changes the frequency of the wire, musician should adjust the tension of the piano string to change the frequency and repeat the process until the beats are no longer be heard.

Q. 3. What type of vibration is produced when the string of a musical instrument is plucked?

Ans. If the string of a musical instrument is plucked perpendicular to the length of the string, transverse waves are produced.

Q. 4. Write the expressions for speeds of transverse wave and longitudinal wave in a stretched string.

Ans. Speed of transverse wave = $\sqrt{\frac{T}{m}}$

Speed of longitudinal wave = $\sqrt{\frac{Y}{\rho}}$

Q. 5. Square of the frequency of transverse vibration is plotted against the tension of a stretched string. What will be the nature of the graph?

Ans. Square of the frequency of transverse vibration is directly proportional to the time period. Hence the graph will be a straight line passing through the origin.